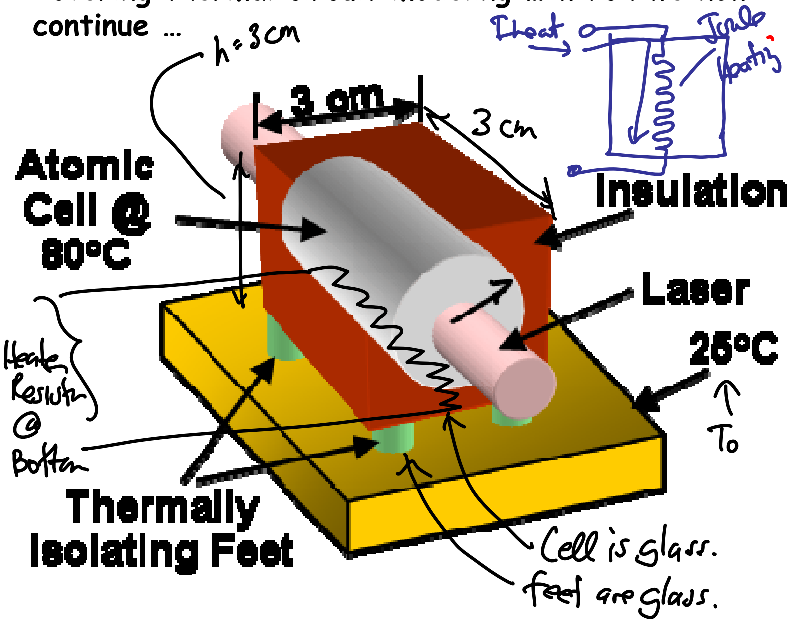
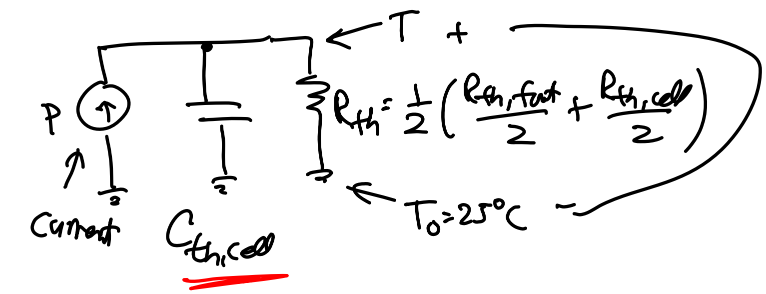
Lecture 4: Benefits of Scaling III

- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
- ----------------------------------
- Last Time:
- Covering thermal circuit modeling … which we now continue …

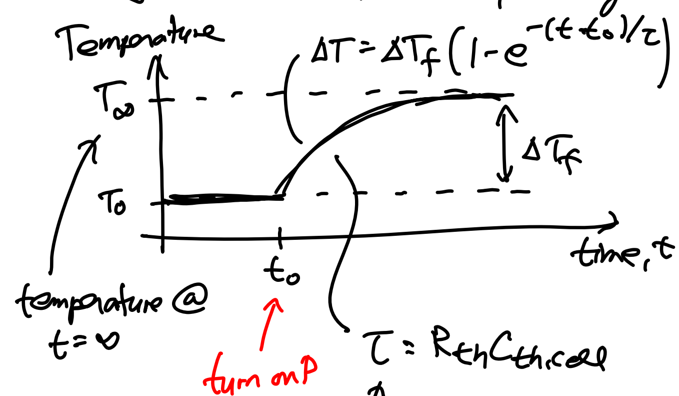


Glass: $\rho_{glass} = 2500\,\mathrm{kg/m^3}$ $\quad K_{glass} = 1.05\,\frac{W}{m\cdot K}$

$C_{p,glass} = 0.5\,J/(g\cdot K)$



$\Delta T = T - T_0 = P R_{th}$ in steady-steady

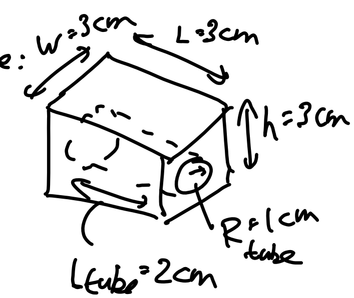


$\tau = R_{th} C_{th,cell}$

time constant determines how fast T_∞ can be achieved

Find $C_{th,cell}$:

