# EE C245 – ME C218
# Introduction to MEMS Design
# Fall 2011

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Lecture Module 11: Equivalent Circuits I



## Lecture Outline

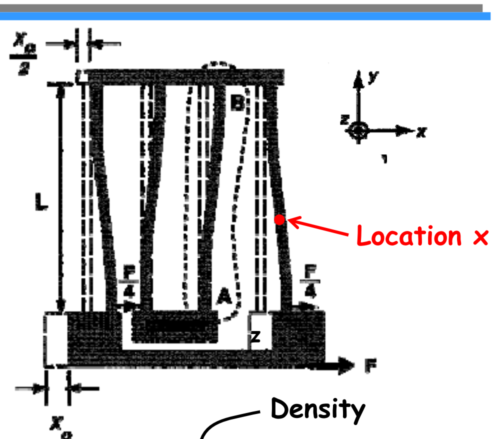
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mass
  - Lumped Stiffness
  - Lumped Damping
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies

## Lumped Parameter Mechanical Equivalent Circuit



## Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity




Maximum Kinetic Energy

Density

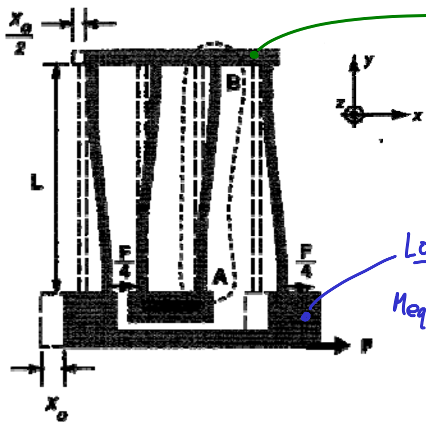
$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^{\ell} V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x

Maximum Velocity Function

## Equivalent Dynamic Mass

- For the folded-beam structure, we've already determined the maximum kinetic energy
- And in our resonance frequency analysis, we've already determined expressions for velocity




Location on the Truss:

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\omega_0^2 X_0^2 \left(\frac{1}{2}\right)\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\frac{1}{2}\left(\frac{1}{4}\right)\omega_0^2 X_0^2}$$

$$\therefore M_{eq(truss)} = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

Location on the Shuttle:

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\omega_0^2 X_0^2 \left(\frac{1}{2}\right)\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\frac{1}{2}\omega_0^2 X_0^2}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$



## Equivalent Dynamic Stiffness & Damping

- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{K_{eq}(x) = \omega_o^2 M_{eq}(x)}$$

⇒ large equiv. mass & large stiffness go hand-in-hand

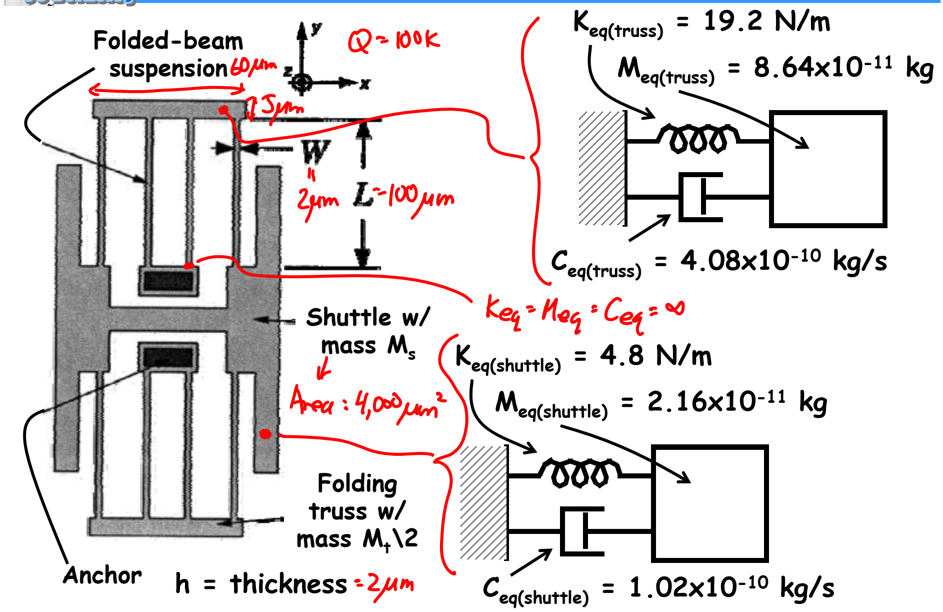
- And damping also follows readily from knowledge of Q or other loss measurands

