Lecture 23-24: Comb Drive & Equivalent Circuits II

- **Announcements:**
- Reminder: 2nd project slide due this Friday
- Module 13 now online
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- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- ------------------------------------
- Equivalent Circuits II Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- **Last Time:**
- Finished Electrical Stiffness
- Just started comb-drive

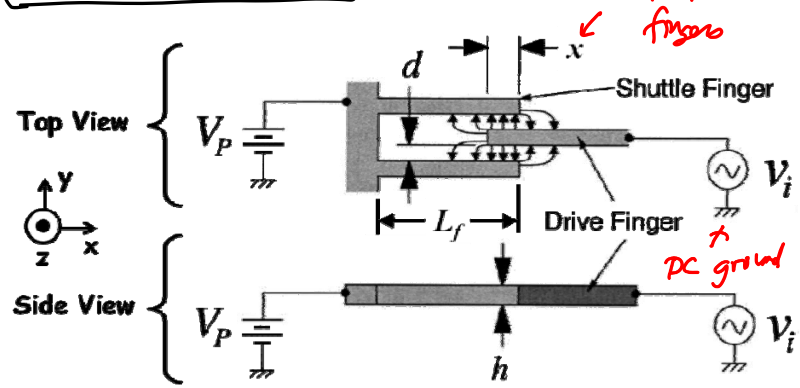
Project Remarks

Parameter	SOA (before)	After Scaling
Power Cons.	50W	1mW

- Be able to create a table like the above comparing state-of-the-art (SOA) performance with scaled performance
- It would be nice to show at least 10x better performance

- Go through slide 35 in Module 12, looking at the advantages of comb-drive

Electrostatic Comb Drive



$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$ Need $C(x)$.

$$\left[C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$

linear! No dependence on x! (ideally)

$\therefore$ no electrical stiffness k_e!

$$F_d = \frac{1}{\cancel{2}}\frac{\cancel{2}\epsilon_0 h}{d}(\cancel{V_P^2} - 2V_P v_i + \cancel{v_i^2})$$

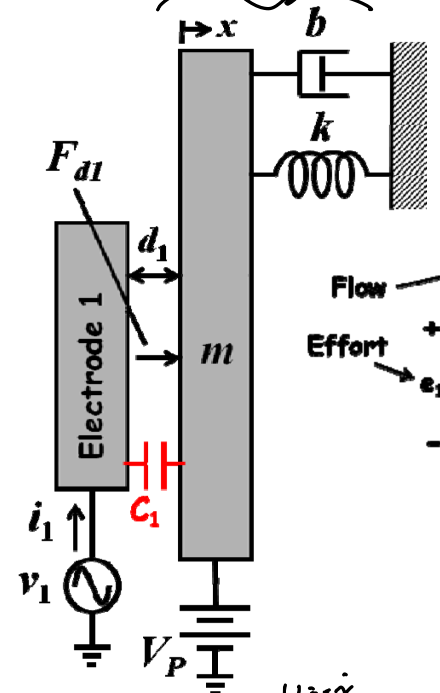
Can balance out by symmetrically placed electrodes

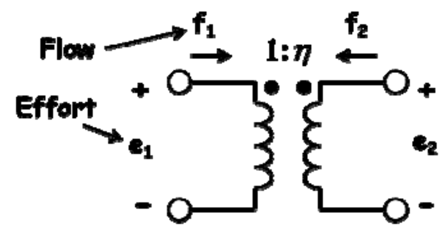
$v_i \ll V_P$ also

$$\boxed{F_d = -2V_P \frac{\epsilon_0 h}{d} v_i}$$

- Go through the rest of Module 12, starting from slide 38
- Start Module 13 and go through slides 1-9

Input Electrical Equiv. Ckt.

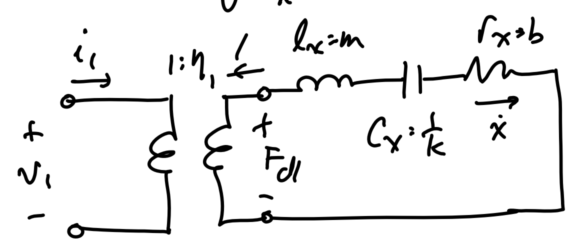




$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

$e_2 = \eta e_1$



$$\underbrace{F_{d1}}_{e_2} = -\underbrace{V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

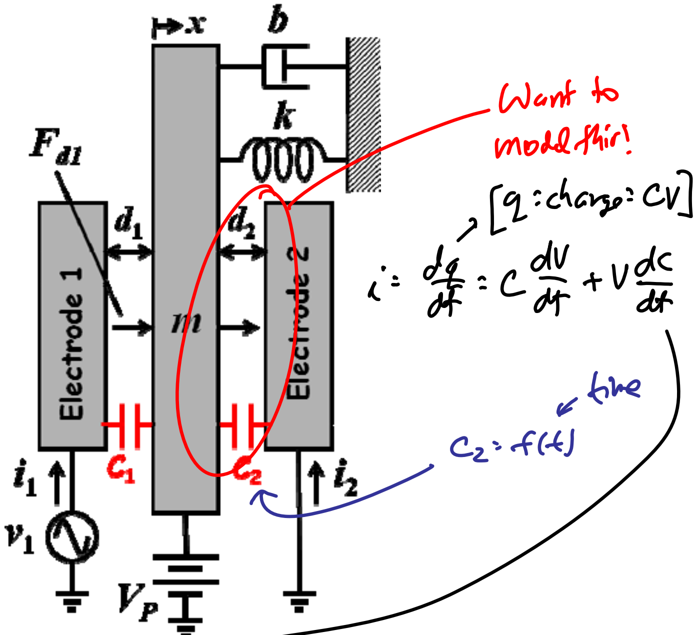
$\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|$

$\frac{\epsilon_0 A}{d_0^2}$

$d \downarrow$ gap $\downarrow \rightarrow \eta_1 \uparrow\uparrow$

$V_P \uparrow \rightarrow \eta_1 \uparrow$

Output Current Into Ground



[q : charge = CV]

$$i = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt}$$

$$i_2 = \cancel{C_2(x,t)\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

0

$$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form:

$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

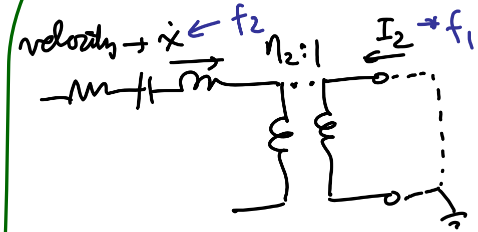
motional current

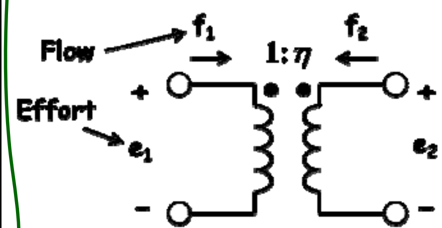
proportional to displacement!

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X \rightarrow -V_P\frac{\partial C}{\partial x}\dot{x}$$

velocity

90° phase lag (+) (+) $\rightarrow I_2 = (-)$ when $x = (+)$





$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

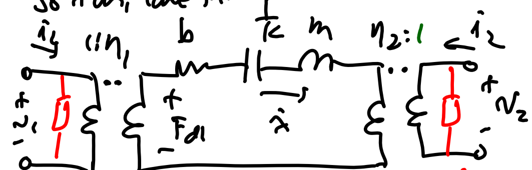
Describing Matrix

$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

$$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$$

$$\therefore \eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|$$

So far, have this:



... but not done ... → need to model parasitics