⇒ first find the cell volume:



$V_{cell} = hWL - \pi R_{tube}^2 \cdot L_{tube}$

$= 20.7\,cm^3$

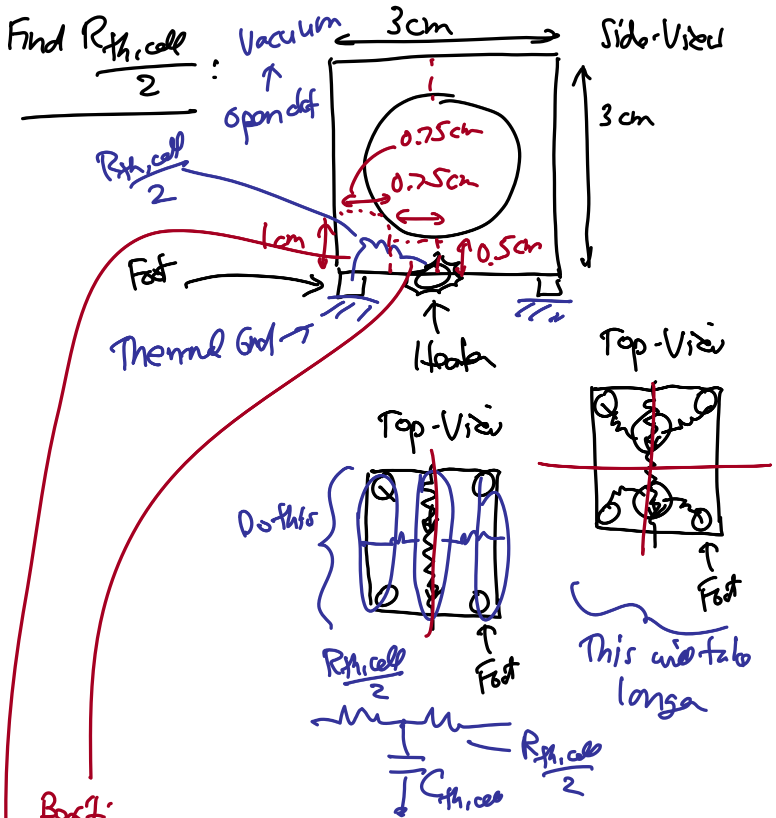
specific heat

$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$

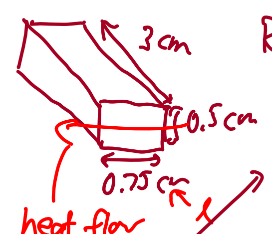
$= (2500\,\frac{\cancel{kg}}{\cancel{m^3}})(1000\,\frac{\cancel{g}}{\cancel{kg}})(\frac{1}{100^3}\,\frac{\cancel{m^3}}{\cancel{cm^3}})(20.7\,\cancel{cm^3})(0.5\,\frac{J}{\cancel{g}\cdot K})$

$\Rightarrow C_{th,cell} = 25.9\,J/K$

Find $\frac{R_{th,cell}}{2}$:



Box 1:



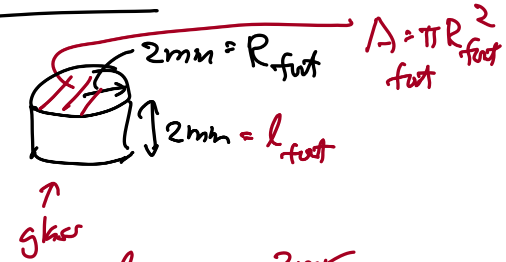
$$R_{th,Box1} = \frac{\ell}{kA} = \frac{\frac{3}{4}}{k\left(\frac{1}{2}\right)(3)}$$