$$Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \longrightarrow \boxed{C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}}$$

($C_{eq}(x)$ ← damping)

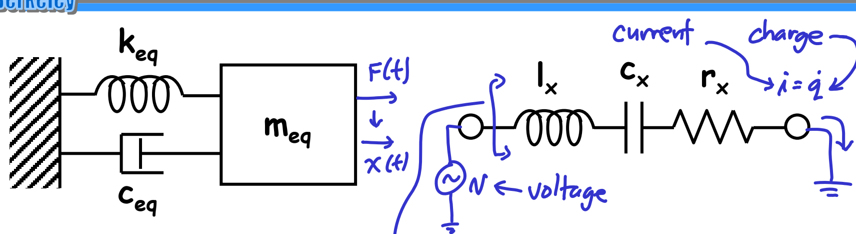
- With mass, stiffness, and damping ⇨ lumped parameter equivalent circuit

## Get Potential Energy & Frequency






## Electromechanical Analogies




$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos\omega t$

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

⇒ using phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ by analogy:

| | | |
|---|---|---|
| $F \rightarrow v$ | $m_{eq} \rightarrow l_x$ | $c_{eq} \rightarrow r_x$ |
| $\dot{x} \rightarrow i$ | $k_{eq} \rightarrow \frac{1}{C_x}$ | |

[Parameter Relationships in the Current Analogy]

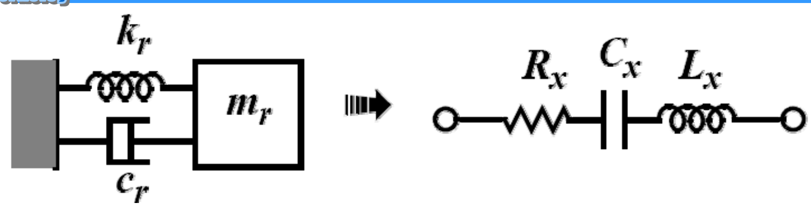
$v(t) = V\cos\omega t \rightarrow i(t) = I\cos\omega t$

Impedance looking in:

$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega C_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/C_x)}{j\omega} i + r_x i$$

## Electromechanical Analogies (cont)

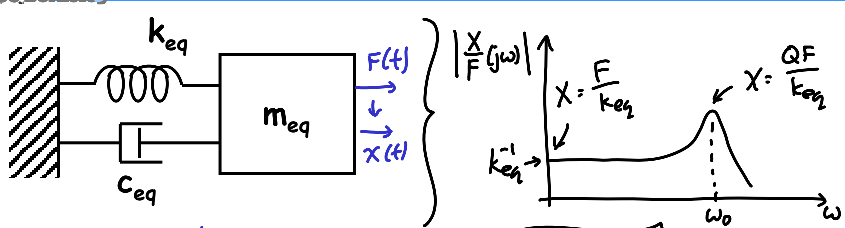



- Mechanical-to-electrical correspondence in the current analogy:

| Mechanical Variable | Electrical Variable |
|---|---|
| Damping, $c$ | Resistance, $R$ |
| Stiffness$^{-1}$, $k^{-1}$ | Capacitance, $C$ |
| Mass, $m$ | Inductance, $L$ |
| Force, $f$ | Voltage, $V$ |
| Velocity, $v$ | Current, $I$ |



