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EE C245 – ME C218
Introduction to MEMS Design
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Lecture Module 7: Mechanics of Materials

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Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

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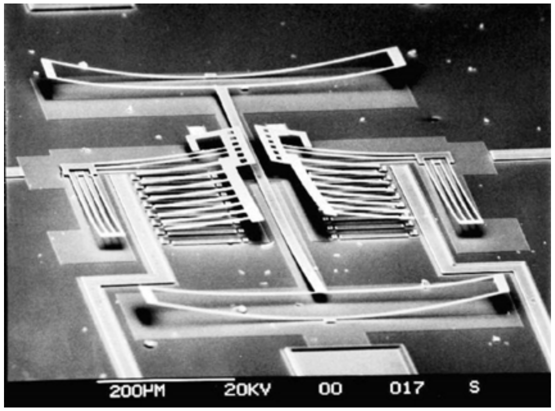
Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



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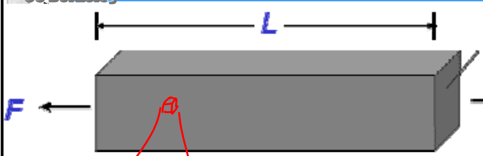
Elasticity

Normal Stress (1D)

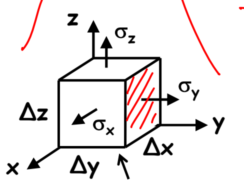


If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \left\{\begin{matrix}\text{Force per}\\ \text{unit area}\end{matrix}\right\} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

standard mks unit

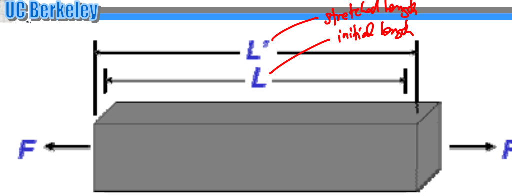


Differential volume element

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

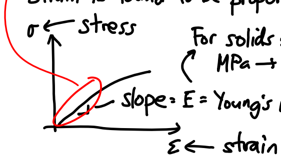
Strain (1D)



$$\text{Strain} = \left\{\begin{matrix}\text{Fractional Change}\\ \text{in Length}\end{matrix}\right\} = \varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$$

Sometimes a unit called the "microstrain" is used, where $10^{-6} = 1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress

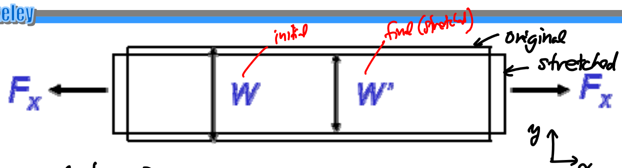


For solids: MPa → GPa

$\sigma = \varepsilon E$ → $\boxed{\varepsilon = \frac{\sigma}{E} \quad [\text{unitless}]}$

Thus, the units of E are the same as σ → Pa

The Poisson Ratio



Apply normal stress to a free-standing object → uniaxial strain → but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\Delta W}{W} = -\nu\varepsilon_x$$

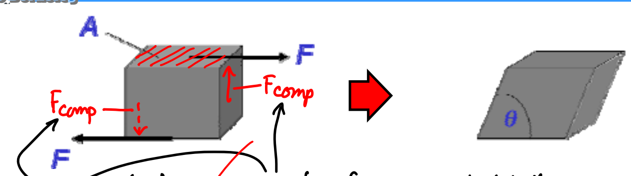
ν = Poisson ratio [unitless]

typical values: 0 → 0.5

⇒ inorganic solids: 0.2 → 0.3

⇒ elastomers (e.g., rubber): ~0.5

Shear Stress & Strain (1D)



Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.) — nothing rotates

$$\text{Shear Stress} = \left\{\begin{matrix}\text{Force Per Unit Area}\\ \text{Parallel to the Surfaces}\end{matrix}\right\} = \tau = \frac{F}{A} \quad [Pa]$$

Generates a shear strain:

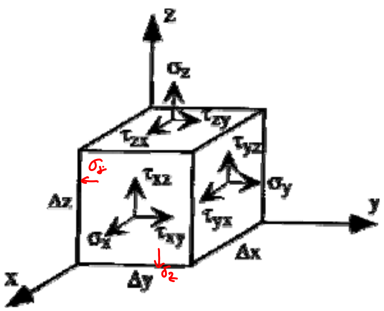
$$\text{Shear Strain} = \theta = \frac{\tau}{G} \leftarrow G \triangleq \text{shear modulus}$$

← differential volume element

$$G = \frac{E}{2(1+\nu)}$$

2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:

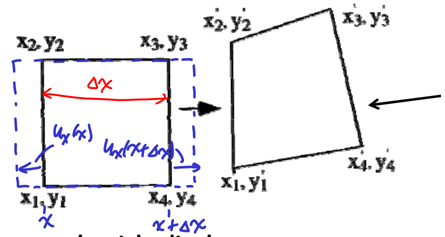


Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$

2D Strain

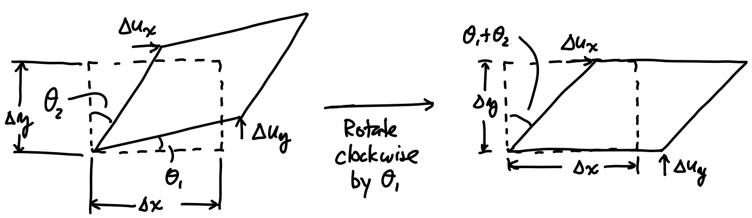
- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation



Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

2D Shear Strain



⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

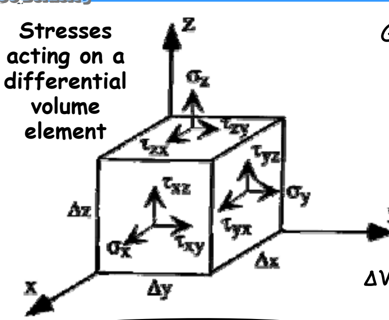
↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

For small amplitude deformations.

Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element



Given an x-directed uniaxial stress, σ_x:

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume ΔV

$$\Delta V = \Delta x\Delta y\Delta z(1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x\Delta y\Delta z$$
$$= \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] ⇒ $\Delta V = \Delta x\Delta y\Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$[(1+mx)^n \approx 1+nmx]$ ⇒ $\approx \Delta x\Delta y\Delta z\left[\cancel{1} + \epsilon_x - 2\nu\epsilon_x - \underbrace{2\nu\epsilon_x^2}_{\text{negl.}} - \cancel{1}\right]$

$$\boxed{\Delta V \approx \Delta x\Delta y\Delta z(1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z \underbrace{- \nu\left(\sigma_x + \sigma_y\right)}_{\text{Basically, add in off-axis strains from normal stresses in other directions}}\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$