

Lecture 25: Gyroscopes & Sensing Circuits

- **Announcements:** Modules 14 & 15 online
- What if this course were offered spring, not fall?
- Reminder: 3rd project slide due Nov. 30
- HW#7 online and due Thursday, Dec. 6
- Project Outbrief Signups: pick two days
 - ↳ Tuesday, Dec. 11, afternoon? Okay.
 - ↳ Wednesday, Dec. 12? Okay.

Reading: Senturia, Chpt. 6, Chpt. 14

Lecture Topics:

↳ **Input Modeling**

- Force-to-Velocity Equiv. Ckt.
- Input Equivalent Ckt.

↳ **Current Modeling**

- Output Current Into Ground
- Input Current
- Complete Electrical-Port Equiv. Ckt.

↳ **Impedance & Transfer Functions**

Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21

Lecture Topics:

↳ **Gyroscopes**

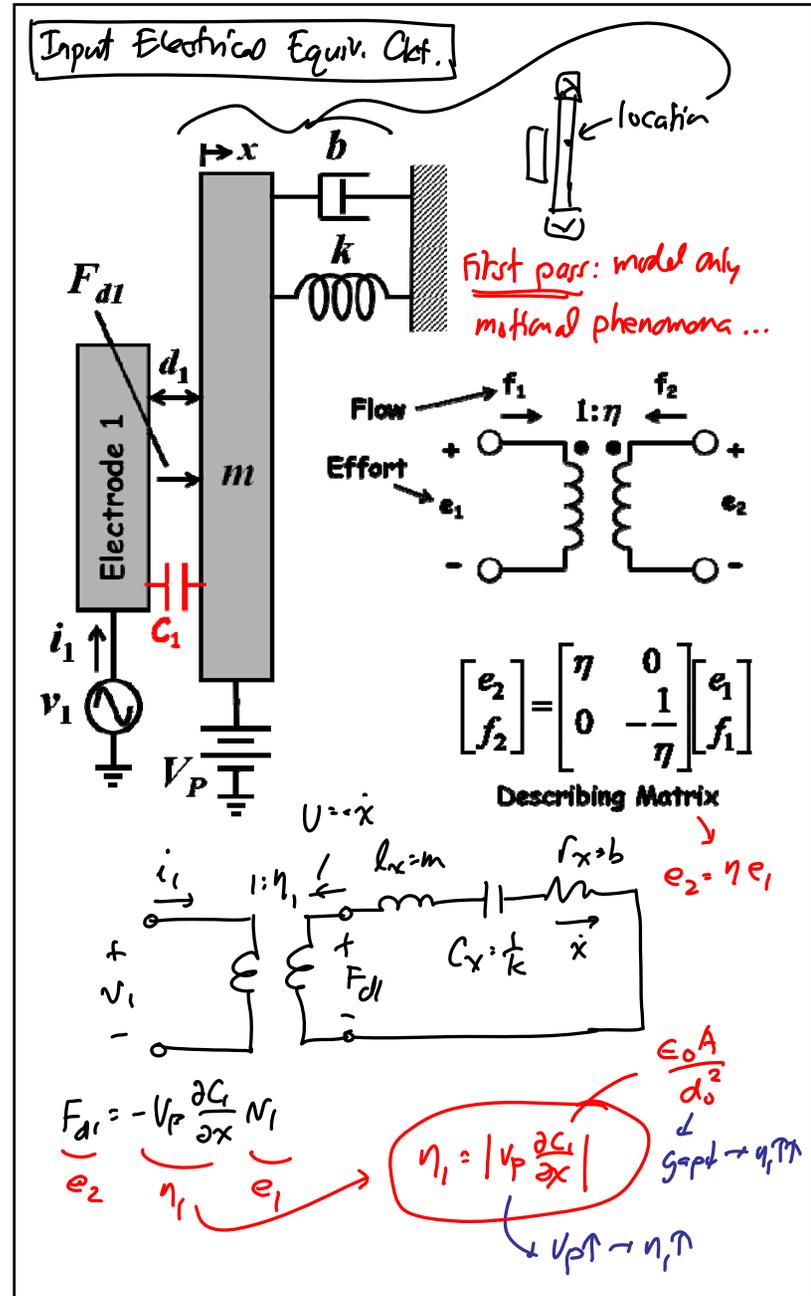
Reading: Senturia, Chpt. 14

Lecture Topics:

↳ **Detection Circuits**

- Velocity Sensing
- Position Sensing

Last Time:



Output Current Into Ground

Want to model this!