Box 2:

$$R_{th,Box2} = \frac{\ell}{kA} = \frac{\frac{3}{4}}{k(1)(3)}$$

$$\frac{R_{th,cell}}{2} = R_{th,Box1} + R_{th,Box2} = \frac{3}{8}\frac{1}{k} = \frac{3}{8}\frac{1}{(1.05)} \times \left(100\frac{cm}{m}\right) = \underline{35.7\ K/W}$$

Find $R_{th,foot}$:



$$R_{th,foot} = \frac{\ell_{foot}}{kA_{foot}} = \frac{2mm}{\left(1.05\frac{W}{m \cdot K}\right)\pi(2mm)^2} = \underline{151.6\ K/W}$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right) \Rightarrow R_{th} = \underline{55.8\ K/W}$$

→ Find power req'd to maintain $T_\infty = 80°C$ in steady-state:

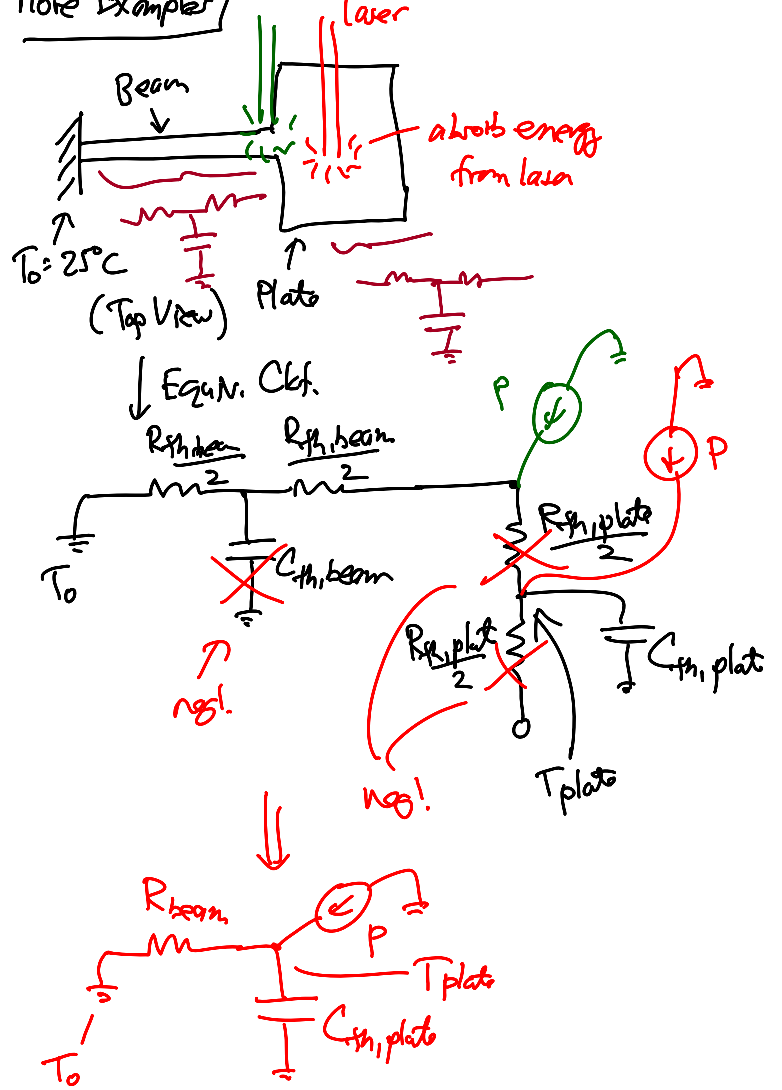
$$P = \frac{T_\infty - T_o}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

⇒ Find the time constant:

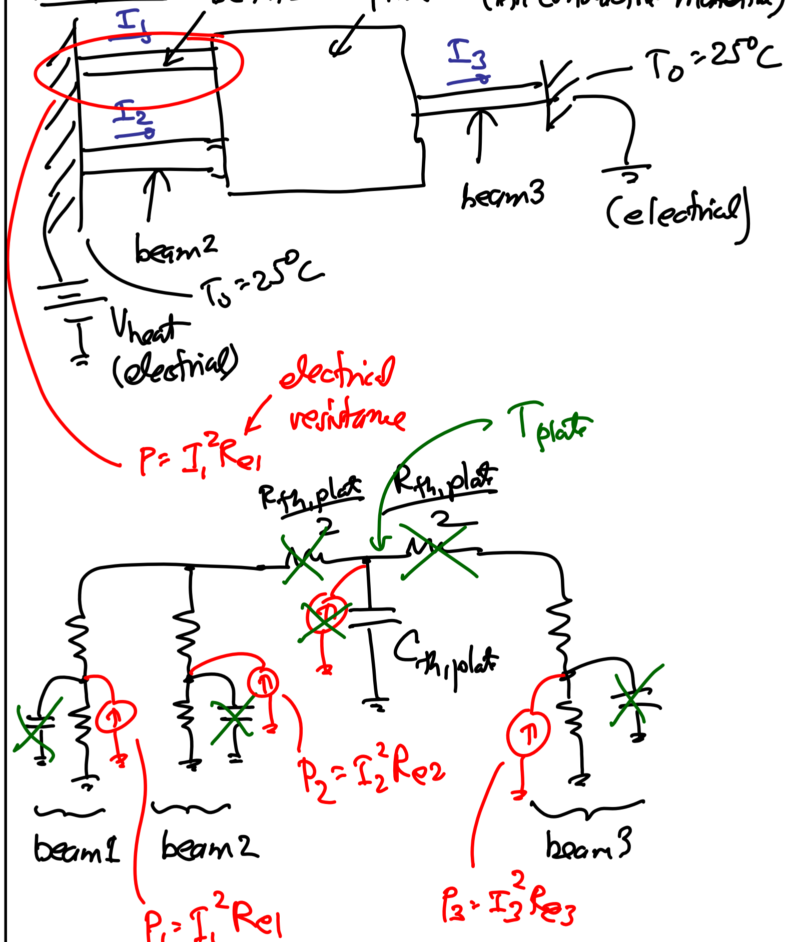
$$\tau = R_{th} C_{th,cell} = 24\ min.$$

If takes ~3τ to reach steady-state
∴ must wait 72 min. before using this atomic clock!

More Examples!

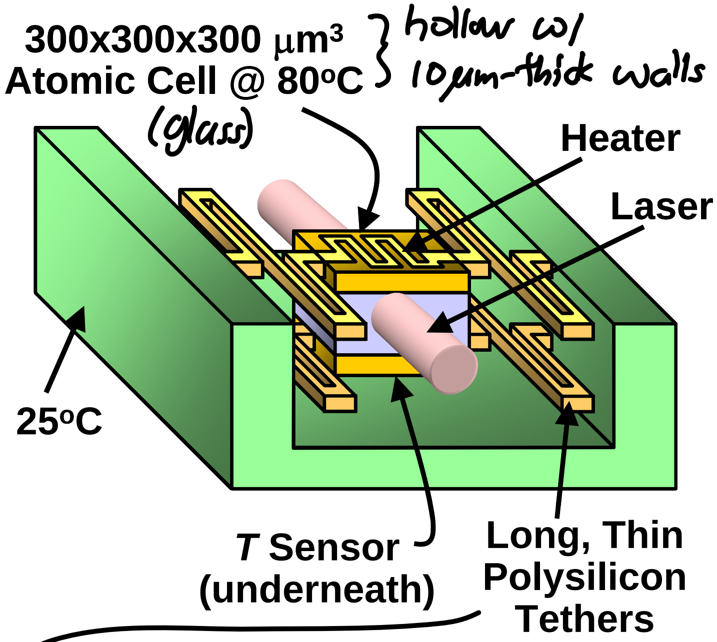


Example 2:



To analyze:

1. $I_3 = I_1 + I_2$
2. $I_3 = \frac{V_{heat}}{R_{e,tot}}$
3. Use superposition → handle one current source at a time, then add to get T_{plate}



500µm-long, 10µm-thick, 20µm-wide

$$V_{cell} = (300\mu)(300\mu)(300\mu) - (280\mu)(280\mu)(280\mu)$$
$$= 5.048\times10^{-12}\,m^3$$

