

EE C245 - ME C218

Introduction to MEMS Design


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Lecture Module 7: Mechanics of Materials


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Outline

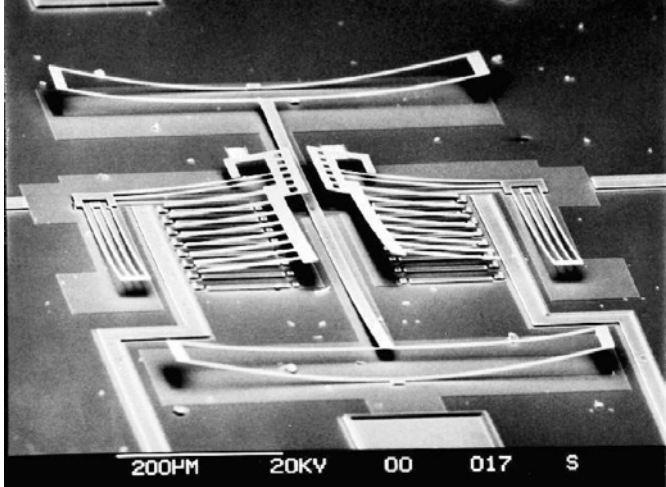
- **Reading: Senturia, Chpt. 8**
- **Lecture Topics:**
 - ↗ **Stress, strain, etc., for isotropic materials**
 - ↗ **Thin films: thermal stress, residual stress, and stress gradients**
 - ↗ **Internal dissipation**
 - ↗ **MEMS material properties and performance metrics**

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Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



200µm 20KV 00 017 S

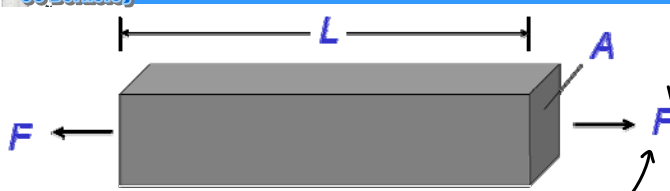
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Elasticity

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Normal Stress (1D)

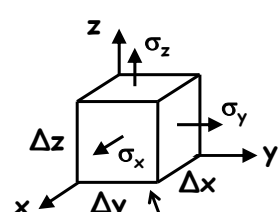


If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

$$\text{Stress} = \left\{ \begin{array}{l} \text{Force per} \\ \text{unit area} \end{array} \right\} = \sigma = \frac{F}{A} \quad [N/m^2 = Pa]$$

↗ standard mks unit



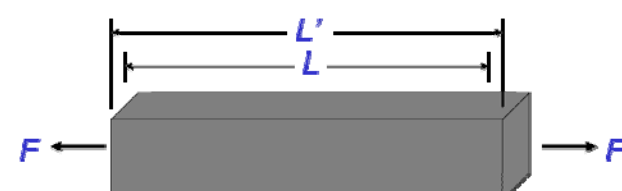
⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

Differential volume element

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Strain (1D)

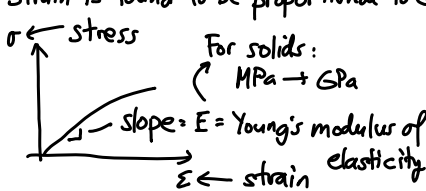


Sometimes a unit called the "microstrain" is used, where

$1 \mu\epsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

$$\text{Strain} = \left\{ \begin{array}{l} \text{Fractional Change} \\ \text{in Length} \end{array} \right\} = \epsilon = \frac{L' - L}{L} = \frac{\Delta L}{L} \quad [\text{unitless}]$$

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress



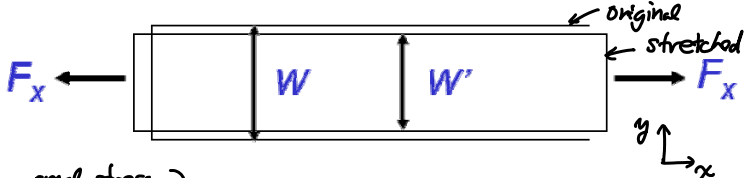
For solids: MPa → GPa

Thus, the units of E are the same as $\sigma \rightarrow Pa$

$$\sigma = E \epsilon \rightarrow \boxed{\epsilon = \frac{\sigma}{E} \quad [\text{unitless}]}$$

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The Poisson Ratio



Apply normal stress to a free-standing object

- uniaxial strain
- but also get contraction in directions transverse to the uniaxial strain

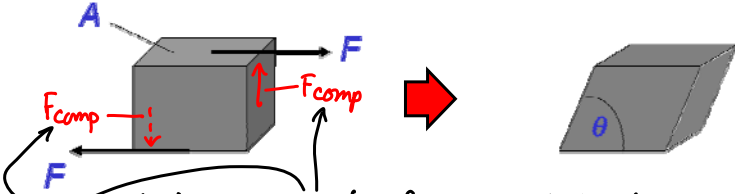
⇒ contraction creates a (-) strain:

$$\epsilon_y = \frac{W' - W}{W} = \frac{\Delta W}{W} = -\nu \epsilon_x$$

ν = Poisson ratio [unitless]
 ↳ typical values: 0 → 0.5
 ⇒ inorganic solids: 0.2 → 0.3
 ⇒ elastomers (e.g., rubber): ~0.5

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Shear Stress & Strain (1D)



Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)


Shear Stress = $\left\{ \begin{array}{l} \text{Force per Unit Area} \\ \text{Parallel to the Surfaces} \end{array} \right\} = \tau = \frac{F}{A} \quad [\text{Pa}]$

↳ Generates a shear strain:

$$\text{Shear Strain} = \theta = \frac{\tau}{G} \leftarrow G \triangleq \text{shear modulus}$$

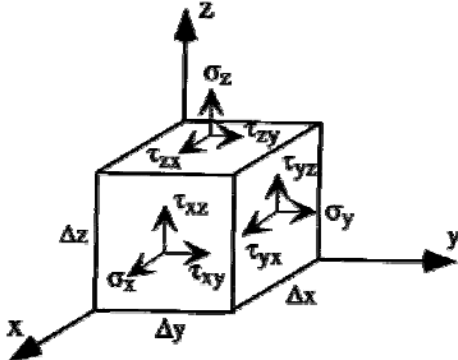
$$G = \frac{E}{2(1+\nu)}$$

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2D and 3D Considerations


- **Important assumption:** the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
 - ↳ Every σ must have an equal σ in the opposite direction on the other side of the element
 - ↳ For no net torque, the shear forces on different faces must also be matched as follows:



Stresses acting on a differential volume element

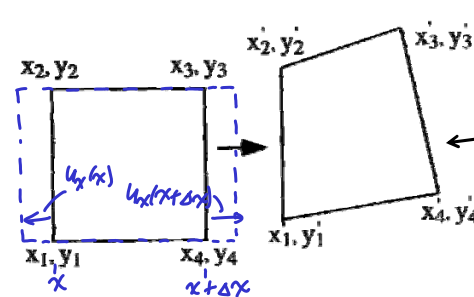
$\tau_{xy} = \tau_{yx}$
 $\tau_{xz} = \tau_{zx}$
 $\tau_{yz} = \tau_{zy}$

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2D Strain

- In general, motion consists of
 - ↳ rigid-body displacement (motion of the center of mass)
 - ↳ rigid-body rotation (rotation about the center of mass)
 - ↳ Deformation relative to displacement and rotation



Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: $\epsilon_x = \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$

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2D Shear Strain

⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation
 ↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x} \right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

↑
For small amplitude deformations.