$i = \frac{dq}{dt} = C \frac{dV}{dt} + V \frac{dC}{dt}$ [q: charge = CV]

$C_2 = f(t)$

In phasor form:

$$I_2(j\omega) = -j\omega V_p \frac{\partial C_2}{\partial x} \Delta x$$

↑ motion current ↑ proportional to displacement!

$I_2(j\omega) = -j\omega V_p \frac{\partial C_2}{\partial x} \Delta x \rightarrow -V_p \frac{\partial C}{\partial x} \dot{x}$

90° phase lag (+) (+) → $I_2 = (-)$ when $x = (+)$

Flow → f_1 ← f_2

Effort + e_1 + e_2

Describing Matrix

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

$f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2$

$[f_1 = I_2, f_2 = \dot{x}] \Rightarrow I_2 = -\eta \dot{x}$

$\therefore \eta_2 = |V_p \frac{\partial C_2}{\partial x}|$

So far, here's it:

... but not done... → need to model parasitics

Input Current Expression

$b = \frac{k}{\omega_0 Q} = \frac{\sqrt{km}}{Q}$
 $\omega_0 Q = b$

Due to mass motion

Get $I_i(j\omega)$:

$$i_i(t) = C_i(x,t) \frac{dV_i(t)}{dt} + V_i(t) \frac{dC_i(x,t)}{dt}$$

$$[V_i(t) = v_i - v_p] \Rightarrow i_i = C_i \frac{dv_i}{dt} + (v_i - v_p) \frac{\partial C_i}{\partial x} \frac{\partial x}{\partial t}$$

$v_p \gg v_i \Rightarrow$

$$\therefore I_i(j\omega) = \underbrace{j\omega C_i v_i}_{\text{feedthrough current}} + \underbrace{j\omega v_i \frac{\partial C_i}{\partial x} x - j\omega v_p \frac{\partial C_i}{\partial x} x}_{\text{motion current}}$$

@DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k} V_p \left(\frac{\partial C_i}{\partial x} \right) v_i$

@resonance: $x = \frac{Q F_{d1}}{jk} = \frac{-Q}{jk} V_p \frac{\partial C_i}{\partial x} v_i = X$

Thus, @ resonance

$$I_i(j\omega) = j\omega_0 C_i v_i + j\omega_0 \left(V_p \frac{\partial C_i}{\partial x} \right) \frac{2Q}{jk} v_i$$

$$= j\omega_0 C_i v_i + \omega_0 \frac{Q}{k} \eta_{er} v_i = j\omega_0 C_i v_i + \frac{\eta_{er}^2}{b} v_i$$

feedthrough current
 motion current
 $|I_i(j\omega)|$
 i_x
 ω_0
 ω
 90° phase-shifted from v_i
 in phase w/ v_i

This is a capacitor in shunt w/ the input:

This is an effective resistance seen "looking into electrodes" @ ω_0

cl. @ resonance
 static electrode to shuttle overlap capacitance

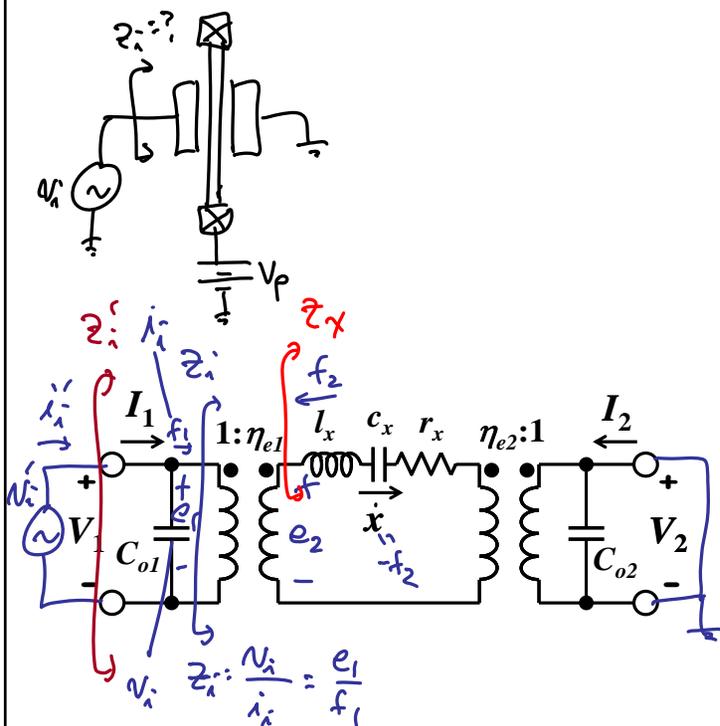
What is the value of the Motional Resistance?

