

EE C245 - ME C218

Introduction to MEMS Design


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Lecture Module 12: Capacitive Transducers

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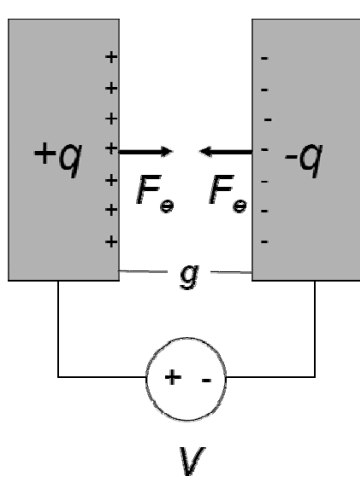
Lecture Outline

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
 - ↗ Energy Conserving Transducers
 - Charge Control
 - Voltage Control
 - ↗ Parallel-Plate Capacitive Transducers
 - Linearizing Capacitive Actuators
 - Electrical Stiffness
 - ↗ Electrostatic Comb-Drive
 - 1st Order Analysis
 - 2nd Order Analysis

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Basic Physics of Electrostatic Actuation

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- **Goal:** Determine gap spacing g as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
 - ↪ Change the charge q
 - ↪ Change the separation g

$$\Delta W(q, g) = V \Delta q + F_e \Delta g$$

$$dW = V dq + F_e dg$$

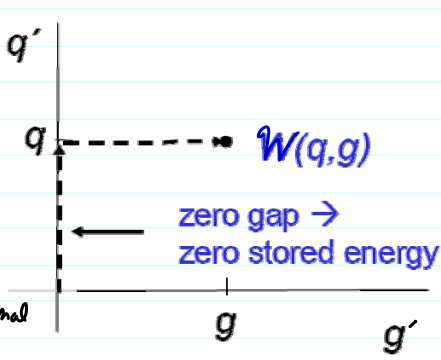
- **Note:** We assume that the plates are supported elastically, so they don't collapse

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Stored Energy

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• Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap



$W = 0 + \int_0^g F_e dg'$

No change in charge: $dq = 0$

$F_e = \left(\frac{q}{2}\right) \epsilon = \frac{1}{2} \frac{q^2}{\epsilon A}$
(independent of g)

$\therefore W = \int_0^g F_e dg' = F_e g \Big|_0^g = F_e g$

$\therefore W(g) = \frac{1}{2} \frac{q^2}{\epsilon A} g$

For a capacitor C :
 $q = CV \rightarrow V = \frac{q}{C}$

$\therefore W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq$
 $= \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{q^2}{\epsilon A} g = W(g)$

Work done to charge C to q at fixed gap

Same answer when

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Charge-Control Case

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- Having found stored energy, we can now find the force acting on the plates and the voltage across them:

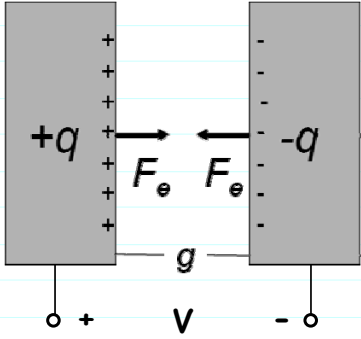
From $dW = Vdq + F_e dg$:

⇒ Force is given by:

$$F_e = \left. \frac{\partial W(q, g)}{\partial g} \right|_{q=\text{const.}} = \frac{\partial}{\partial g} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right)$$

∴ $F_e = \frac{1}{2} \frac{q^2}{\epsilon A} \Rightarrow$ independent of gap spacing!

⇒ Voltage is given by:

$$V = \left. \frac{\partial W(q, g)}{\partial q} \right|_{g=\text{const.}} = \frac{\partial}{\partial q} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right) = \frac{qg}{\epsilon A} \therefore V = \frac{q}{C} \Rightarrow \text{consistent w/ what we already know} \checkmark$$


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Voltage-Control Case

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- Practical situation: We control V
 - Charge control on the typical sub-pF MEMS actuation capacitor is difficult
 - Need to find F_e as a partial derivative of the stored energy $W = W(V, g)$ with respect to g with V held constant? But can't do this with present $W(q, g)$ formula
 - Solution:** Apply Legendre transformation and define the co-energy $W'(V, g)$

Effort (e.g., force, voltage, ...)

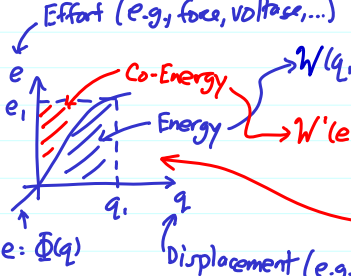
Co-Energy $W'(e) = \int_0^{e_1} q de = \int_0^{q_1} \Phi(q) dq$

Energy $W(q) = \int_0^q e dq = \int_0^{e_1} \Phi^{-1}(e) de$


Displacement (e.g., displacement, charge)

⇒ Can define co-energy as: $W'(e) = eq - W(q)$ (from this plot)

For a linear system, these will be equal.



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Co-Energy Formulation

- For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

$$W'(V, g) = qV - W(q, g)$$

Differentially, this becomes:

$$dW'(V, g) = (q dV + V dq) - dW(q, g)$$

But $dW(q, g) = F_e dg + V dq$

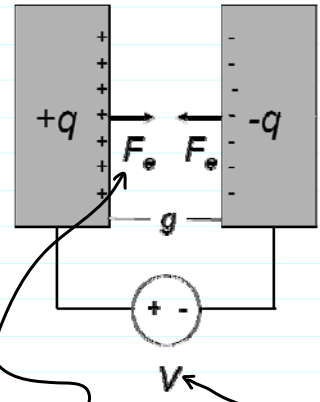
$dW'(V, g) = q dV - F_e dg$

Working Co-Energy Expression


From which:

Charge, $Q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_{g = \text{const.}}$

Force, $F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_{V = \text{const.}}$ ⇒ this gives force as a function of applied voltage



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Electrostatic Force (Voltage Control)

- Find co-energy in terms of voltage (w/ gap held constant)

$$W' = \int_0^V q(g, V') dV' = \int_0^V \left(\epsilon \frac{A}{g} \right) V' dV' = \frac{1}{2} \left(\frac{\epsilon A}{g} \right) V^2 = \frac{1}{2} CV^2$$

(as expected)

- Variation of co-energy with respect to gap yields electrostatic force:

$$F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = - \frac{1}{2} \left(- \frac{\epsilon A}{g^2} \right) V^2 = \frac{1}{2} \frac{C}{g} V^2$$

strong function of gap!