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Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element

Given an x-directed uniaxial stress, σ_x :

$$\begin{aligned} \Delta x &\rightarrow \Delta x (1 + \epsilon_x) \\ \Delta y &\rightarrow \Delta y (1 - \nu \epsilon_x) \\ \Delta z &\rightarrow \Delta z (1 - \nu \epsilon_x) \end{aligned}$$

↓ The resulting change in volume ΔV

$$\begin{aligned} \Delta V &= \Delta x \Delta y \Delta z (1 + \epsilon_x)(1 - \nu \epsilon_x)^2 - \Delta x \Delta y \Delta z \\ &= \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - \nu \epsilon_x)^2 - 1] \end{aligned}$$


{Assume small strains} $\Rightarrow \Delta V = \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - 2\nu \epsilon_x) - 1]$

$[(1 + m x)^n \approx 1 + n m x] \Rightarrow \approx \Delta x \Delta y \Delta z [1 + \epsilon_x - 2\nu \epsilon_x - 2\nu \epsilon_x^2 - 1]$

$\Delta V \approx \Delta x \Delta y \Delta z (1 - 2\nu) \epsilon_x$

For $\nu = 0.5$ (rubber) \rightarrow no ΔV !
 $\nu < 0.5 \rightarrow$ finite ΔV

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Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\begin{aligned}\epsilon_x &= \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] & \gamma_{xy} &= \frac{1}{G} \tau_{xy} \\ \epsilon_y &= \frac{1}{E} [\sigma_y - \nu(\sigma_z + \sigma_x)] & \gamma_{yz} &= \frac{1}{G} \tau_{yz} \\ \epsilon_z &= \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] & \gamma_{zx} &= \frac{1}{G} \tau_{zx}\end{aligned}$$

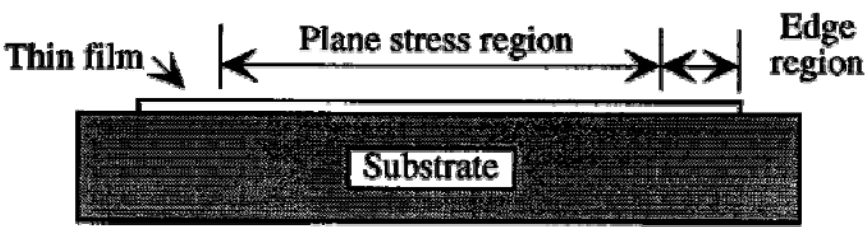
Basically, add in off-axis strains from normal stresses in other directions

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Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)



- At regions more than 3 thicknesses from edges, the top surface is stress-free $\rightarrow \sigma_z = 0$
- Get two components of in-plane stress:

$$\begin{aligned}\epsilon_x &= (1/E)[\sigma_x - \nu(\sigma_y + 0)] \\ \epsilon_y &= (1/E)[\sigma_y - \nu(\sigma_x + 0)]\end{aligned}$$

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Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane $\rightarrow \sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$
where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

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Edge Region of a Tensile ($\sigma > 0$) Film

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero


Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here

Discontinuity of stress at the attached corner \rightarrow stress concentration

Peel forces that can peel the film off the surface

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Linear Thermal Expansion


- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

$$\left. \begin{array}{l} \text{Linear thermal} \\ \text{expansion coefficient} \end{array} \right\} \triangleq \alpha_T = \frac{d\varepsilon_x}{dT} \quad [\text{Kelvin}^{-1}]$$

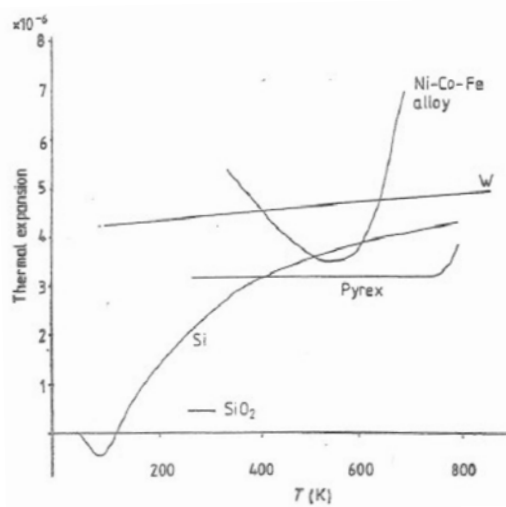
Remarks:

- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of $\mu\text{strain/K}$, where $10^{-6} \text{ K}^{-1} = 1 \mu\text{strain/K}$
- In 3D, get volume thermal expansion coefficient $\longrightarrow \frac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

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α_T As a Function of Temperature



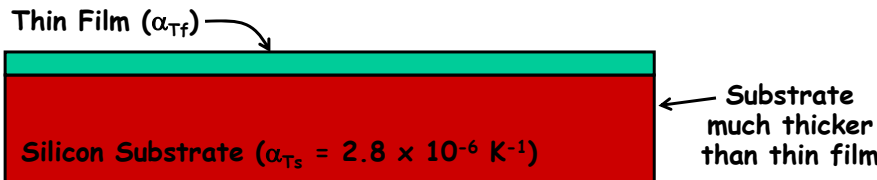
[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

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Thin-Film Thermal Stress




- Assume film is deposited stress-free at a temperature T_r , then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)
 $\epsilon_s = -\alpha_{Ts} \Delta T$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate: $\epsilon_{f, \text{free}} = -\alpha_{Tf} \Delta T$ ↪ over

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Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate.

$$\epsilon_{f, \text{attached}} = -\alpha_{Ts} \Delta T$$

Thus:

$$\text{Thermal Mismatch Strain} = \epsilon_{f, \text{mismatch}} = (\alpha_{Tf} - \alpha_{Ts}) \Delta T$$

↪ Note that this is biaxial strain
 ↪ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f, \text{mismatch}} = \left(\frac{E}{1-\nu} \right) \epsilon_{f, \text{mismatch}}$$


Ex. Thin-film is polyimide → $\alpha_{Tf} = 70 \times 10^{-6} \text{ K}^{-1}$, $E = 46 \text{ GPa}$
 deposited @ 250°C , then cooled to RT: 25°C → $\Delta T = 225 \text{ K}$ e.g., SiO_2

$$\epsilon_{f, \text{mismatch}} = (70 - 2.8) \mu (225) = 1.5 \times 10^{-2}$$

$$\sigma_{f, \text{mismatch}} = (46) (1.5 \times 10^{-2}) = 69 \text{ MPa}$$


↪ stress is (+), ∴ tensile
 [(-) would be compressive]