$$R_{xl} = \frac{V_1}{I_1} = \frac{k}{\omega_0 Q \eta_{e1}^2} = \frac{m \omega_0}{Q \eta_{e1}^2} = \frac{b}{\eta_{e1}^2} = R_{xl}$$

"motion"

$$R_{xl} = \frac{\sqrt{k m}}{Q} \frac{1}{\eta_{e1}^2}$$

Input Impedance Into Port 1

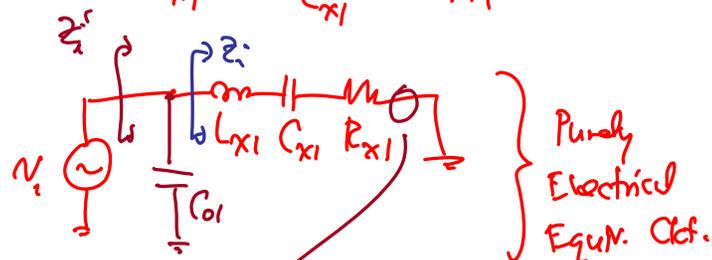


$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{aligned} e_2 &= \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 &= -\frac{1}{\eta} f_1 \rightarrow f_1 = \eta f_2 \end{aligned}$$

$$\frac{e_1}{f_1} = \frac{e_2}{\eta} \left(\frac{1}{-\eta f_2} \right) = -\frac{1}{\eta^2} \frac{e_2}{f_2} \rightarrow \frac{V_1}{i_1} = z_i = -\frac{1}{\eta_{e1}^2} \frac{F d_2}{(-\dot{x})}$$

$$\therefore z_i = \frac{1}{\eta_{e1}^2} z_x$$

$$\begin{aligned} z_i &= \frac{1}{\eta_{e1}^2} (j\omega L_x + \frac{1}{j\omega C_x} + r_x) \\ &= j\omega \underbrace{\frac{L_x}{\eta_{e1}^2}}_{L_{x1}} + \frac{1}{j\omega \underbrace{(\eta_{e1}^2 C_x)}_{C_{x1}}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}} \end{aligned}$$



Can feed into SPICE!

$$Q^2 = 4kTR_{x1} \rightarrow \text{model noise completely!}$$

X-former Impedance Analysis

Input Impedance Into Port 2 (w/ Port 1 Grounded)

$$z_i = \frac{V_1}{I_1} = \frac{z_x}{n^2} = j\omega \left(\frac{L_x}{n^2} \right) + \frac{1}{j\omega \left(n^2 C_x \right)} + \frac{r_x}{n^2}$$

$$L_{x2} \quad C_{x2} \quad R_{x2}$$

Port 1 to Port 2 Transconductance

$G = \frac{i_o}{V_i}$

$$\dot{x} = \frac{1}{n_1} i_i$$

$$i_o = n_2 \dot{x} \Rightarrow i_o = \frac{n_2}{n_1} i_i = \frac{n_2}{n_1} \left(\frac{V_i}{Z_i} \right)$$

$$= \frac{n_2}{n_1} V_i \left[\frac{1}{j\omega L_x + \frac{1}{j\omega C_x} + r_x} \right]$$

$$\therefore \frac{i_o}{V_i}(j\omega) = \frac{n_2 n_1}{j\omega L_x + \frac{1}{j\omega C_x} + r_x}$$

$$= \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12} \right]^{-1}$$

$L_{x12} = \frac{R_x}{\eta_1 \eta_2}$, $C_{x12} = \eta_1 \eta_2 C_x$, $R_{x12} = \frac{r_x}{\eta_1 \eta_2}$

Got the Bode plot:

$$\frac{i_o(s)}{v_i(s)} = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s(\frac{1}{L_x})}{s^2 + \frac{1}{L_x C_x} + s(\frac{R_x}{L_x})}$$

$\left[\frac{1}{L_x C_x} = \omega_0^2, Q = \frac{\omega_0 L_x}{R_x} \Rightarrow \frac{R_x}{L_x} = \frac{\omega_0}{Q} \right]$

$$\frac{i_o(s)}{v_i(s)} = \frac{1}{R_x} \frac{s(\frac{\omega_0}{Q})}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2} = \frac{1}{R_x} \textcircled{H}(s)$$

Gain term: $\frac{1}{R_x}$
 Freq. Shaping Term: $\frac{s(\frac{\omega_0}{Q})}{s^2 + s(\frac{\omega_0}{Q}) + \omega_0^2}$
 resonance magnitude: $\frac{1}{R_x}$

Bandpass or BiQuad

$\Delta\omega = \frac{\omega_0}{Q}$ → later on, will just solve everything @ resonance, then multiply by the proper freq. shaping term.

