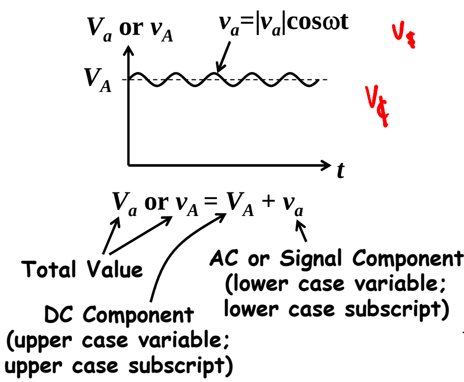
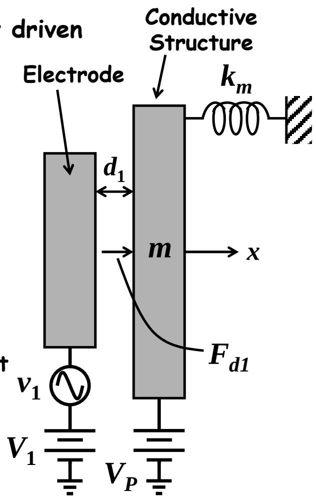
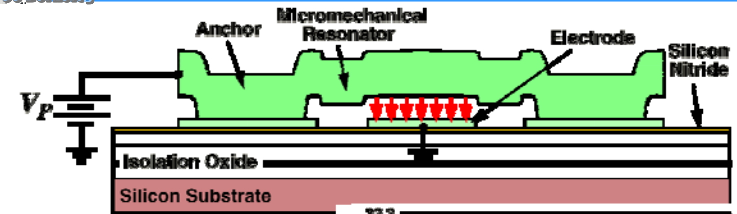
Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:





Voltage-Controllable Center Frequency

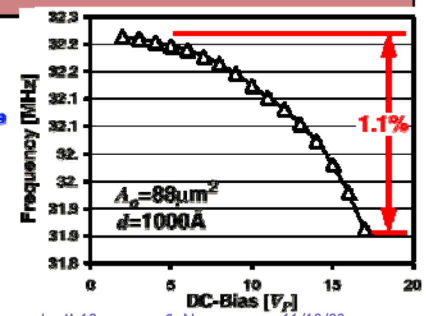


- Quadrature force ⇒ voltage-controllable electrical stiffness:

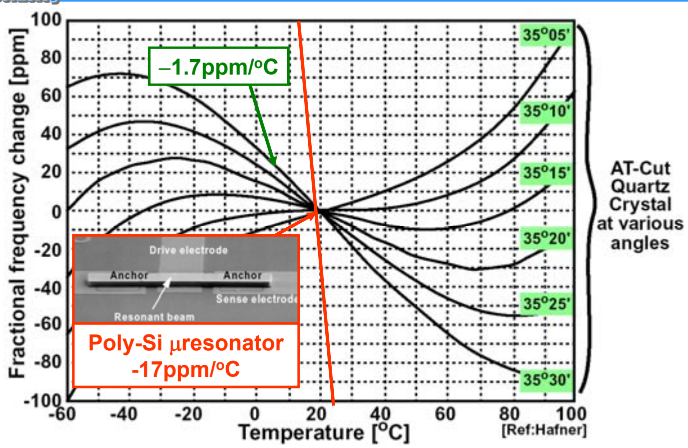
$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

Electrode Overlap Area; Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



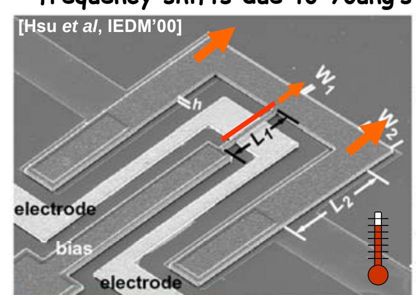
Microresonator Thermal Stability

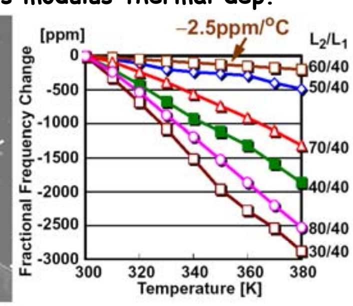


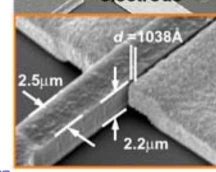
- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

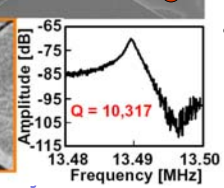
Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.





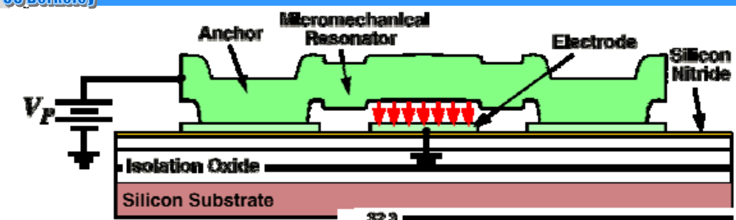




- *Problems*:
  - stress relaxation
  - compromised design flexibility

[Hsu *et al* IEDM 2000]

Voltage-Controllable Center Frequency



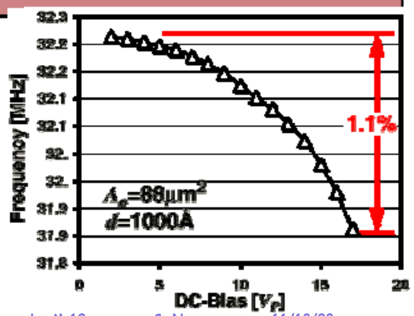
- Quadrature force ⇒ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