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MEMS Material Properties

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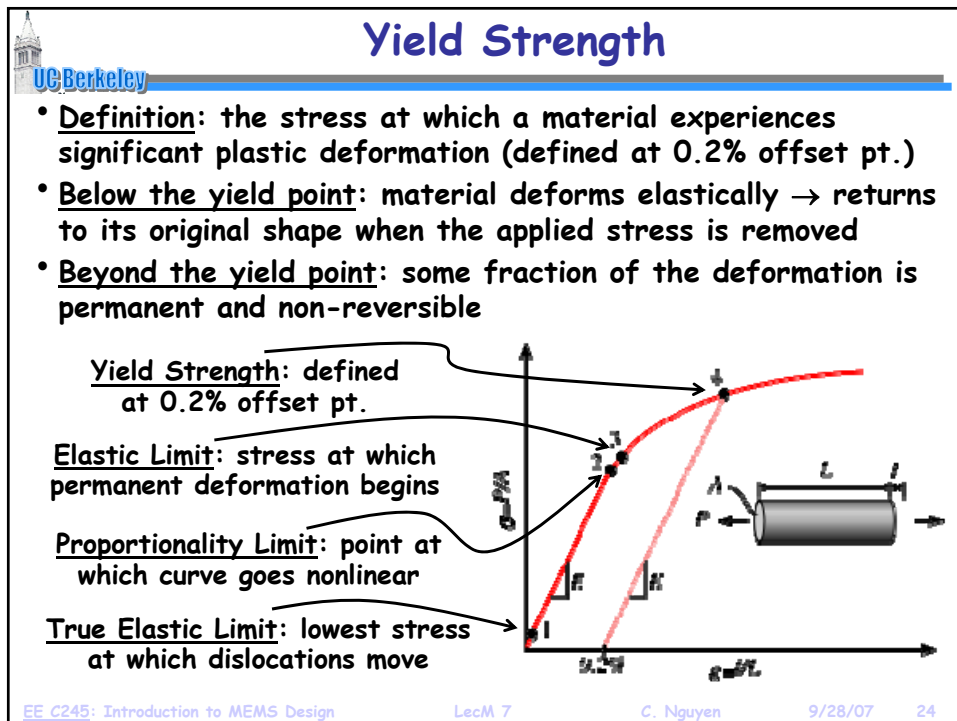
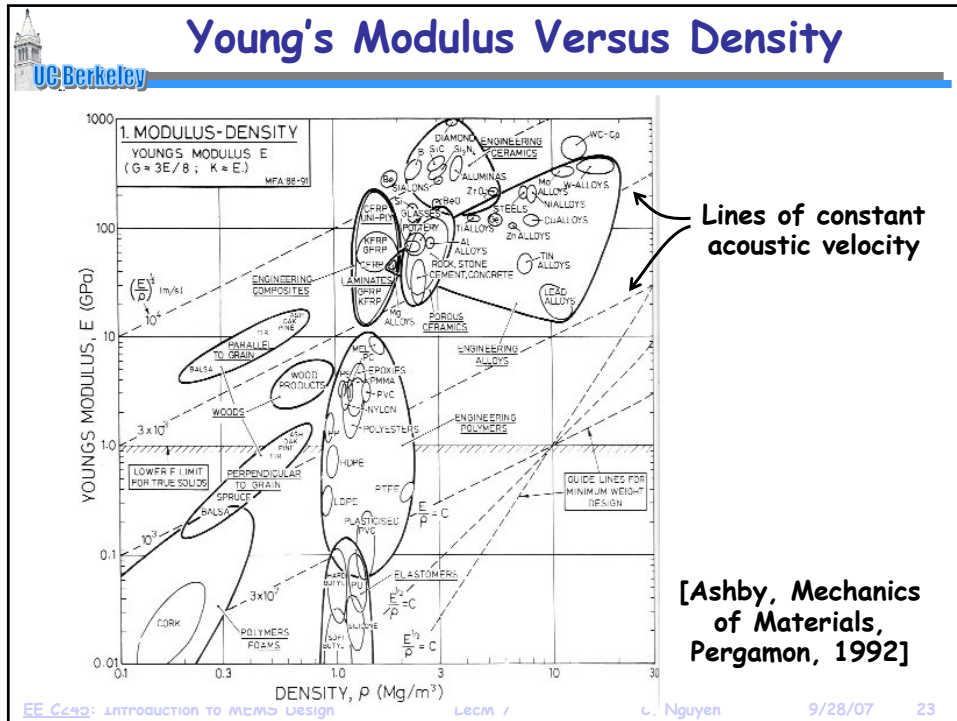
Material Properties for MEMS

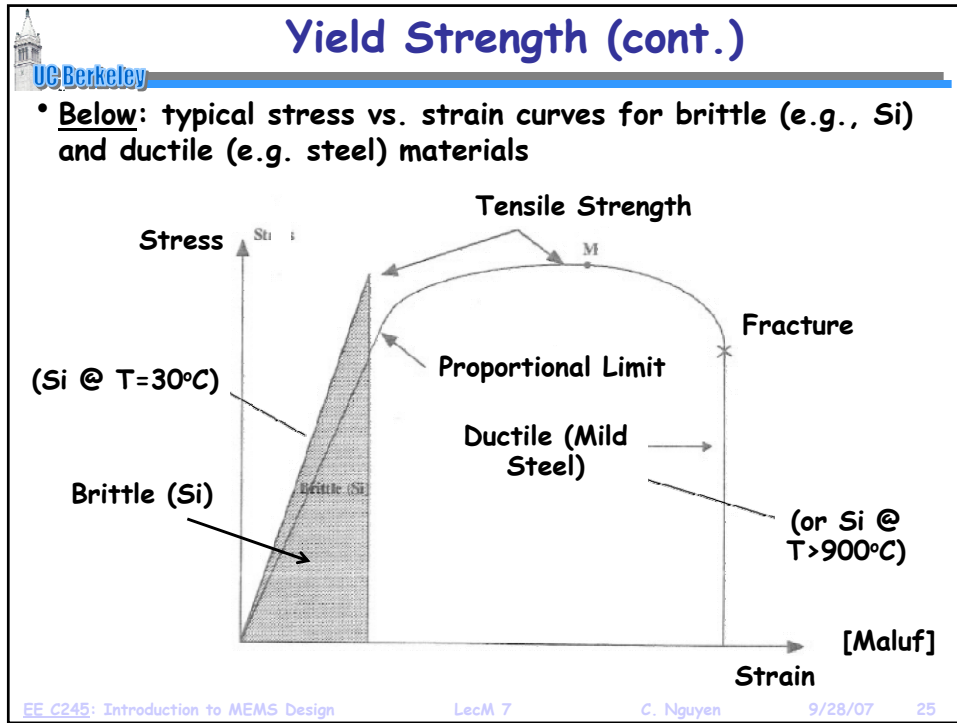
Material	Density, ρ , Kg/m ³	Modulus, E, GPa	E/ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: (m/s)²
↓
 $\sqrt{E/\rho}$ is acoustic velocity

[Mark Spearing, MIT]