- Variation of co-energy with respect to voltage yields charge:

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = \left(\frac{\epsilon A}{g} \right) V = CV \quad \text{as expected}$$

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Spring-Suspended Capacitive Plate

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Charge Control of a Spring-Suspended C

Force generated by charge q supplied by current I :

$$F_e = \left. \frac{\partial W(q, g)}{\partial g} \right|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring: $F_{\text{spring}} = kz = F_e$ (@ equilibrium)

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k} = g$$

initial gap

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \frac{q}{\epsilon A} \left(g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k} \right) = V \Rightarrow V \downarrow \text{ as } g \downarrow$$

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Voltage Control of a Spring-Suspended C

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Again, $F_{\text{spring}} = kz = F_e$
But now:

$$F_e = \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = \frac{1}{2} \frac{\epsilon A}{g^2} V^2$$

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{\epsilon A V^2}{g^2 k} = g}$$

cubic nonlinearity in g!

Charge: (for a stable gap)

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = CV = \boxed{\frac{\epsilon A}{g} V = q}$$

Feedback!
If $V \uparrow \rightarrow g \downarrow \rightarrow F_e \uparrow$
If loop gain > 1 , then this will go unstable!

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Stability Analysis

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- Net attractive force on the plate:

$$F_{\text{net}} = \underbrace{\frac{\epsilon A V^2}{2g^2}}_{F_e} - \underbrace{k(g_0 - g)}_{F_{\text{spring}}}$$

- An increment in gap dg leads to an increment in net attractive force dF_{net}

$$dF_{\text{net}} = \frac{\partial F_{\text{net}}}{\partial g} dg = \left[-\frac{\epsilon A V^2}{g^3} + k \right] dg$$

$F_{\text{net}} \downarrow \rightarrow dF_{\text{net}} = (-)$


If $g \downarrow \rightarrow dg = (-)$, then for stability need \rightarrow otherwise, plate collapses

Thus, need this \Rightarrow

$$\boxed{k > \frac{\epsilon A V^2}{g^3}}$$

(for a stable uncollapsed state)

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Pull-In Voltage V_{PI}

- V_{PI} = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\epsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - k(g_o - g_{PI}) \quad (2)$$

\nwarrow pull-in gap

- Substituting (1) into (2):

$$0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - \frac{\epsilon A V_{PI}^2}{g_{PI}^3} (g_o - g_{PI})$$

$$\frac{g_o - g_{PI}}{g_{PI}} = \frac{1}{2} \rightarrow g_o = \frac{3}{2} g_{PI}$$

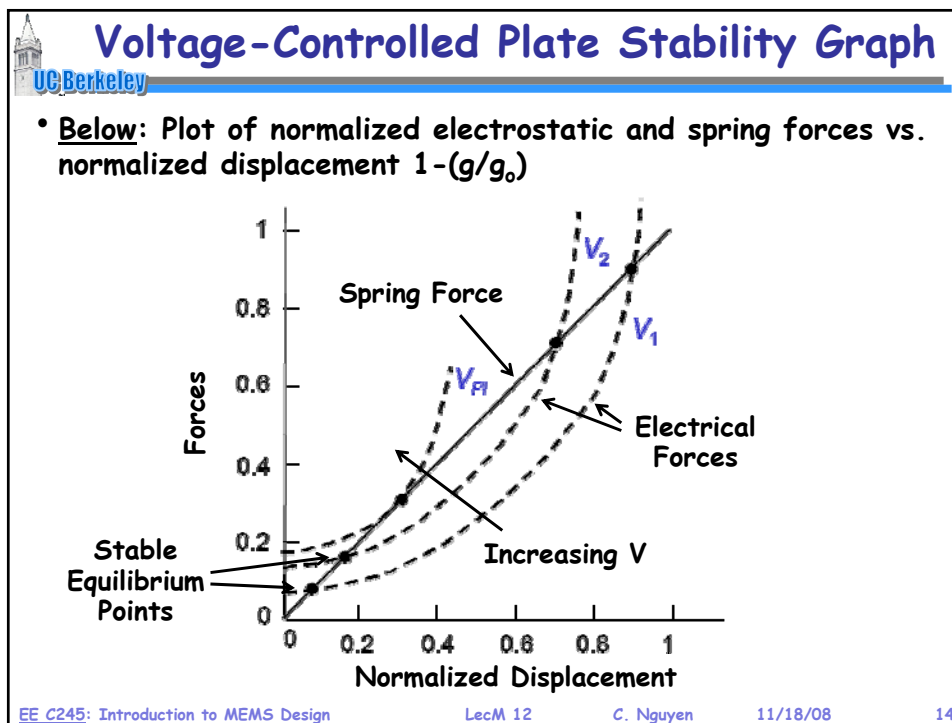
$$\therefore g_{PI} = \frac{2}{3} g_o$$


$$V_{PI} = \sqrt{\frac{k g_{PI}^3}{\epsilon A}}$$

$$\therefore V_{PI} = \sqrt{\frac{8}{27} \frac{k g_o^3}{\epsilon A}}$$

When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!

