Lecture 16: Beam Combos I

- Announcements:
- Midterm is nearing: Thursday, Oct. 27
  - I will pass out materials associated with the midterm, including and information sheet and old exams, this coming Thursday
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- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
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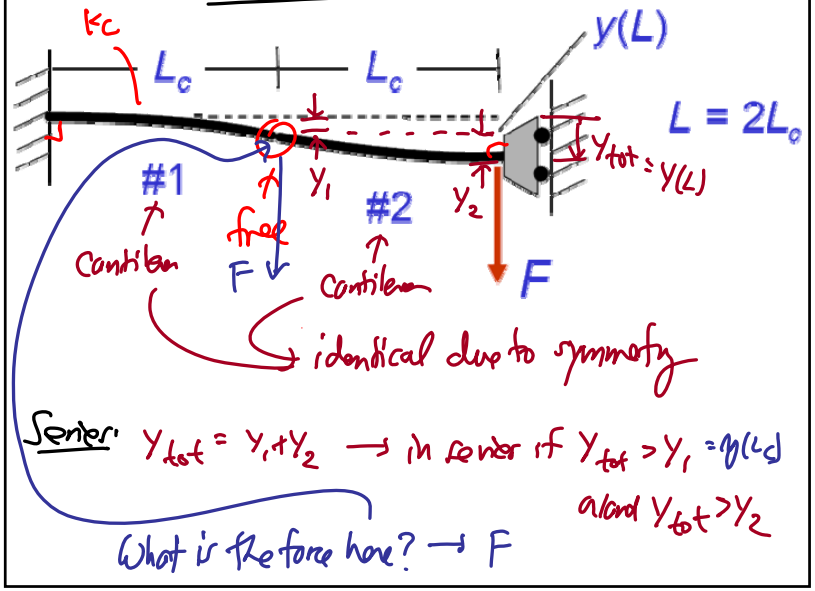
- Last Time: Series Combination of Springs



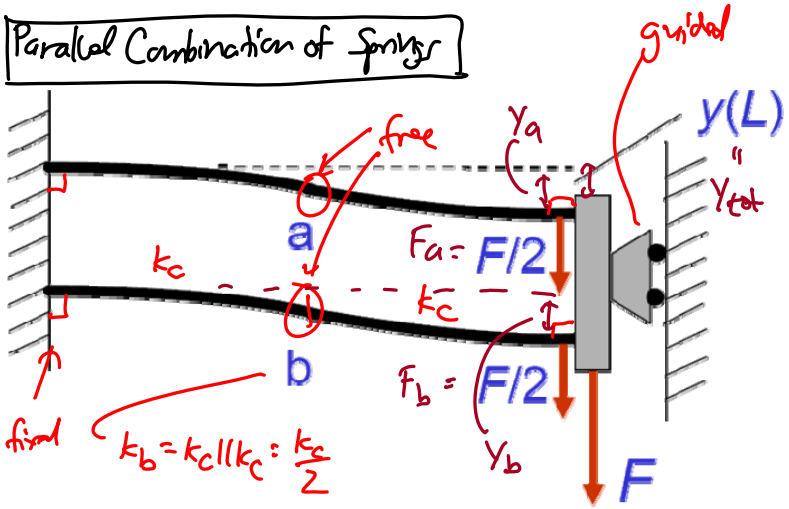
$y(L) = \frac{F}{k_{tot}} = 2y(L_c) = 2\left(\frac{F}{k_c}\right) = F\left(\frac{1}{k_c}+\frac{1}{k_c}\right)$

$k_c = \frac{1}{4}Ew\left(\frac{h}{L_c}\right)^3$

$\frac{1}{k_{tot}} = \frac{1}{k_c}+\frac{1}{k_c} \rightarrow \boxed{k_{tot} = k_c \| k_c}$

Definition for "||": $A\|B = \frac{1}{\frac{1}{A}+\frac{1}{B}} = \frac{AB}{A+B}$

Parallel Combination of Springs



Parallel: $y_{tot} = y_a = y_b$ ($\frac{F}{2}$, $\frac{F}{2}$)

$y_{tot} = y(L) = \frac{F}{k_{tot}} = \frac{F_a}{k_a} = \frac{F_b}{k_b} = \left(\frac{F}{2}\right)\left(\frac{1}{k_a}\right)$