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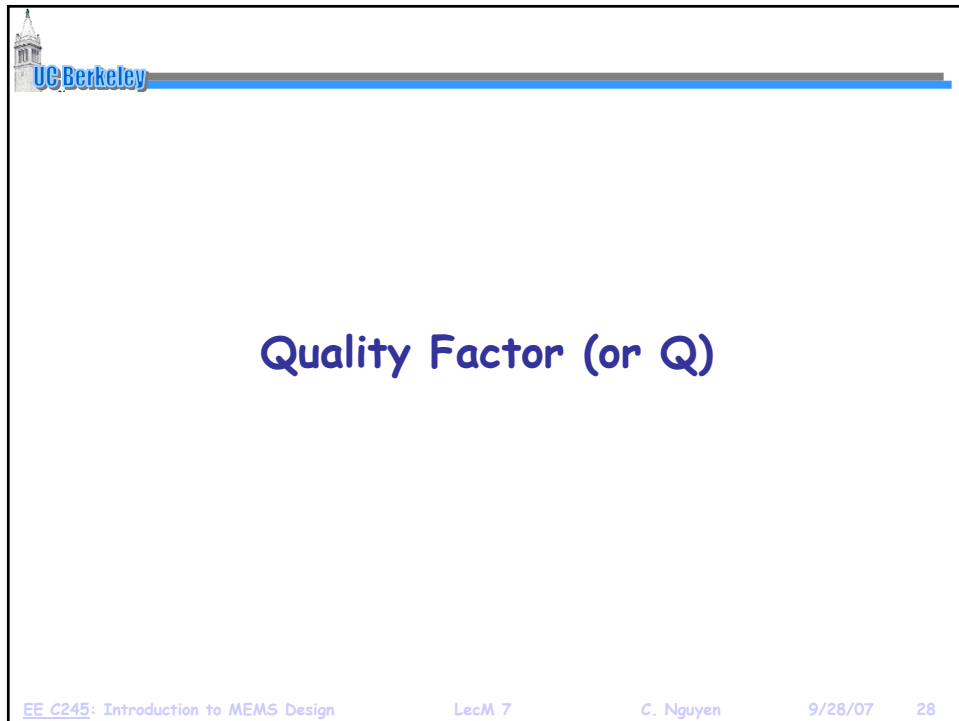
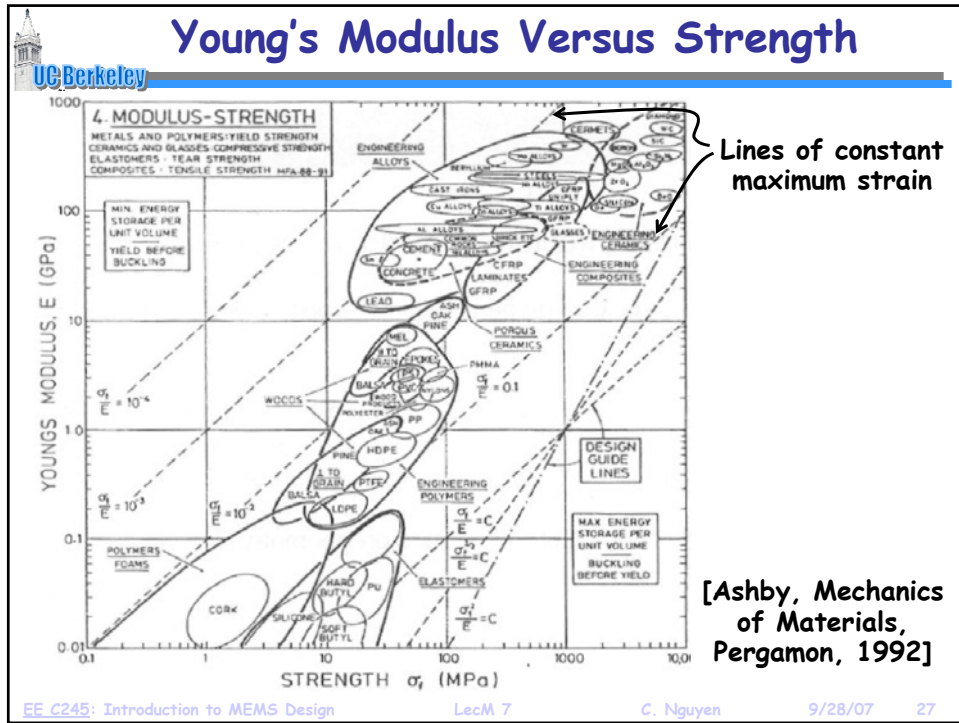
Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f MPa	$\frac{\sigma_f}{E}$ (-) x 10 ⁻³	$\frac{\sigma_f^2}{E}$ MJ/m ³
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

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Clamped-Clamped Beam μ Resonator

Frequency:

Stiffness k_r Young's Modulus E

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_r}{m_r}} = 1.03 \sqrt{\frac{E}{\rho} \frac{h}{L_r^2}}$$

Density ρ

Mass m_r (e.g., $m_r = 10^{-13}$ kg)

Smaller mass \Rightarrow higher freq. range and lower series R_x

Note: If $V_P = 0V \Rightarrow$ device off

$i_o = V_P \frac{dC}{dt}$

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Quality Factor (or Q)

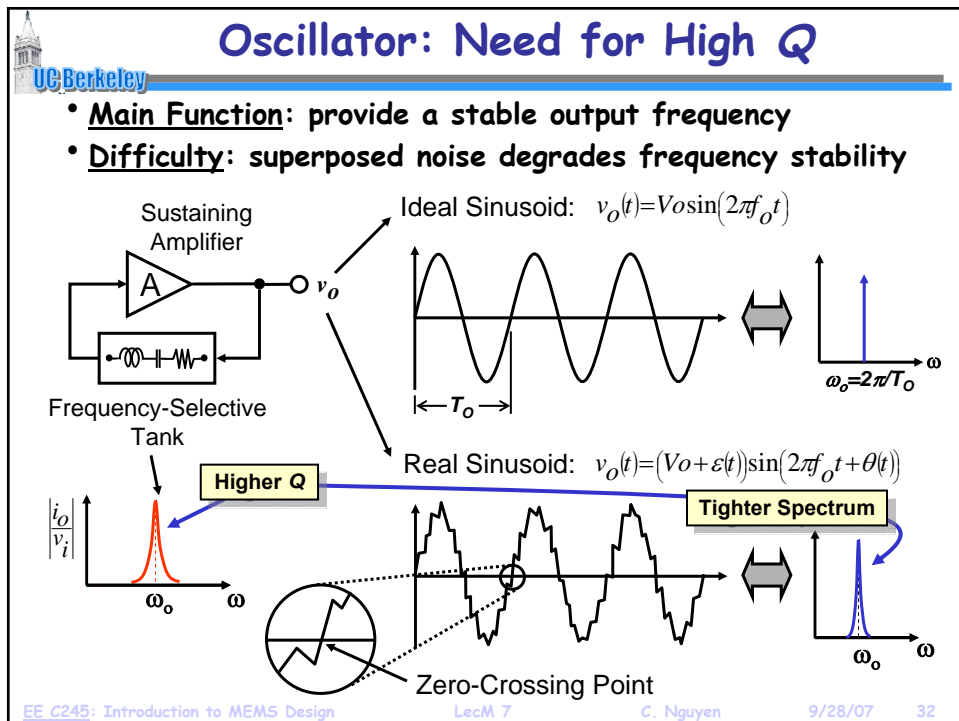
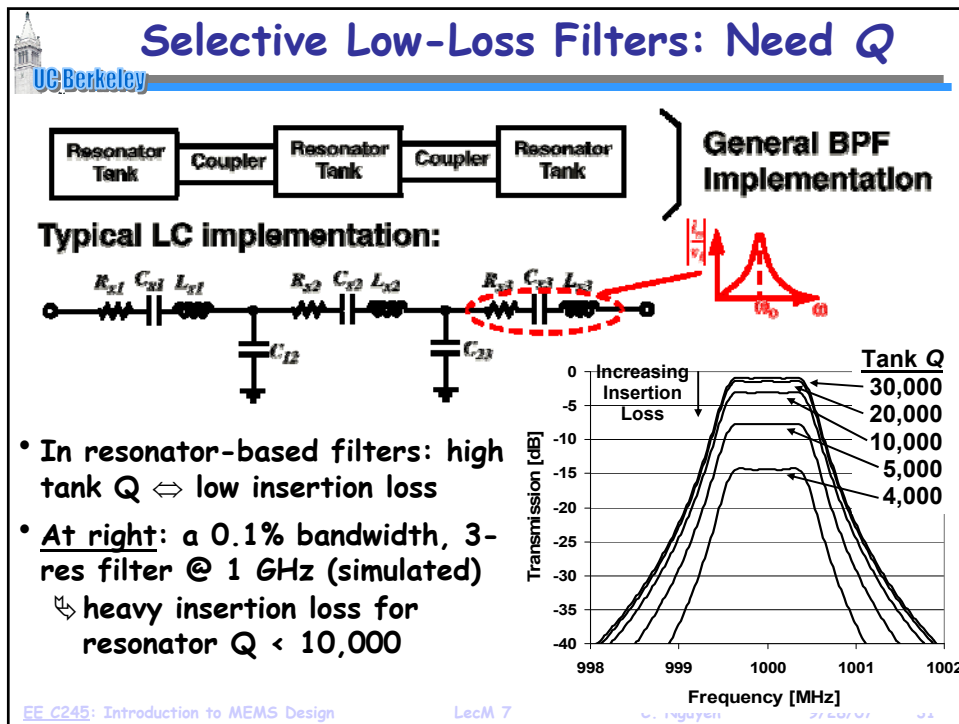
- Measure of the frequency selectivity of a tuned circuit
- **Definition:**