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Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through I^2R losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures


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Problems With Electrostatic Actuators


- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

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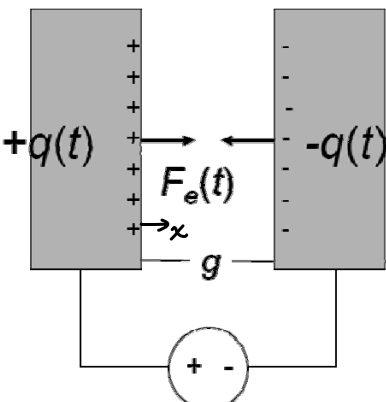
Linearizing the Voltage-to-Force Transfer Function

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Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage V_p together with the intended input (or drive) voltage $v_i(t)$, where $V_p \gg v_i(t)$



$$F_e(t) = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x} \left(\frac{1}{2} C [v(t)]^2 \right)$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} [v(t)]^2 = \frac{1}{2} (V_p + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} [V_p^2 + 2V_p v_i(t) + [v_i(t)]^2] \frac{\partial C}{\partial x}$$

$$[V_p \gg v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2} V_p^2 \frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_p \frac{\partial C}{\partial x} v_i(t)}_{\text{AC drive signal}}$$


$$C(x) = \frac{\epsilon A}{g_0 - x} = C_0 \left(1 - \frac{x}{g_0} \right)^{-1} \approx C_0 \left(1 + \frac{x}{g_0} \right)$$

$[x \ll g_0] \quad \sim \text{const.} \therefore \text{linear}$

$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_0}{g_0} \Rightarrow F_e(t) = \frac{1}{2} \frac{C_0}{g_0} V_p^2 + V_p \frac{C_0}{g_0} v_i(t)$$

$v(t) = V_p + v_i(t)$

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Differential Capacitive Transducer

• The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$

Do the math.

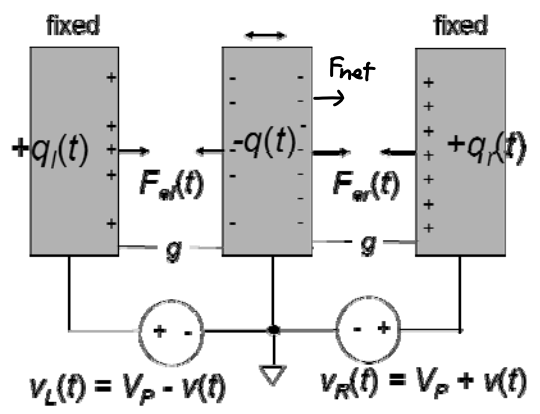
Assume matched gaps.

$$F_{net}(t) = \frac{1}{2} \frac{\partial C}{\partial x} \{ [V_R(t)]^2 - [V_L(t)]^2 \}$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} \{ (V_p^2 + 2V_p v(t) + [v(t)]^2) - (V_p^2 - 2V_p v(t) + [v(t)]^2) \}$$

$\therefore F_{net}(t) = 2V_p \frac{\partial C}{\partial x} v(t) = 2V_p \frac{C_0}{g_0} v(t)$

\Rightarrow Linear w/ $v(t)$!
(gap match limited)




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Remaining Nonlinearity (Electrical Stiffness)

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Parallel-Plate Capacitive Nonlinearity

• **Example:** clamped-clamped laterally driven beam with balanced electrodes

• **Nomenclature:**

V_a or v_A $v_a = |v_a| \cos \omega t$

V_A

V_a or $v_A = V_A + v_a$

Total Value AC or Signal Component (lower case variable; lower case subscript)

DC Component (upper case variable; upper case subscript)

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Parallel-Plate Capacitive Nonlinearity

• **Example:** clamped-clamped laterally driven beam with balanced electrodes

• **Expression for $\partial C / \partial x$:**

$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_0 \left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + \frac{x}{d_1}\right)^{-2}$

[Expand the Taylor series further]

$\frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \dots\right)$

where $A_1 = -\frac{2}{d_1}$

$A_2 = \frac{3}{d_1^2}$

$A_3 = -\frac{4}{d_1^3}$

\vdots

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Parallel-Plate Capacitive Nonlinearity

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- Thus, the expression for force from the left side becomes:

$$F_{d1} = \frac{1}{2} \frac{\partial C}{\partial x} (V_P - V_1 - N_i)^2 = \frac{1}{2} \frac{\partial C}{\partial x} (V_{P1} - N_i)^2$$

[small displacements: $x \ll d_1$]

$$F_{d1} = \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) (1 + A_1 x) (V_{P1}^2 - 2V_{P1}N_i + N_i^2)$$

$$= \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) \{ V_{P1}^2 - 2V_{P1}N_i + N_i^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x N_i + A_1 x N_i^2 \}$$

@ resonance: $x = \frac{Q F_{d1}}{j k} \approx \frac{Q}{j k} \frac{\partial C}{\partial x} V_{P1} N_i$

Thus:

$$N_i = |N_i| \cos \omega_o t \rightarrow x = |x| \sin \omega_o t$$

↑
90° phase-shifted from N_i

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Parallel-Plate Capacitive Nonlinearity

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- Retaining only terms at the drive frequency:

$$F_{d1}|_{\omega_o} = V_{P1} \frac{C_{o1}}{d_1} |v_1| \cos \omega_o t + V_{P1}^2 \frac{C_{o1}}{d_1^2} |x| \sin \omega_o t$$

Drive force arising from the input excitation voltage at the frequency of this voltage


Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
 - A negative spring constant
 - Since it derives from V_P , we call it the electrical stiffness, given by:

$$k_e = V_{P1}^2 \frac{C_{o1}}{d_1^2} = V_{P1}^2 \frac{\epsilon A}{d_1^3}$$

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Electrical Stiffness, k_e

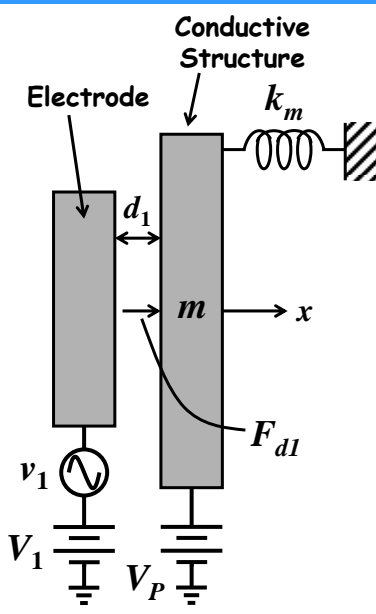
- The electrical stiffness k_e behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$


$$= \sqrt{\frac{k_m}{m} \left(1 - \frac{k_e}{k_m}\right)^{1/2}}$$

$$\omega_o' = \omega_o \left(1 - \frac{V_{P1}^2 \epsilon A}{k_m d_1^3}\right)^{1/2}$$