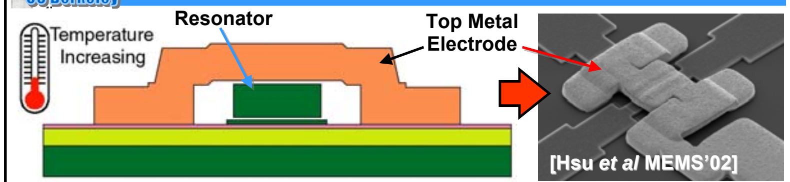
Electrode Overlap Area

Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



Excellent Temperature Stability

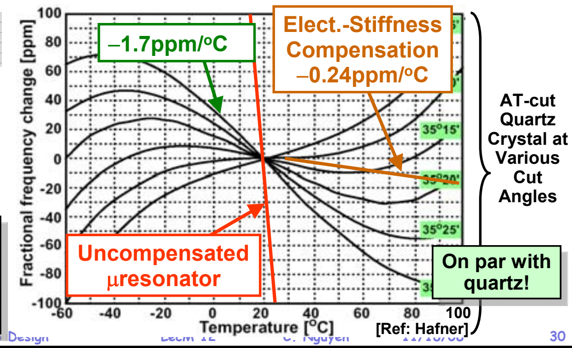


Top Electrode-to-Resonator Gap ⇧

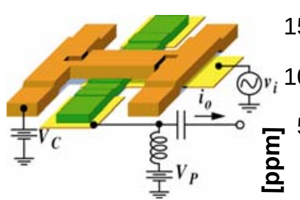
Elect. Stiffness: $k_e \sim 1/d^3$ ⇩

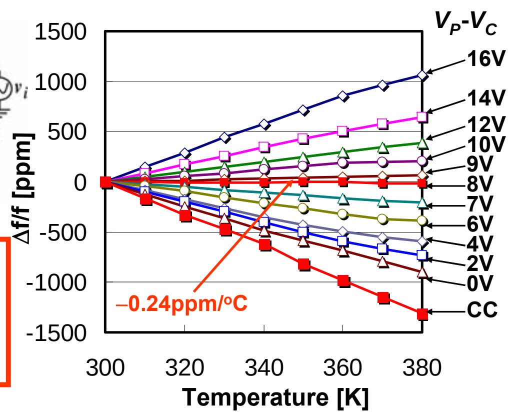
Frequency: $f_o \sim (k_m - k_e)^{0.5}$ ⇧

Counteracts reduction in frequency due to Young's modulus temp. dependence



Measured Δf/f vs. T for k_e-Compensated μResonators





Design/Performance:
f_o =10MHz, Q=4,000
V_P=8V, h_e=4μm
d_o=1000Å, h=2μm
W_r=8μm, L_r=40μm

[Hsu et al MEMS'02]

- Slits help to release the stress generated by lateral thermal expansion ⇒ linear TC_f curves ⇒ –0.24ppm/°C!!!

Can One Cancel k_e w/ Two Electrodes?

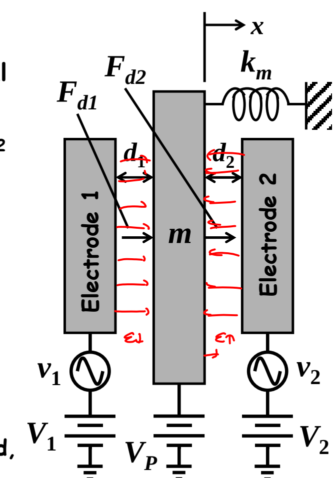
- What if we don't like the dependence of frequency on V_P?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

Subtracts from the F_{d1} term, as expected

$$F_{d2}\Big|_{\omega_o} = -V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t$$

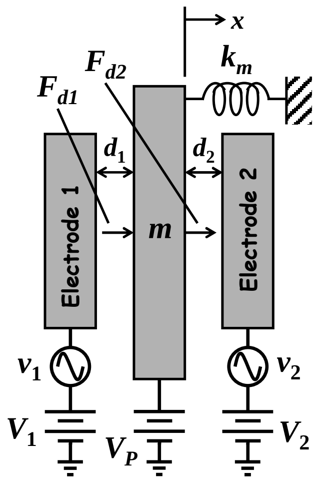
$$+V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t$$

Adds to the quadrature term → k_e's add, no matter the electrode configuration!



Problems With Parallel-Plate C Drive

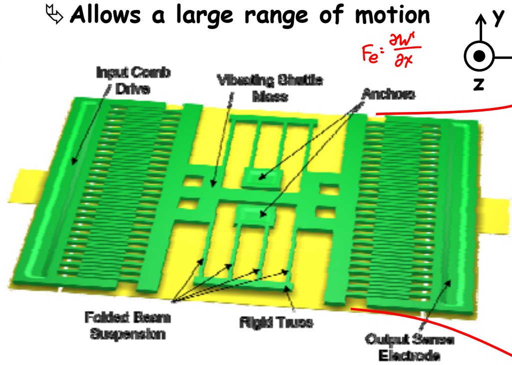
- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_o)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

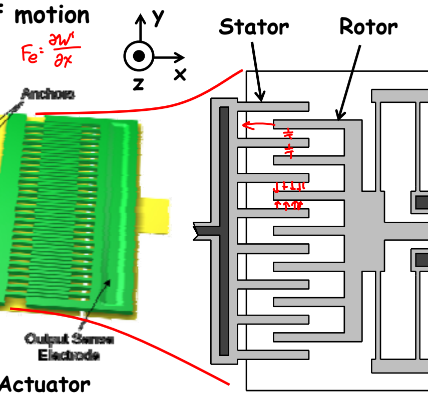


Electrostatic Comb Drive

Electrostatic Comb Drive

- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion





Comb-Driven Folded Beam Actuator