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$
- **Example:** series LCR circuit

$$Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$$
- **Example:** parallel LCR circuit

$$Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o L G}$$

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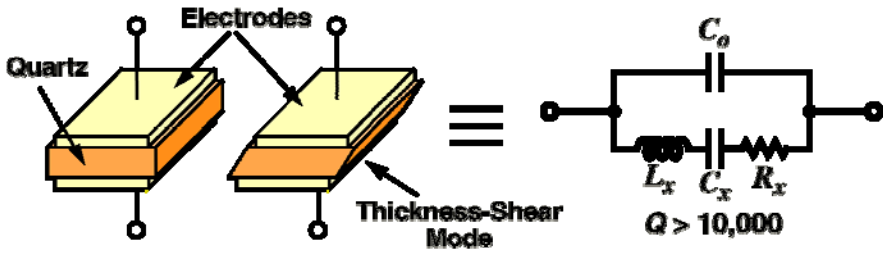


Attaining High Q

Problem: IC's cannot achieve Q 's in the thousands
 ↳ transistors ⇒ consume too much power to get Q
 ↳ on-chip spiral inductors ⇒ Q 's no higher than ~10
 ↳ off-chip inductors ⇒ Q 's in the range of 100's

Observation: vibrating mechanical resonances ⇒ $Q > 1,000$

Example: quartz crystal resonators (e.g., in wristwatches)
 ↳ extremely high Q 's ~ 10,000 or higher ($Q \sim 10^6$ possible)
 ↳ mechanically vibrates at a distinct frequency in a thickness-shear mode



The diagram illustrates a quartz crystal resonator in the thickness-shear mode. It shows a quartz plate with electrodes on both sides. The mechanical vibration is represented by an equivalent circuit model consisting of a series combination of an inductor L_x , a capacitor C_x , and a resistor R_x , all in parallel with a capacitor C_0 . The quality factor Q is noted as $Q > 10,000$.

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Energy Dissipation and Resonator Q

Material Defect Losses

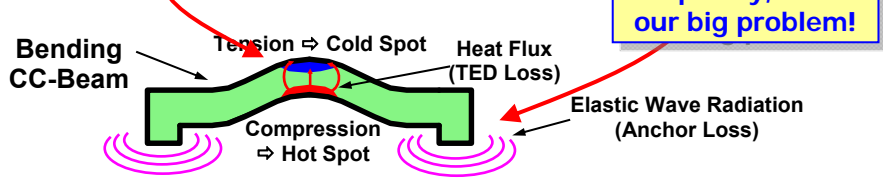
Gas Damping

$$\frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}$$

Thermoelastic Damping (TED)


Anchor Losses

At high frequency, this is our big problem!



The diagram shows a bending cross-coupled (CC) beam. It highlights the tension and compression regions, which create cold and hot spots respectively. Heat flux (TED loss) is shown flowing from the hot spot to the cold spot. Elastic wave radiation (Anchor loss) is shown at the base of the beam.

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Thermoelastic Damping (TED)

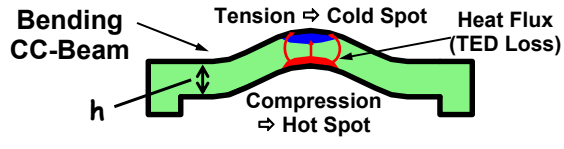
- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

$$\zeta = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2 \left[\frac{f_{TED} f}{f_{TED}^2 + f^2} \right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$




Bending CC-Beam
h

Tension ⇒ Cold Spot
Heat Flux (TED Loss)
Compression ⇒ Hot Spot

ζ = thermoelastic damping factor
 α = thermal expansion coefficient
 T = beam temperature
 E = elastic modulus
 ρ = material density
 C_p = heat capacity at const. pressure
 K = thermal conductivity
 f = beam frequency
 h = beam thickness
 f_{TED} = characteristic TED frequency

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TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

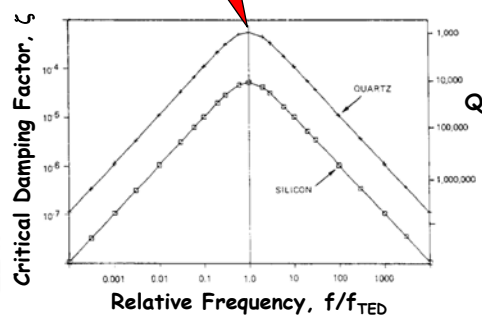
ρ = material density
 C_p = heat capacity at const. pressure
 K = thermal conductivity
 h = beam thickness
 f_{TED} = characteristic TED frequency

- Governed by
 - ↳ Resonator dimensions
 - ↳ Material properties

TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10 ¹² dyne/cm ²
Material density	2.33	2.60	g/cm ³
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10 ⁷ dyne/°K/s
Peak damping @ 300°K	1.06	11.34	10 ⁻⁴

Peak where Q is minimized

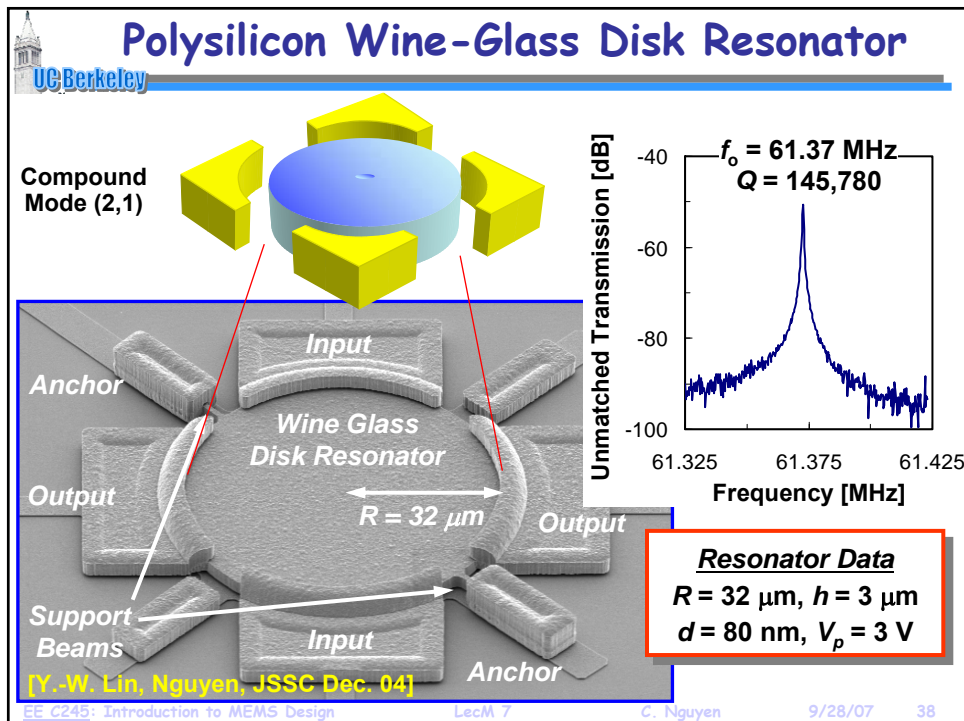
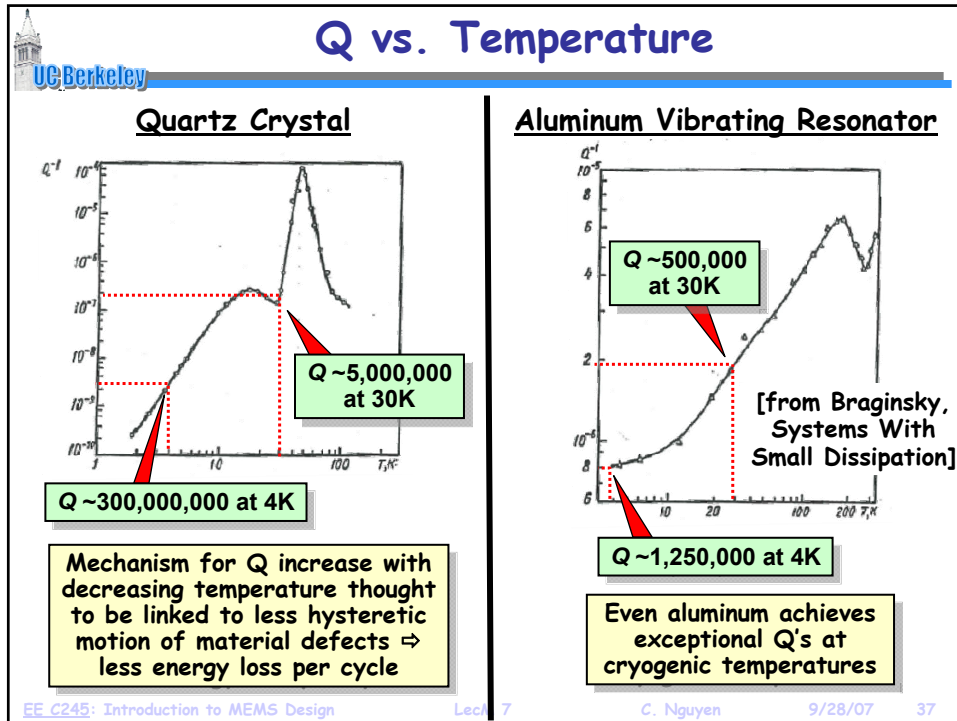