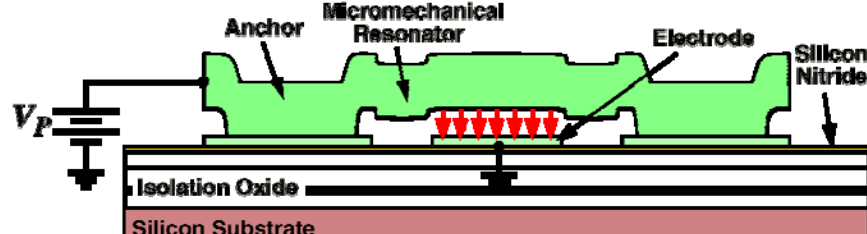
Frequency is now a function of dc-bias V_{P1}



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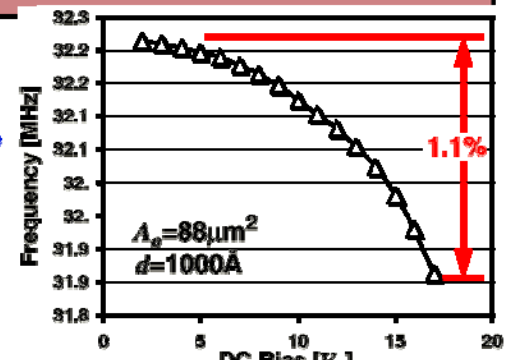
Voltage-Controllable Center Frequency



- Quadrature force \Rightarrow voltage-controllable electrical stiffness:

$$k_e = \frac{\epsilon_o A_o}{d^3} V_P^2$$

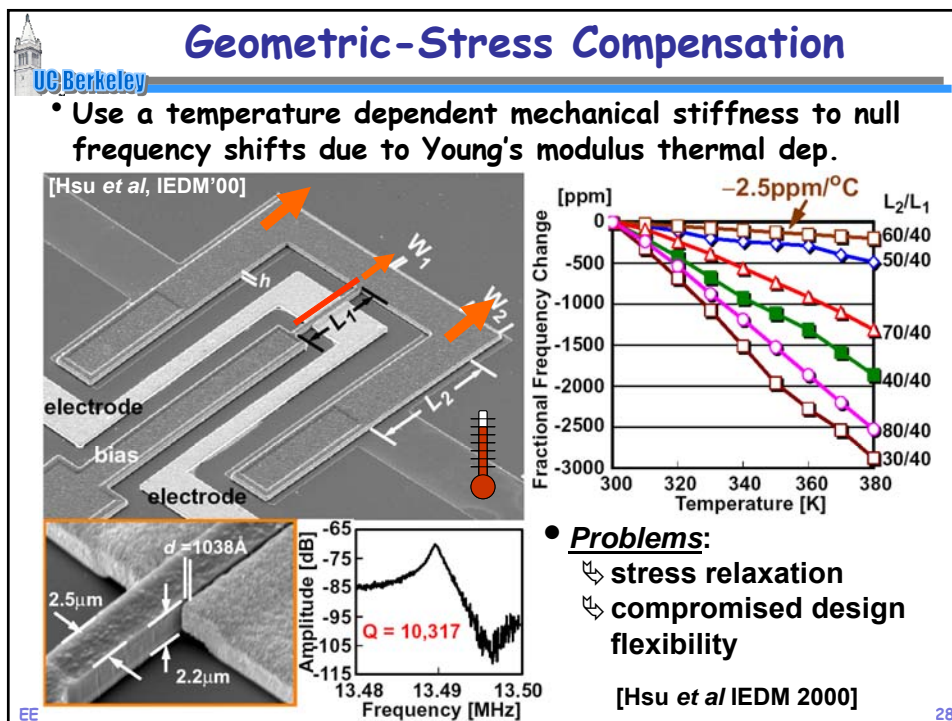
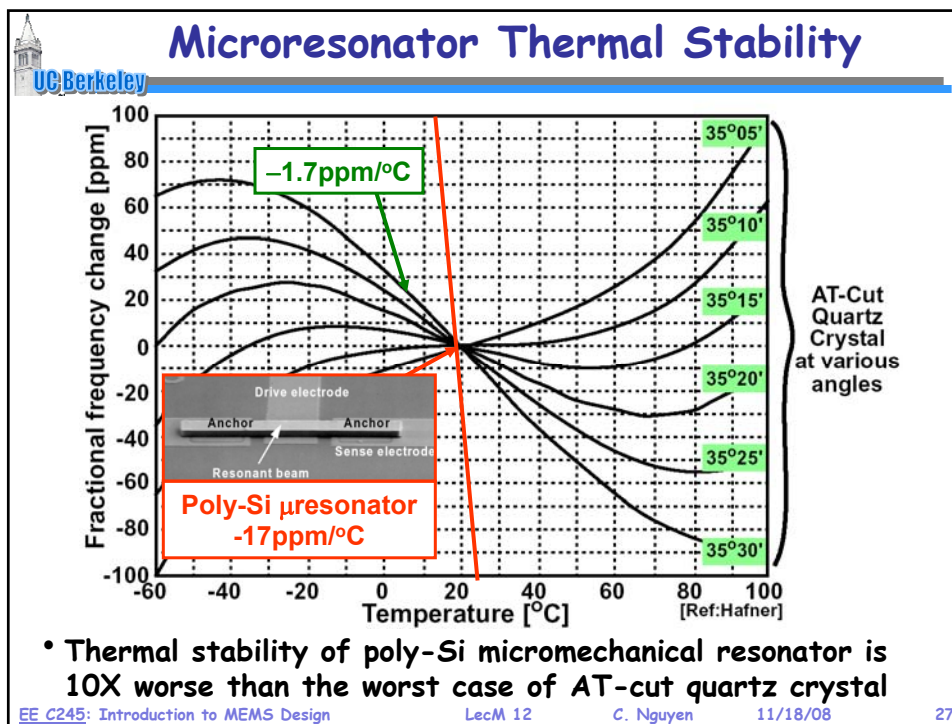
Electrode Overlap Area A_o
Gap d