## Bandpass Biquad Transfer Function




$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

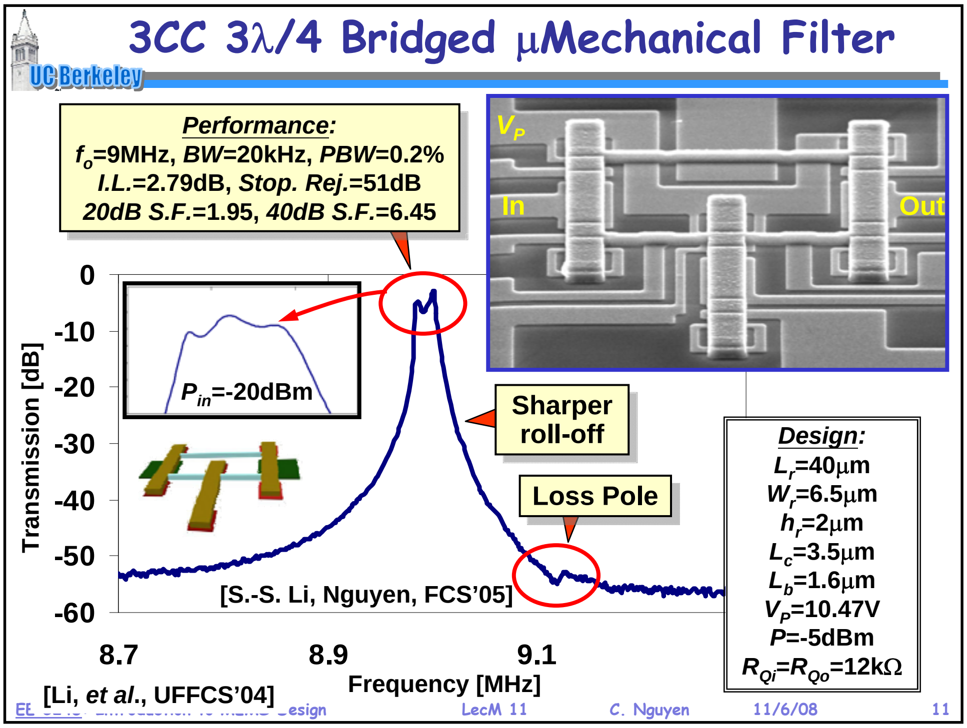
$\Rightarrow$ converting to full phasor form:

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{j\omega}(j\omega X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_o}\right)^2 + 1 + j\frac{\omega}{Q\omega_o}\right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_o^2,\ Q = \frac{m_{eq}\omega_o}{c_{eq}} = \frac{k_{eq}}{\omega_o c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_o\right]$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega}{Q\omega_o}}}$$