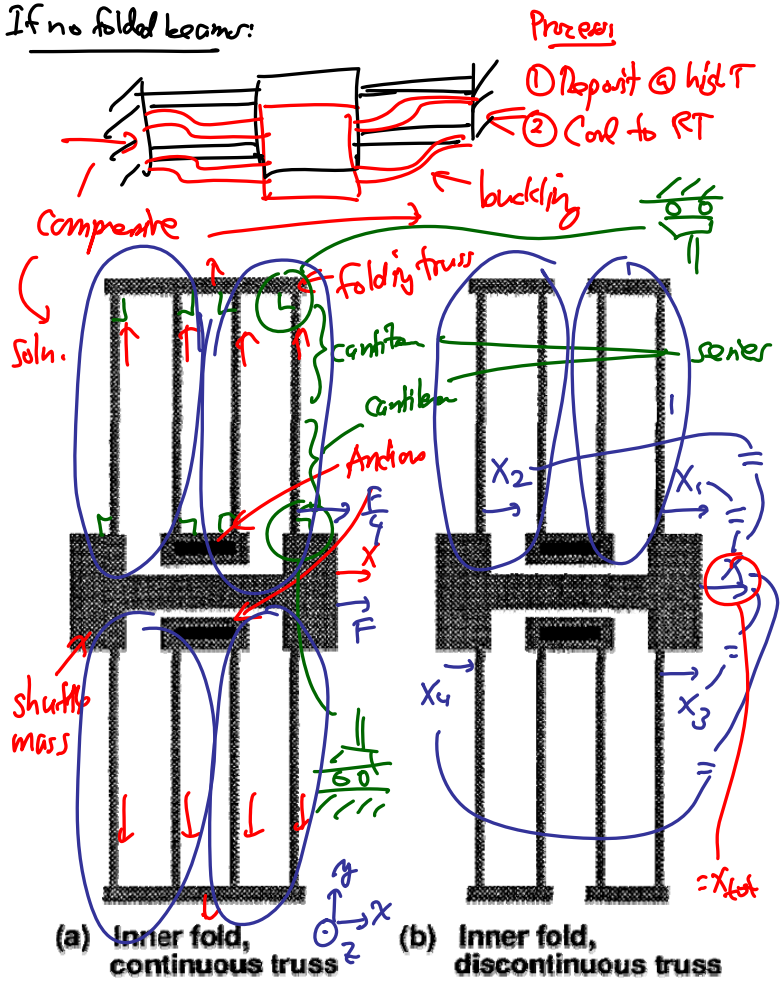
$\boxed{k_{tot} = 2k_a}$

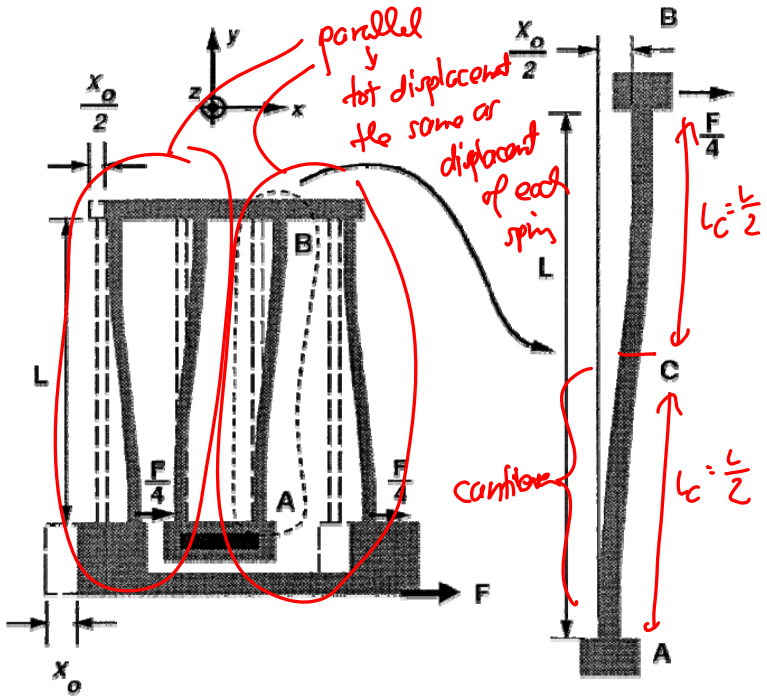
In general: $k_{tot} = k_{cl} + k_b$

⇒ Springs in parallel add!