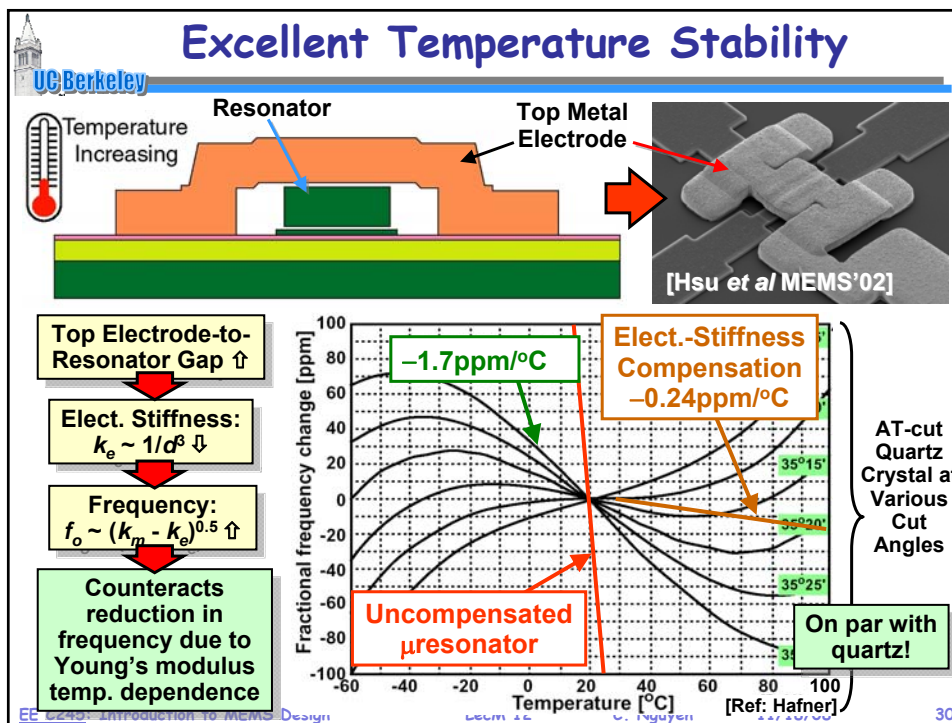
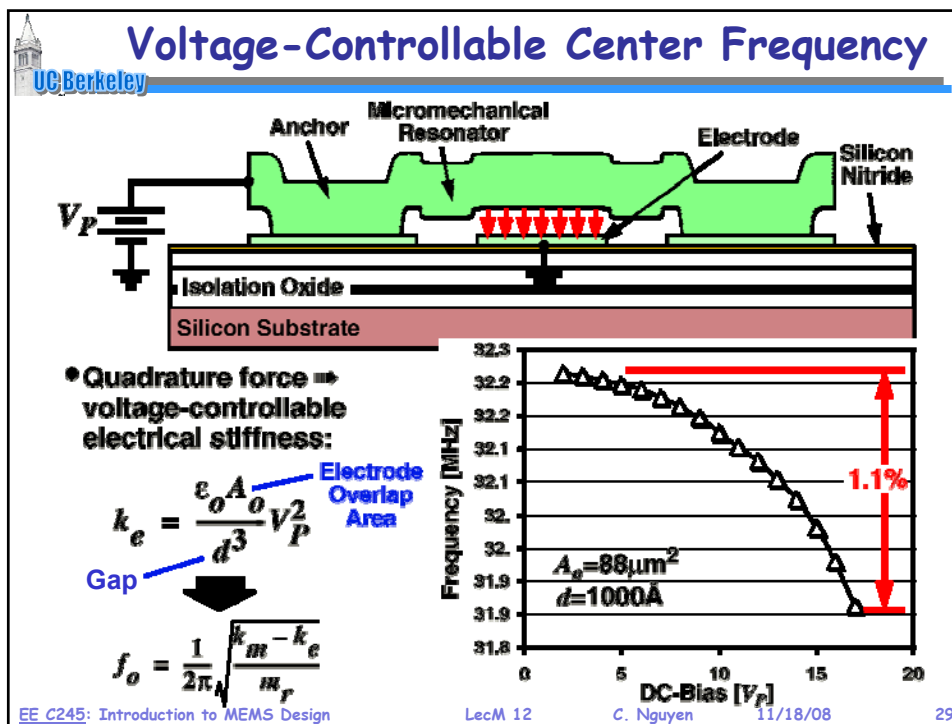
$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_m - k_e}{m_r}}$$