3CC 3λ/4 Bridged µMechanical Filter
UC Berkeley
Performance:
fo=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45
VP
In
Out
Pin=-20dBm
Sharper roll-off
Loss Pole
Design:
Lr=40µm
Wr=6.5µm
hr=2µm
Lc=3.5µm
Lb=1.6µm
VP=10.47V
P=-5dBm
RQi=RQo=12kΩ
Transmission [dB]
0
-10
-20
-30
-40
-50
-60
8.7
8.9
9.1
Frequency [MHz]
[S.-S. Li, Nguyen, FCS'05]
[Li, et al., UFFCS'04]
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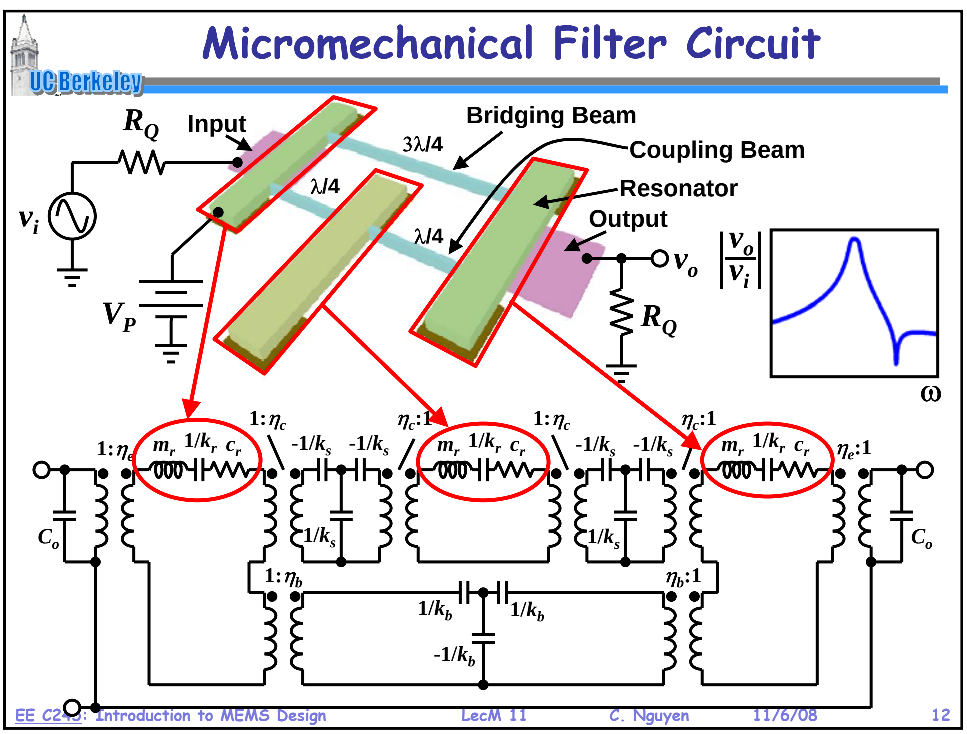
Micromechanical Filter Circuit
UC Berkeley
RQ
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
λ/4
vi
VP
vo
RQ
|vo/vi|
ω
1:ηe
mr 1/kr cr
1:ηc
-1/ks
-1/ks
ηc:1
1:ηc
ηc:1