For EE's: springs behave like capacitors

If no folded beams:





From last time:

$$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c}\right)$$

$$x(y) = \frac{F_c y^2}{6EI_z}(3L_c - y)$$

$$k_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3} = \frac{1}{4} Eh \frac{w^3}{L_c^3}$$

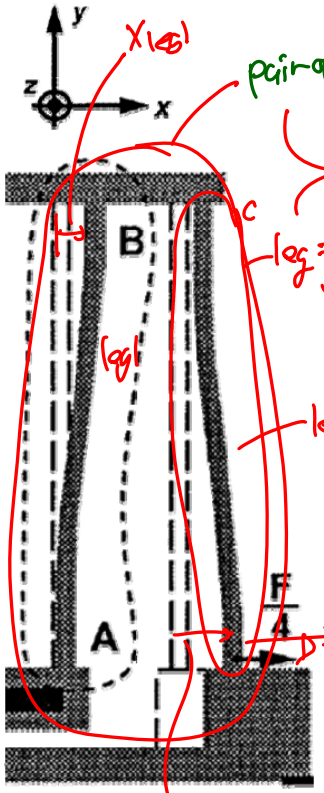
($I_z = \frac{1}{12} h w^3$; L_c = cantilever length)

⇒ insert k_c:

$$k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3} \Rightarrow k_c = 2Eh\left(\frac{w}{L}\right)^3$$

$L_c = \frac{L}{2} \rightarrow L = 2L_c$

full beam length k_c equation



x_{leg1}

pair of legs

series connected!