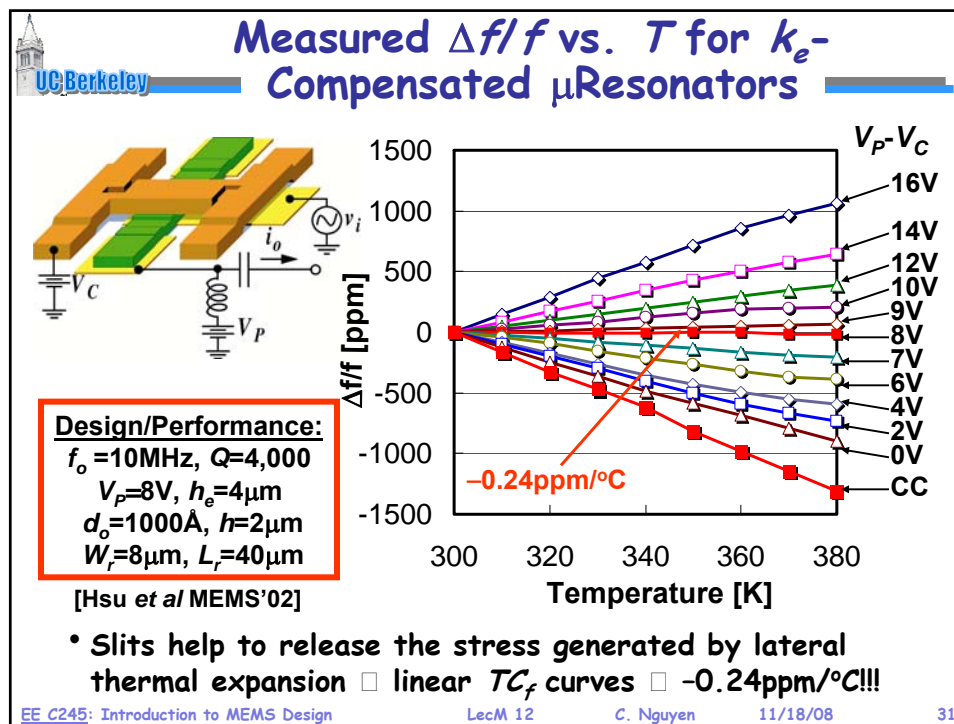
DC-Bias [VP]	Frequency [MHz]
0	32.25
5	32.22
10	32.15
15	32.05
20	31.90

$A_o = 88 \mu\text{m}^2$
 $d = 1000 \text{\AA}$

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Can One Cancel k_e w/ Two Electrodes?


- What if we don't like the dependence of frequency on V_P ?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

Subtracts from the F_{d1} term, as expected

$$F_{d2}|_{\omega_o} = -V_{P2} \frac{C_{o2}}{d_2} |v_2| \cos \omega_o t + V_{P2}^2 \frac{C_{o2}}{d_2^2} |x| \sin \omega_o t$$

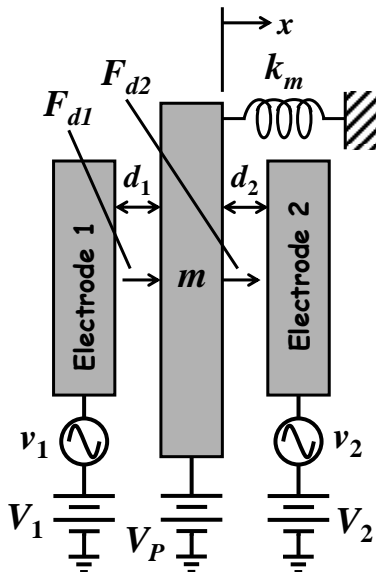
Adds to the quadrature term $\rightarrow k_e$'s add, no matter the electrode configuration!

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Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
 - ↪ Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
 - ↪ Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_0)
 - ↪ Noise can alias due to nonlinearity
- Range of motion is small
 - ↪ For larger motion, need larger gap ... but larger gap weakens the electrostatic force
 - ↪ Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



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Electrostatic Comb Drive

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Electrostatic Comb Drive

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- Use of comb-capacitive transducers brings many benefits
 - ↳ Linearizes voltage-generated input forces
 - ↳ (Ideally) eliminates dependence of frequency on dc-bias
 - ↳ Allows a large range of motion

Comb-Driven Folded Beam Actuator

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Comb-Drive Force Equation (1st Pass)

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Top View

Side View

$$C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \left[\frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$

$$F_d = \frac{\partial W}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} (V_P - V_i)^2 = \frac{2}{2} \frac{\epsilon_0 h}{d} (V_P^2 - 2V_P V_i + V_i^2) \approx -2V_P \frac{\epsilon_0 h}{d} V_i = F_d$$

When $V_i = (+) \rightarrow F_d = (-)$ ✓

↳ But wait! This ignores other practical effects! (No dependence on x ! LINEAR!)

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Lateral Comb-Drive Electrical Stiffness

Top View

Side View

Shuttle Finger

Drive Finger

V_P

V_i

d

x

L_f

h

$\# \text{ drive fingers}$

- Again: $C(x) = \frac{2N\epsilon h x}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\epsilon h}{d}$
- No $(\partial C / \partial x)$ x -dependence \rightarrow no electrical stiffness: $k_e = 0!$
- Frequency immune to changes in V_P or gap spacing!

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Typical Drive & Sense Configuration

2-port Lateral Microresonator

N_f : # shuttle fingers

V_1

V_2

V_P

i_1

i_2

F_{d1}

F_{d2}

Simple Analysis:

$$F_{d1} = \frac{1}{2} \frac{\partial C_1}{\partial x} (V_1 - V_{P1})^2 = \frac{1}{2} \left(-\frac{\epsilon_0 h}{d_1} \right) (V_1^2 - 2V_{P1}V_1 + V_{P1}^2) (2N_f)$$

$$F_{d2} = \frac{1}{2} \frac{\partial C_2}{\partial x} (V_2 - V_{P2})^2 = \frac{1}{2} \left(\frac{\epsilon_0 h}{d_2} \right) (V_2^2 - 2V_{P2}V_2 + V_{P2}^2) (2N_f)$$

For $V_1 = V_2, V_1 = -V_2$

$$F_{net} = 2(2N_f) \left(\frac{\epsilon_0 h}{d} \right) V_{P1} V_1$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left(\frac{\epsilon_0 h}{d} \right) (V_2^2 - V_1^2 - 2(V_{P2}V_2 - V_{P1}V_1) + V_{P2}^2 - V_{P1}^2) (2N_f)$$

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Comb-Drive Force Equation (2nd Pass)

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- In our 1st pass, we accounted for
 - Parallel-plate capacitance between stator and rotor
- ... but neglected:
 - Fringing fields
 - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

The diagram illustrates a comb-drive structure. On the left, a 3D cross-section shows a rotor (green) with fingers of length L and thickness t moving vertically between stator fingers (grey) separated by gap g . The rotor is at position x relative to the stator. On the right, a 2D top-down view shows the rotor and stator fingers on a ground plane at height z_0 . An electrical circuit model below shows a voltage source V_s connected to the stator and a voltage source V_r connected to the rotor, both referenced to ground.