ηe:1
Co
1/ks
1:ηb
ηb:1
1/kb
1/kb
-1/kb
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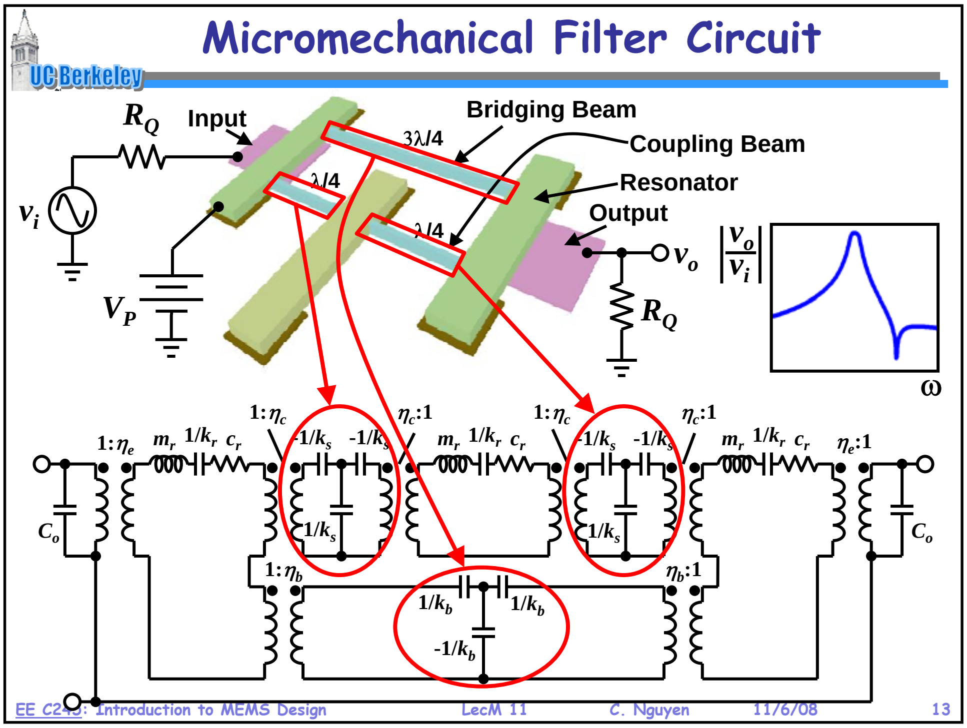
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vi
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1:ηe
mr
1/kr
cr
1:ηc
-1/ks
-1/ks
ηc:1
1/ks
Co
1:ηb
ηb:1
1/kb
1/kb
-1/kb
ηe:1
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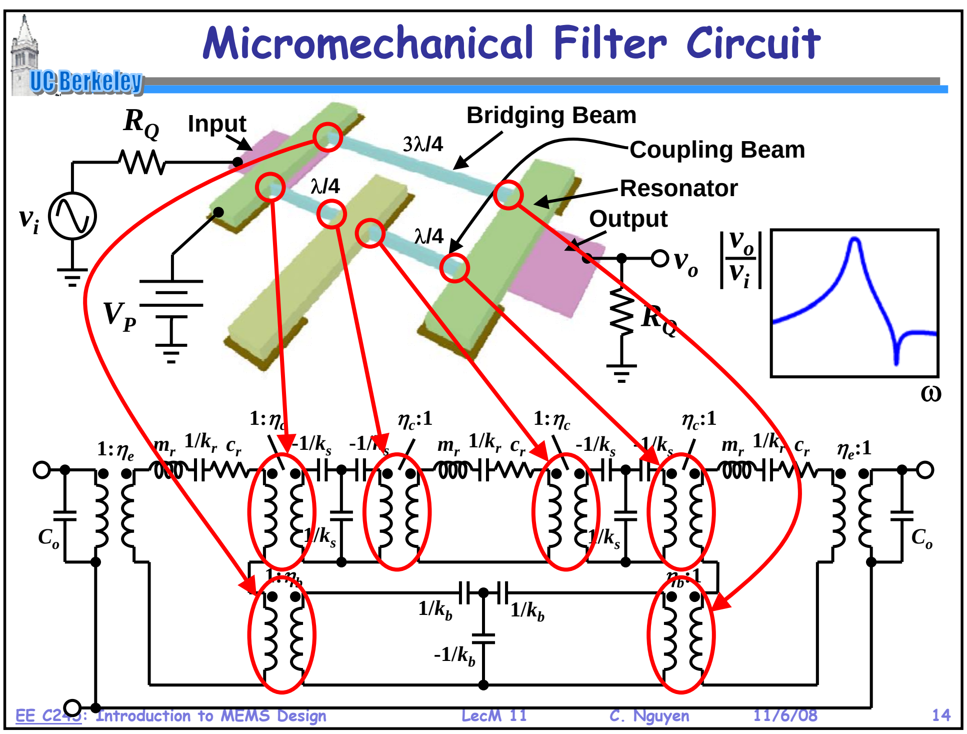
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Bridging Beam
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Resonator
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vi
vo
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ω