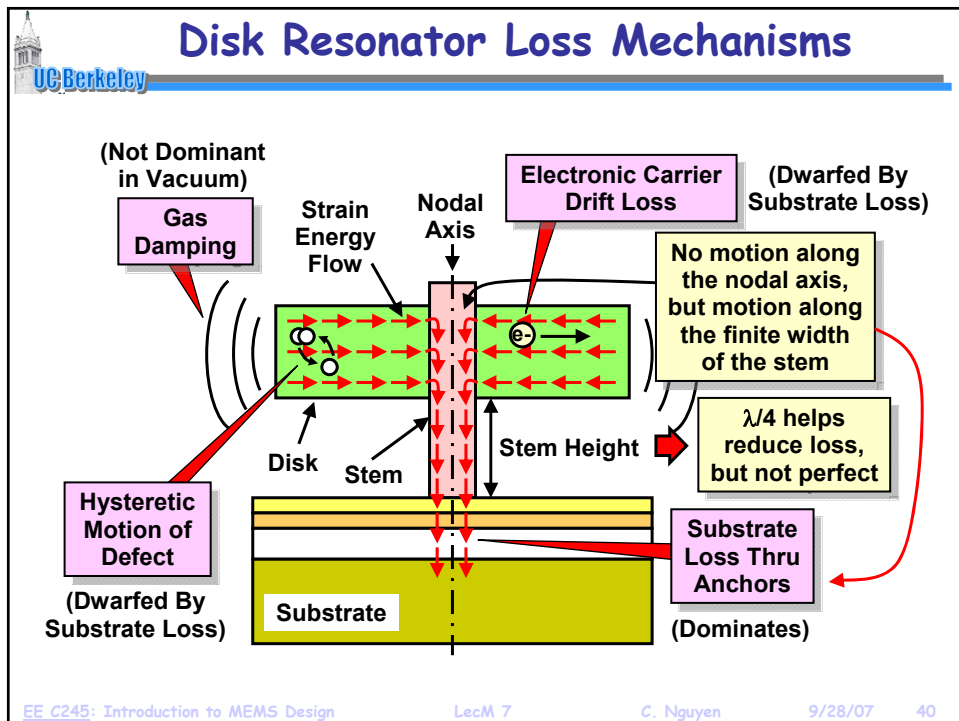
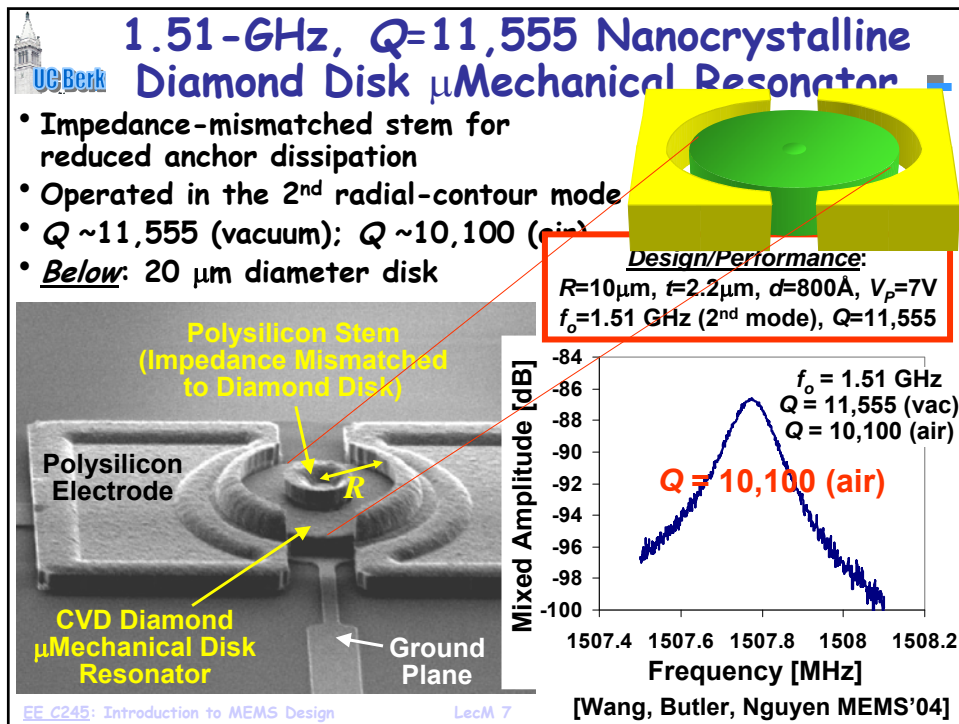
Critical Damping Factor, ζ

Relative Frequency, f/f_{TED}

[from Roszhart, Hilton Head 1990]

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MEMS Material Property Test Structures

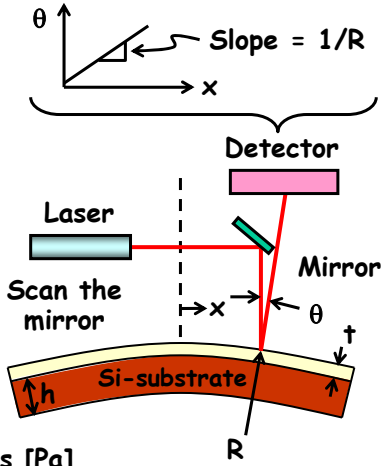
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Stress Measurement Via Wafer Curvature


- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R , then apply:

$$\sigma = \frac{E'h^2}{6Rt}$$



σ = film stress [Pa]
 E' = $E/(1-\nu)$ = biaxial elastic modulus [Pa]
 h = substrate thickness [m]
 t = film thickness
 R = substrate radius of curvature [m]

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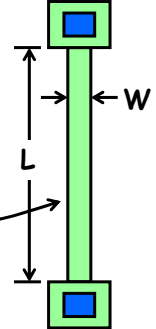
MEMS Stress Test Structure

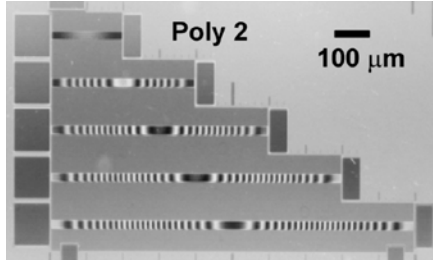
- **Simple Approach:** use a clamped-clamped beam
 - ↳ Compressive stress causes buckling
 - ↳ Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2}{3} \frac{Eh^2}{L^2}$$