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Comb-Drive Force With Ground Plane Correction

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- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

The diagram shows a comb-drive structure with a ground plane at height z_0 . The rotor (r) and stator (s) are shown with their respective capacitances to the ground plane (p). The circuit model shows three capacitors in series: $\frac{1}{2}C_{sp}$ (stator to ground), C_{rp} (rotor to ground), and $\frac{1}{2}C_{rs}$ (stator to rotor). The rotor is connected to a voltage source V_r and the stator is connected to a voltage source V_s .

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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Capacitance Expressions

- Case: $V_r = V_p = 0V$
- C_{sp} depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

Region 2

Region 3

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

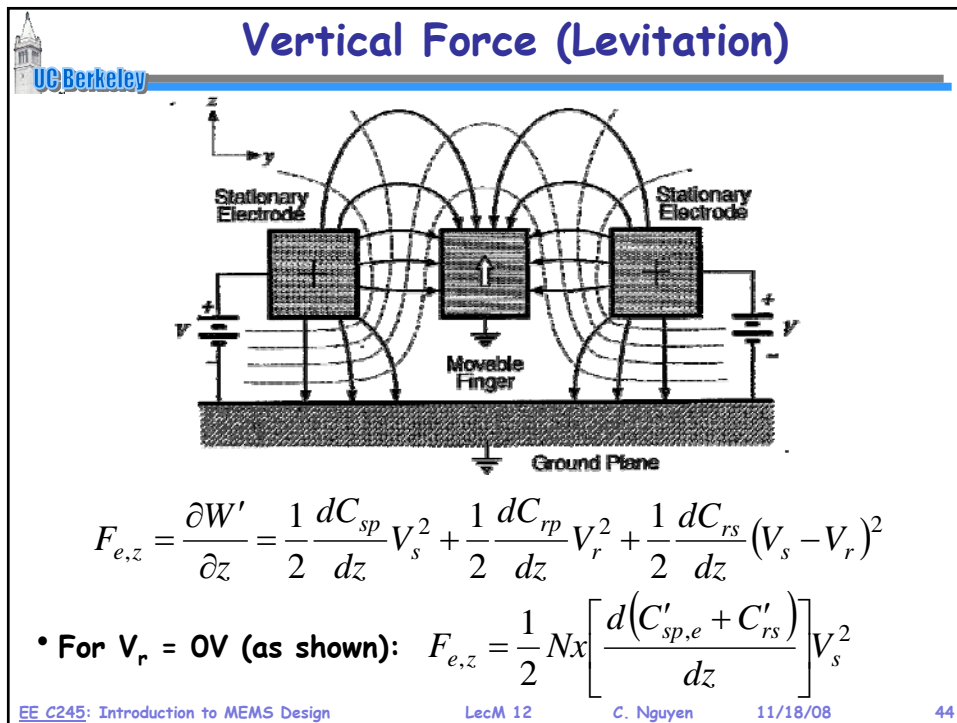
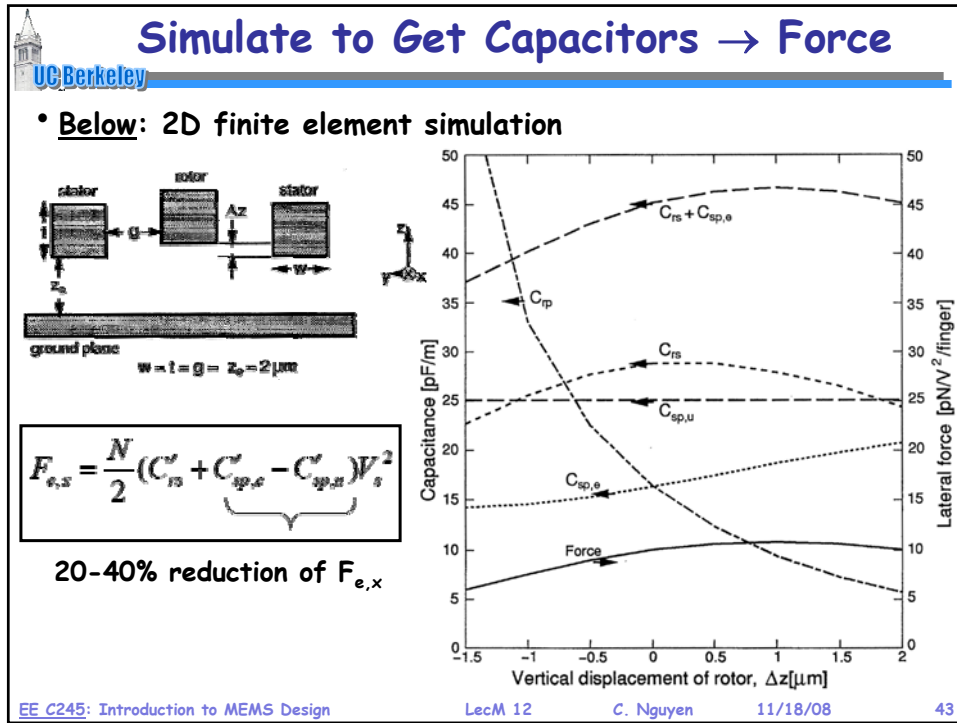
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