1:ηe
mr
1/kr
cr
ηc:1
1:ηc
1/ks
Co
1/kb
1/kb
-1/kb
ηe:1
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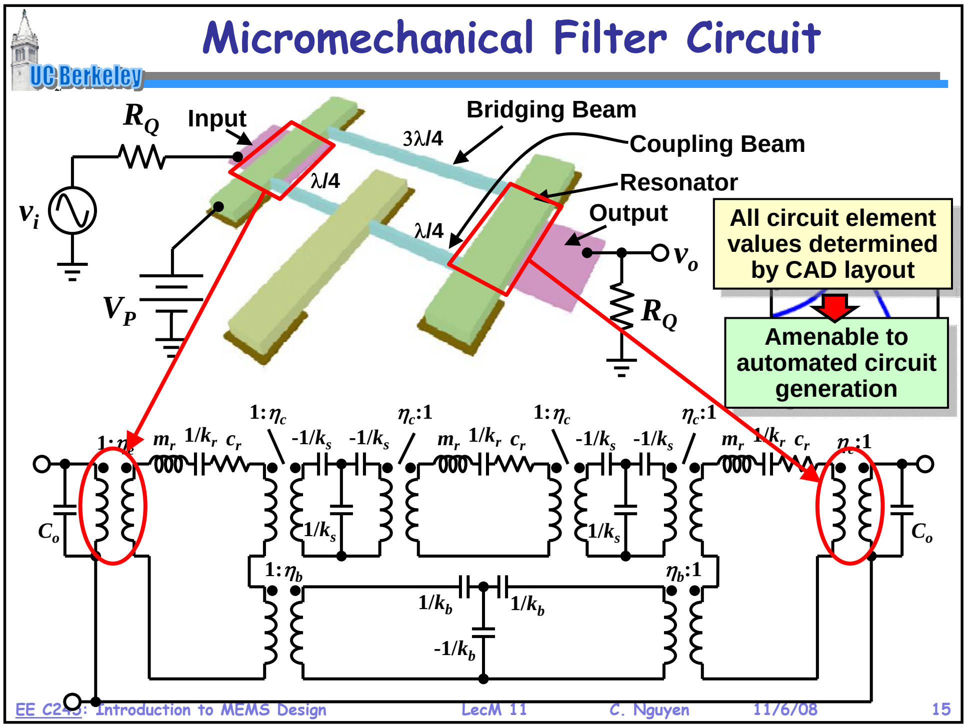
Micromechanical Filter Circuit
UC Berkeley
$R_Q$
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
$v_i$
Output
λ/4
$v_o$
$V_P$
$R_Q$
All circuit element values determined by CAD layout
Amenable to automated circuit generation
$1:\eta_c$
$\eta_c:1$
$1:\eta_c$
$\eta_c:1$
$1:\eta_e$
$m_r$ $1/k_r$ $c_r$
$-1/k_s$ $-1/k_s$
$m_r$ $1/k_r$ $c_r$
$-1/k_s$ $-1/k_s$
$m_r$ $1/k_r$ $c_r$
$\eta_e:1$
$C_o$
$1/k_s$
$1/k_s$
$C_o$
$1:\eta_b$
$\eta_b:1$
$1/k_b$
$1/k_b$
$-1/k_b$