E = Young's modulus [Pa]
 $I = (1/12)Wh^3$ = moment of inertia
 L, W, h indicated in the figure

- ↳ **Limitation:** Only compressive stress is measurable

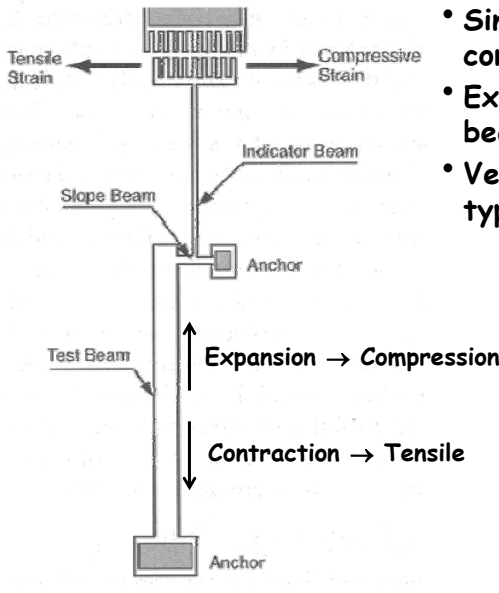




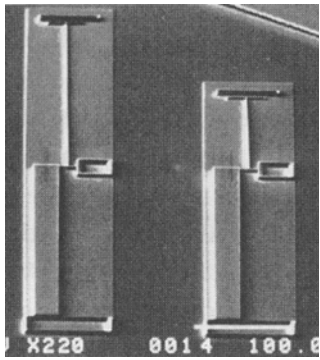
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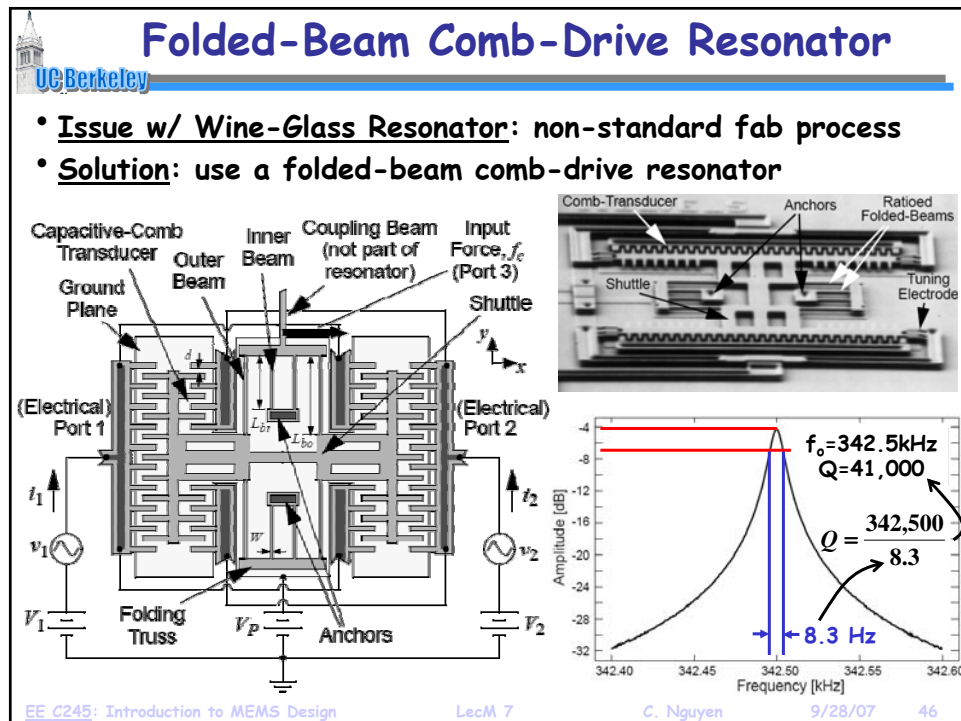
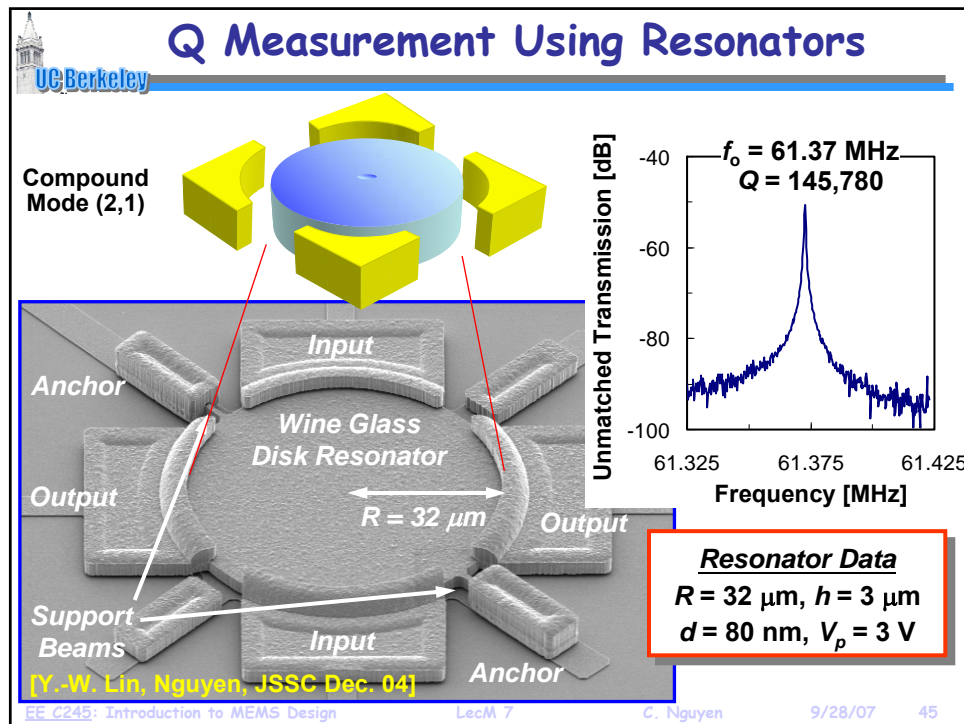
More Effective Stress Diagnostic



- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress

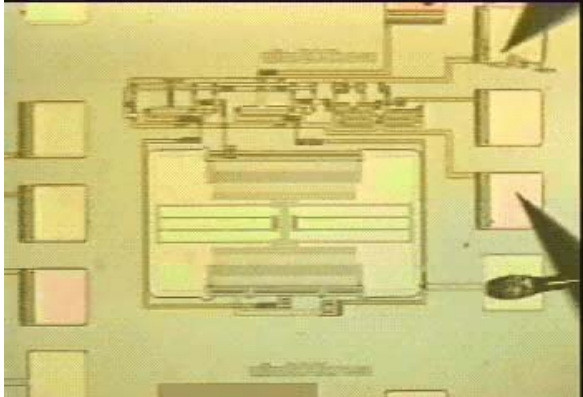


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Comb-Drive Resonator in Action