↳ of course, much smaller than macro

$$C_{th,cell} = \rho_{glass} V_{cell} c_{p,glass}$$
$$= \left(2500\,\frac{kg}{m^3}\right)\left(5.048\times10^{-12}\,m^3\right)$$
$$\times \left(500\,\frac{J}{kg\cdot K}\right)$$

$$\Rightarrow C_{th,cell} = 6.31\times10^{-6}\,\frac{J}{K}$$

↳ 4 million x smaller than macro!

$$R_{th,supp} = \frac{L_{supp}}{k_{polySi}\cdot W_{supp}\cdot h_{supp}}$$
$$= \frac{500\mu}{\left(30\,\frac{W}{m\cdot K}\right)(20\mu)(10\mu)} = 83,333\ K/W$$

548x larger

and...

$$P = \frac{(80-25)}{83,333} = 2.64\,mW$$ ← 548x smaller!

$$\tau = 0.13\,s$$ ← 7300x faster!

All due to scaling!

What makes all of this possible? → scaling!

① Scaling reduces $C_{th} \sim \ell^3 \sim S^3$

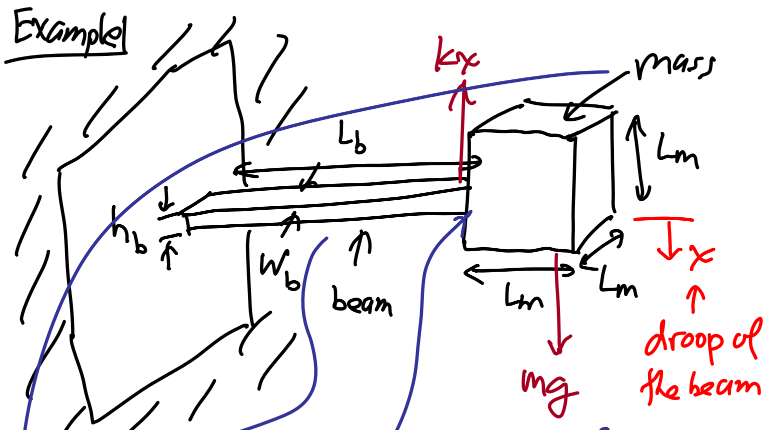
↳ $S\downarrow \rightarrow C_{th}\downarrow\downarrow\downarrow$

② Scaling allows the use of long, thin tethers $\rightarrow R_{th}\uparrow$

↳ tethers can support "more" when things are scaled!

over

Example



$k \triangleq$ stiffness @ this pt. $= \frac{1}{4} E W_b \frac{h_b^3}{L_b^3} \sim S \frac{S^3}{S^3} \sim S$
(in x-direction)

$m \triangleq \text{mass} = \rho L_m^3 \sim S^3 \rightarrow S\downarrow \rightarrow m\downarrow\downarrow\downarrow$

@ static equilibrium:

Force due to Gravity = Spring Force

acceleration due to gravity

$$mg = kx$$

$$x = \frac{m}{k} g \sim \frac{S^3}{S} \sim S^2$$

As dimensions scale smaller, the droop $x\downarrow$

$R_{th} \sim \frac{L_b}{W_b h_b} \rightarrow$ want to raise this
$\rightarrow$ how fast does $R_{th}\uparrow$ as $S\downarrow$ for a constant droop x?

$*$ — $\underbrace{\rho L_m^3}_{m} g = \underbrace{\frac{1}{4} E W_b \frac{h_b^3}{L_b^3}}_{k} x$ ← const.

$$\frac{L_b}{W_b h_b} = \frac{1}{4} E \frac{h_b^2}{L_b^2} x \frac{1}{\rho L_m^3 g} \sim \frac{\cancel{S^2}}{\cancel{S^2}} \frac{1}{S^3} \sim \frac{1}{S^3}$$

as $S\downarrow \rightarrow \frac{L_b}{W_b h_b}\uparrow\uparrow\uparrow \rightarrow R_{th}\uparrow\uparrow\uparrow$
(for constant droop)