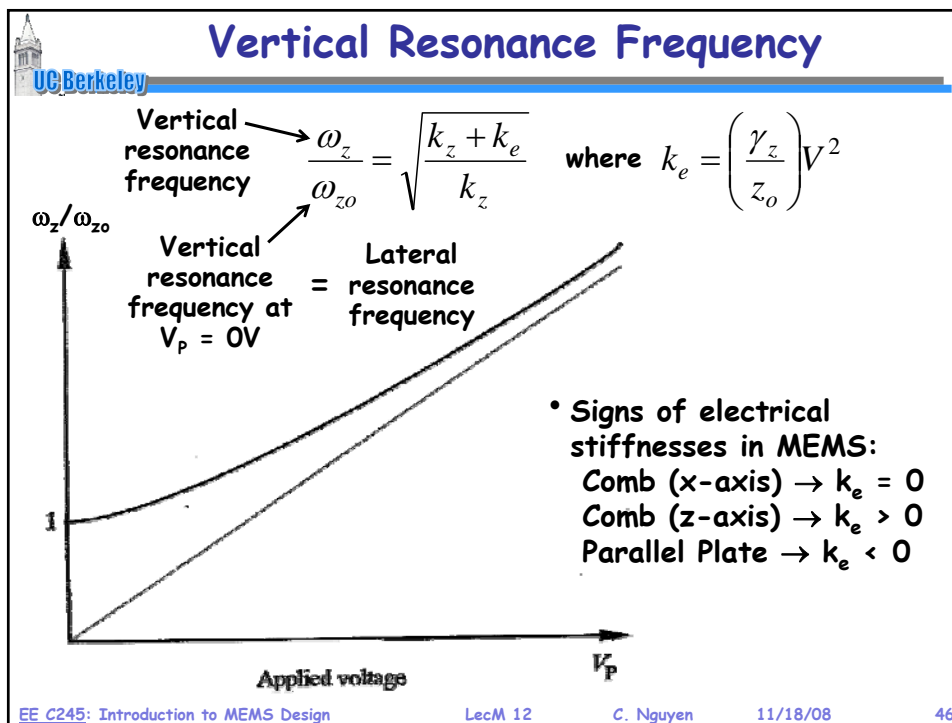
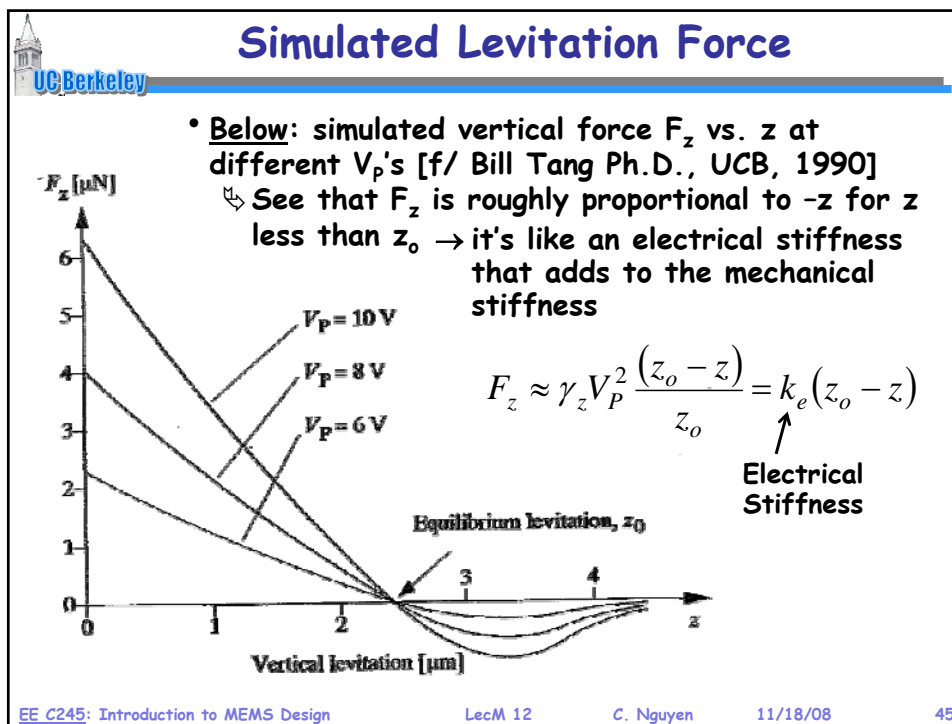
$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,e} - C'_{sp,u}) V_s^2$$

(for $V_r = V_p = 0$)

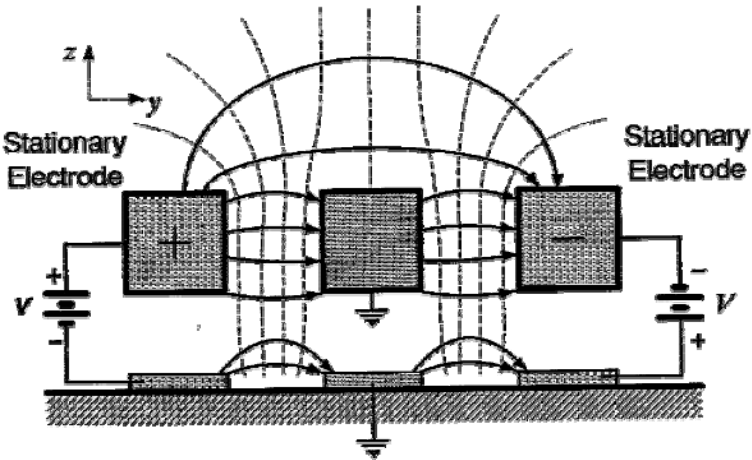
[Gary Fedder, Ph.D., UC Berkeley, 1994]

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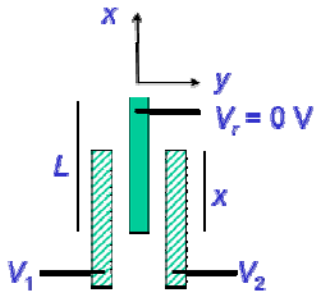
Suppressing Levitation



- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

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Force of Comb-Drive vs. Parallel-Plate



gap = $g = 1 \mu\text{m}$,
thickness = $t = 2 \mu\text{m}$
finger length = $L = 100 \mu\text{m}$
overlap length $x = 75 \mu\text{m}$

- Comb drive (x-direction)
 $\Rightarrow V_1 = V_2 = V_s = 1\text{V}$

$$F_{e,x} = \frac{\epsilon_0 t}{g} V_s^2$$
- Differential Parallel-Plate (y-direction)
 $\Rightarrow V_1 = 0\text{V}, V_2 = 1\text{V}$

$$F_{e,y} = \frac{1}{2} \frac{\epsilon_0 t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\epsilon_0 t x}{g^2} V_2^2}{\frac{2 \epsilon_0 t}{g} V_s^2} = \frac{1}{2} \frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity

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