Q Measurement Using Resonators

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Compound Mode (2,1)

Anchor

Input

Output

Output

Wine Glass Disk Resonator

$R = 32~\mu m$

Support Beams

Input

Anchor

[Y.-W. Lin, Nguyen, JSSC Dec. 04]

f_o = 61.37 MHz
Q = 145,780

Unmatched Transmission [dB]

-40
-60
-80
-100

61.325   61.375   61.425
Frequency [MHz]

Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V

Folded-Beam Comb-Drive Resonator

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- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

Capacitive-Comb Transducer
Outer Beam
Inner Beam
Coupling Beam (not part of resonator)
Input Force, f_c (Port 3)
Ground Plane
Shuttle

(Electrical) Port 1
(Electrical) Port 2

i_1 i_2
v_1 v_2
V_1 V_2

Folding Truss
V_P
Anchors

Comb-Transducer   Anchors   Ratioed Folded-Beams
Shuttle   Tuning Electrode

Amplitude [dB]
-4
-8
-12
-16
-20
-24
-28
-32

f_o=342.5kHz
Q=41,000

$$Q = \frac{342{,}500}{8.3}$$

8.3 Hz

342.40 342.45 342.50 342.55 342.60
Frequency [kHz]

Comb-Drive Resonator in Action

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- **Below**: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

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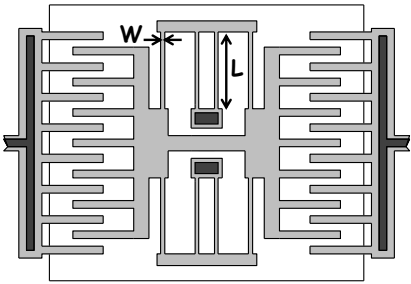
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Measurement of Young's Modulus

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- Use micromechanical resonators
  - ↳ Resonance frequency depends on E
  - ↳ For a folded-beam resonator:

Young's modulus

Resonance Frequency = $f_o = \left[\dfrac{4Eh(W/L)^3}{M_{eq}} \right]^{1/2}$

Equivalent mass

h = thickness

W→ ←

L

- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

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Elastic Constants in Crystalline Materials

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- Get different elastic constants in different crystallographic directions → 81 of them in all
  - ↳ Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

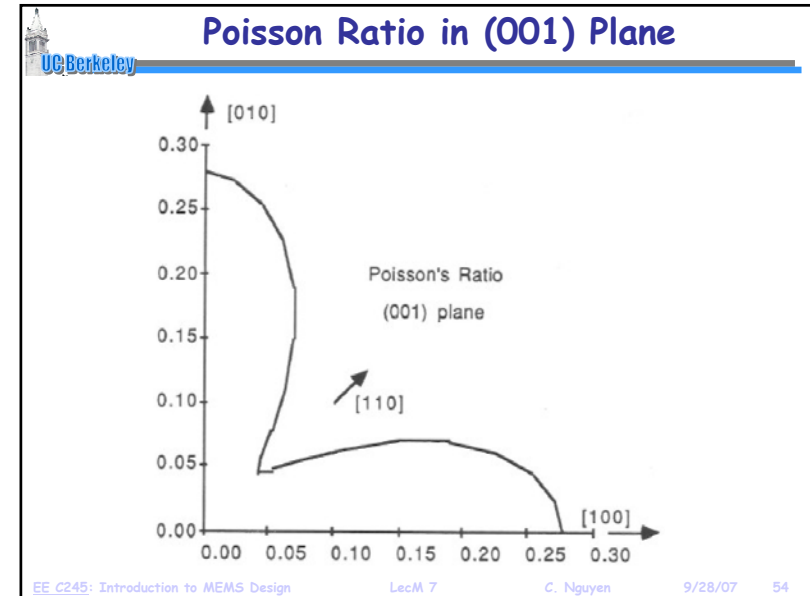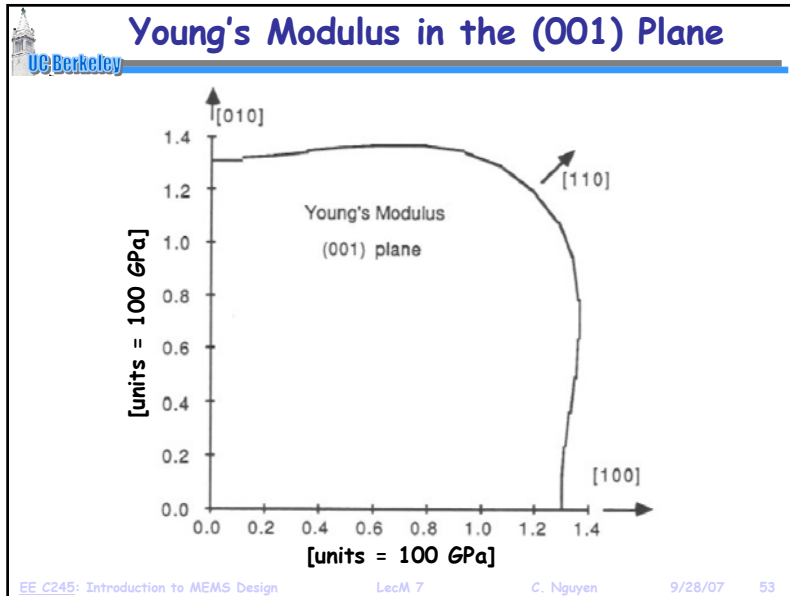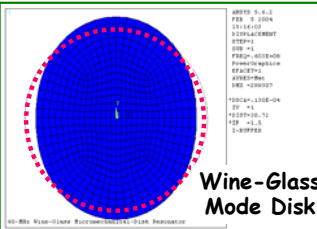Stresses   Stiffness Coefficients   Strains

Stiffness Coefficients of Silicon

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- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

where $\begin{cases} C_{11} = 165.7 \text{ GPa} \\ C_{12} = 63.9 \text{ GPa} \\ C_{44} = 79.6 \text{ GPa} \end{cases}$

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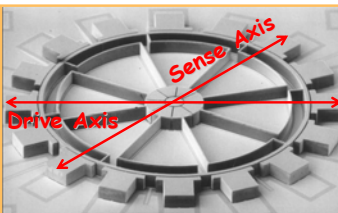## Young's Modulus in the (001) Plane

Poisson Ratio in (001) Plane

Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - ⮩ Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - ⮩ Mode matching is required, where frequencies along different axes of a ring must be the same
  - ⮩ Not okay to ignore anisotropic variations, here

Wine-Glass Mode Disk

Ring Gyroscope

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