leg = 2 cantilevers in series

$\rightarrow k_{leg} = \frac{k_c}{2} = k_c \| k_c$

leg2

$F_{pair} = \frac{F}{4}$

$x = \frac{F_{pair}}{k_{pair}} = \frac{F_{pair}}{(k_{leg} \| k_{leg})}$

$x_{tot} = x_{leg1} + x_{leg2}$ — series

$= \frac{F}{4}\left(\frac{1}{k_{leg}} + \frac{1}{k_{leg}}\right)$

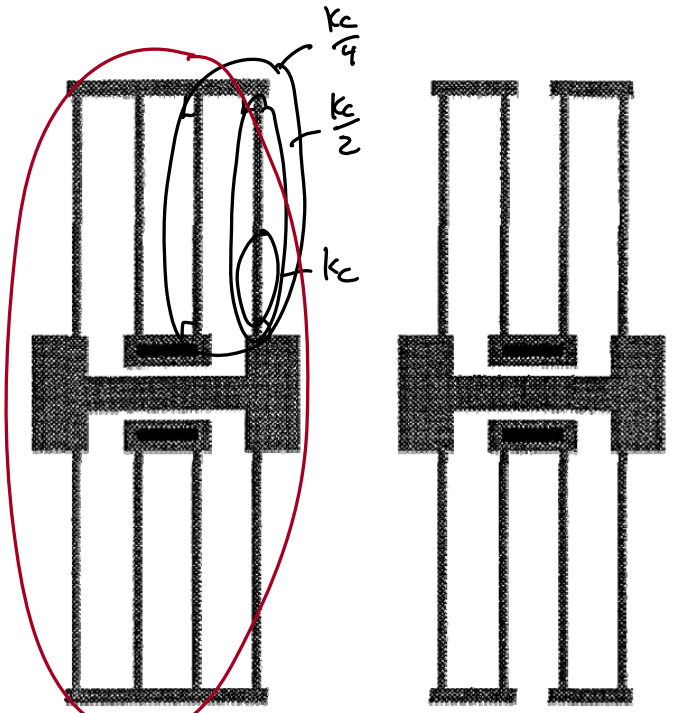
x_{leg2}

From before: $k_{leg} = k_c \| k_c = \frac{k_c}{2}$

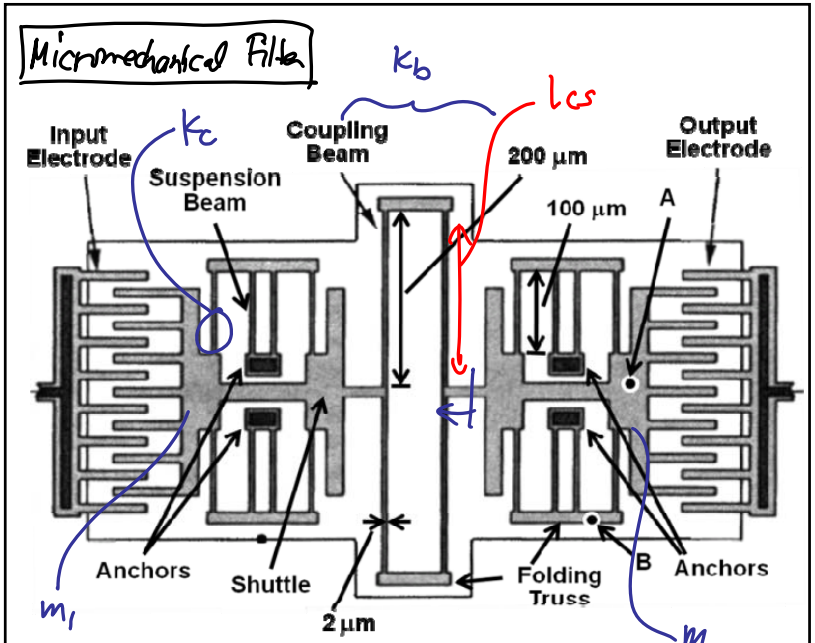
Thus: $x = \left(\frac{F}{4}\right)\left(\frac{2}{k_c} + \frac{2}{k_c}\right) = \frac{F}{k_c} = \frac{F}{k_{tot}}$

$$k_{tot} = k_c = 2Eh\left(\frac{w}{L}\right)^3$$

Better Way to Do It → Just Consider Stiffness!

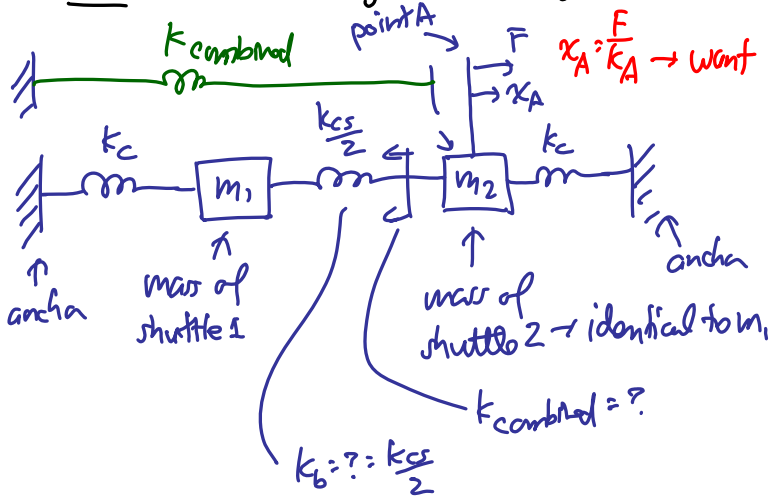


$k_{tot} = 4\left(\frac{k_c}{4}\right) = k_c$ ✓

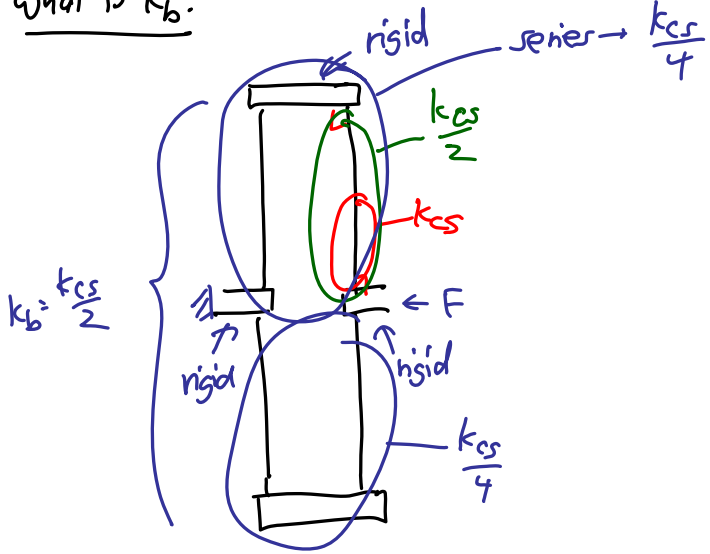


⇒ Find the stiffness @ point A.

Assume: shuttles & folding trusses are rigid.



What is k_b?



$k_A = k_c + k_{combined} = k_c + k_c \| \frac{k_{cs}}{2} = k_A$

where $k_c = \frac{24 EI_z}{L_r^3}$, $k_{cs} = \frac{24 EI_z}{L_{cs}^3}$