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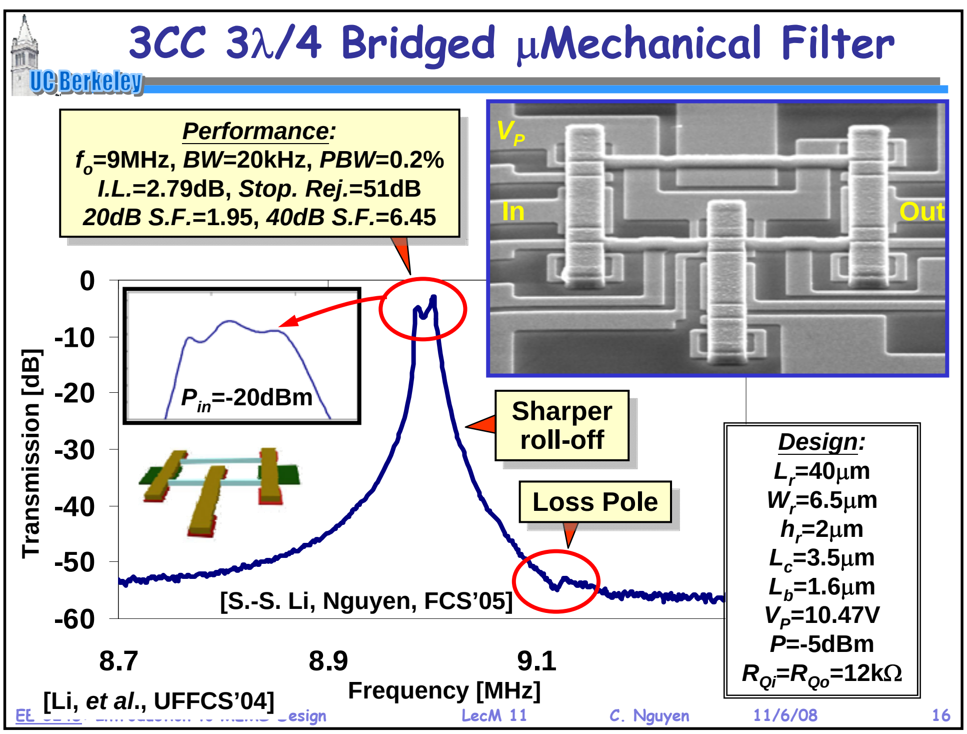
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$V_P$
In
Out
0
-10
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-30
-40
-50
-60
Transmission [dB]
$P_{in}$=-20dBm
Sharper roll-off
Loss Pole
Design:
$L_r$=40μm
$W_r$=6.5μm
$h_r$=2μm
$L_c$=3.5μm
$L_b$=1.6μm
$V_P$=10.47V
P=-5dBm
$R_{Qi}$=$R_{Qo}$=12kΩ
[S.-S. Li, Nguyen, FCS'05]
8.7
8.9
9.1
Frequency [MHz]
[Li, et al., UFFCS'04]
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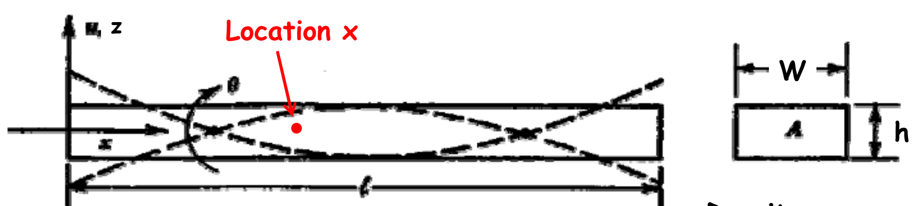
## Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)



## Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity




Density

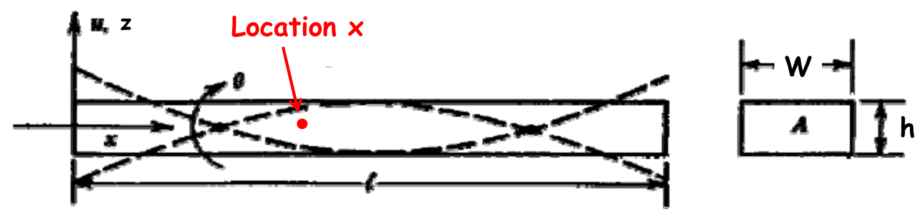
Maximum Kinetic Energy

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^{\ell} V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x

Maximum Velocity Function

## Equivalent Dynamic Mass




- We know the mode shape, so we can write expressions for displacement and velocity at resonance

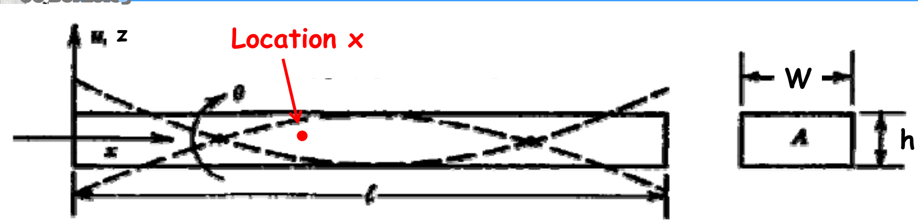
Displacement: $u(x) = B\left[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]$, $\varsigma = \frac{A}{B}$

$$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\frac{1}{2}\rho A \int_0^{\ell} \omega_0^2 [u(x')]^2 dx'}{\frac{1}{2}\omega_0^2 [u(x)]^2}$$

$$M_{eq}(x) = \frac{\rho A \int_0^{\ell} B^2[\varsigma(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{B^2[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$



## Equivalent Dynamic Stiffness & Damping




- Stiffness then follows directly from knowledge of mass and resonance frequency

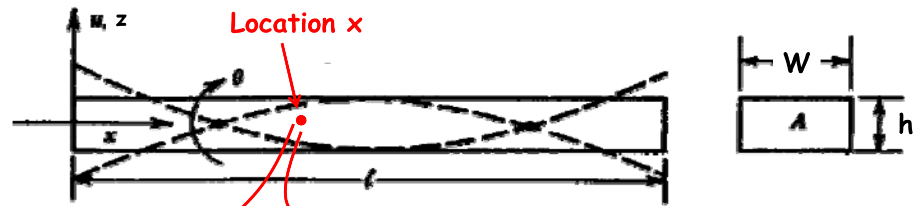
$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

- And damping also follows readily

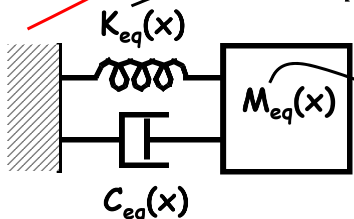
$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \longrightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

($C_{eq}(x)$ ← damping)

## Equivalent Lumped Mechanical Circuit




$$K_{eq}(x) = \omega_o^2 M_{eq}(x)$$




$$M_{eq}(x) = \frac{\rho A \int_o^l [u(x')]^2 dx'}{[u(x)]^2}$$

$$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$$

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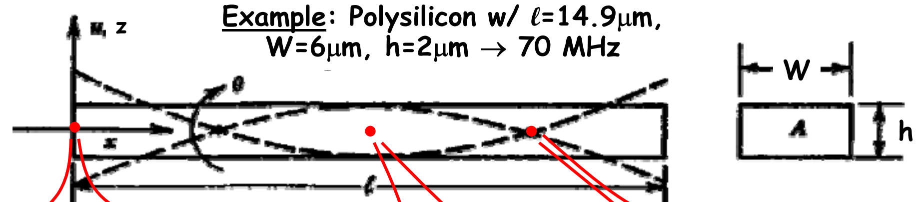
## Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/ ℓ=14.9μm, W=6μm, h=2μm → 70 MHz



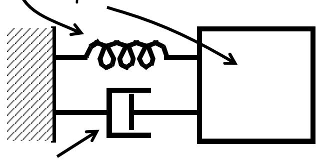

$K_{eq}$(node) = ∞

$M_{eq}$(node) = ∞

$C_{eq}$(node) = ∞

$K_{eq}(0)$ = 19,927 N/m

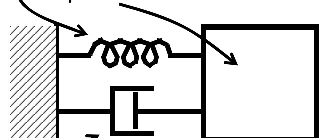
$M_{eq}(0)$ = 1.03x10$^{-13}$ kg



$C_{eq}(0)$ = 5.66x10$^{-9}$ kg/s

$K_{eq}(\ell/2)$ = 53,938 N/m

$M_{eq}(\ell/2)$ = 2.78x10$^{-13}$ kg



$C_{eq}(\ell/2)$ = 1.53x10$^{-8}$ kg/s

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