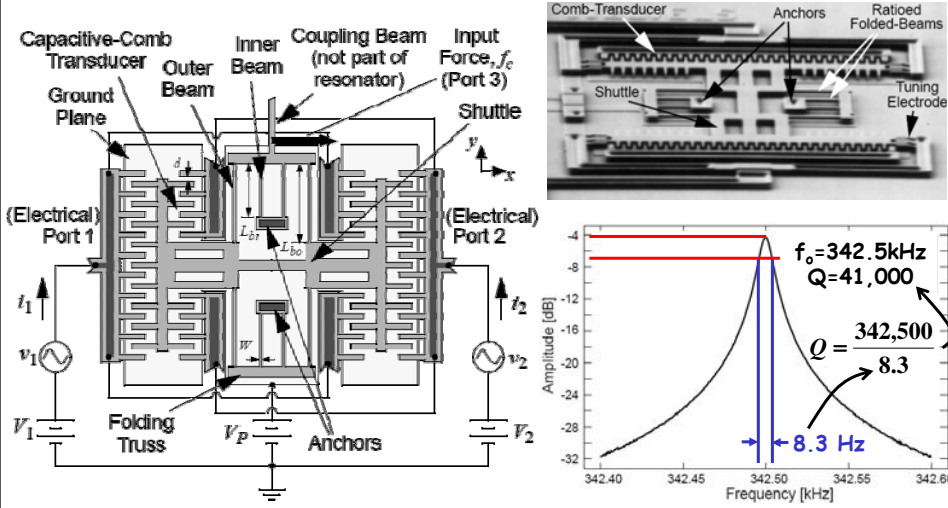
Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



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Folded-Beam Comb-Drive Resonator


Issue w/ Wine-Glass Resonator: non-standard fab process
Solution: use a folded-beam comb-drive resonator



The schematic diagram shows a cross-section of the resonator with labels: Capacitive-Comb Transducer, Ground Plane, Outer Beam, Inner Beam, Coupling Beam (not part of resonator), Input Force, f_c (Port 3), Shuttle, (Electrical) Port 1, (Electrical) Port 2, Folding Truss, Anchors, and V_P . Dimensions L_{b1} , L_{b2} , and w are indicated. The micrograph shows the physical device with labels: Comb-Transducer, Anchors, Ratioed Folded-Beams, Shuttle, and Tuning Electrode.

The graph shows Amplitude [dB] vs Frequency [kHz]. The resonance frequency is $f_o = 342.5 \text{ kHz}$ and the quality factor is $Q = 41,000$. The bandwidth is 8.3 Hz and the Q factor is also indicated as $Q = \frac{342,500}{8.3}$.

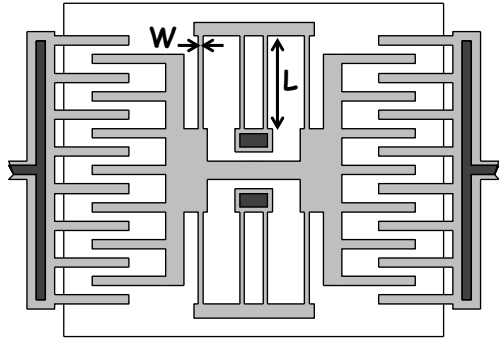
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Measurement of Young's Modulus

- Use micromechanical resonators
 - ↳ Resonance frequency depends on E
 - ↳ For a folded-beam resonator:

h = thickness



$$\text{Resonance Frequency} = f_o = \left[\frac{\overset{\text{Young's modulus}}{4Eh(W/L)^3}}{\underset{\text{Equivalent mass}}{M_{eq}}} \right]^{1/2}$$

- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters


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Anisotropic Materials

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Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
 - ↳ Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{matrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{matrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{matrix}$$

Stresses

Stiffness Coefficients

Strains

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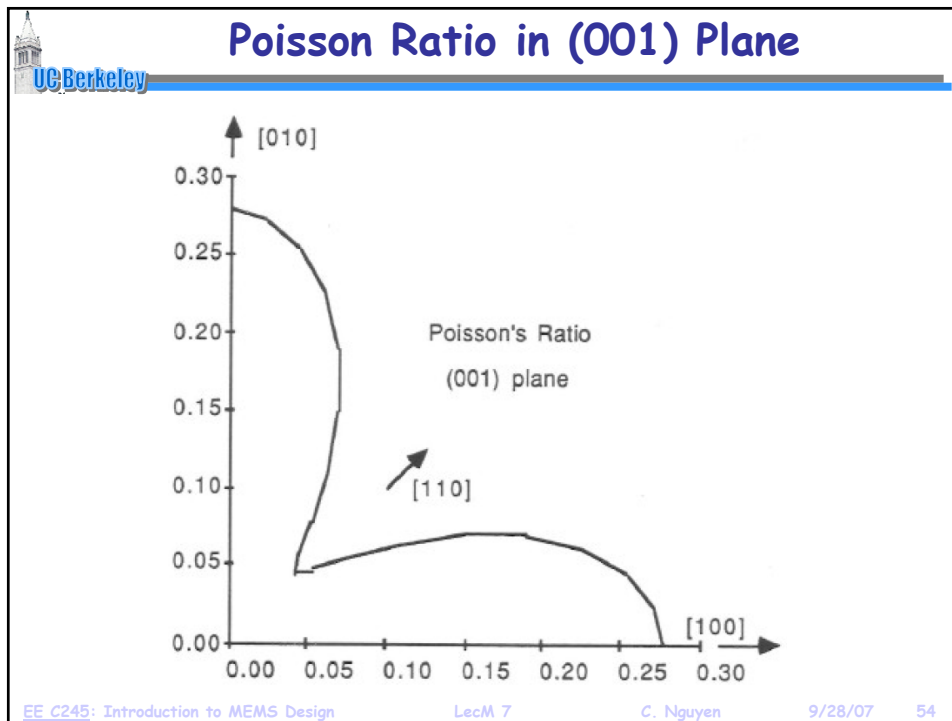
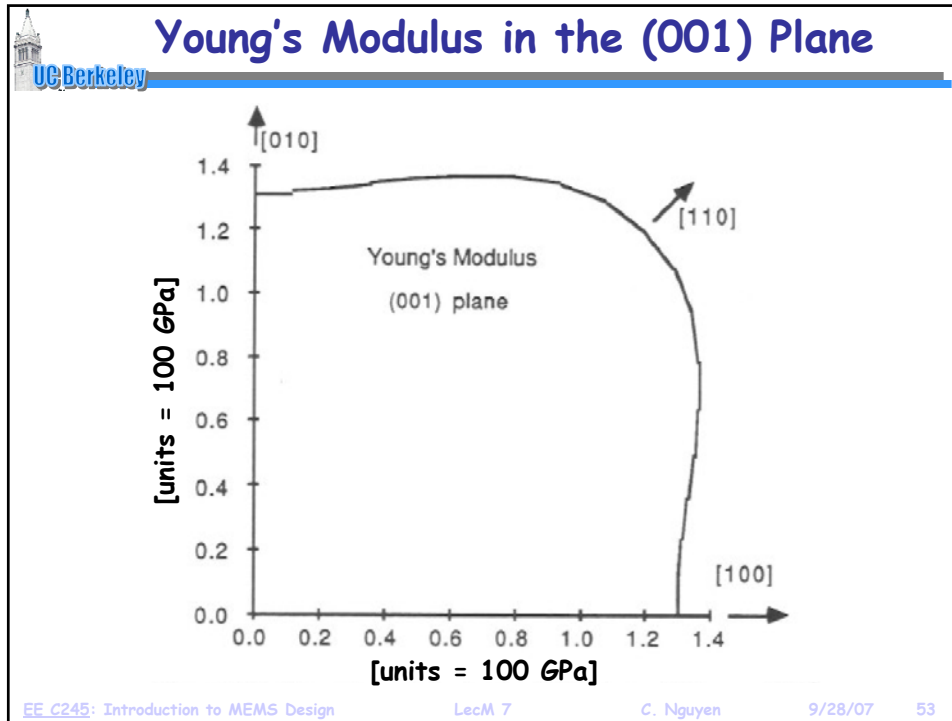
Stiffness Coefficients of Silicon


- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{matrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{zx} \\ \tau_{xy} \end{matrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{matrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{matrix}$$

where $\begin{cases} C_{11} = 165.7 \text{ GPa} \\ C_{12} = 63.9 \text{ GPa} \\ C_{44} = 79.6 \text{ GPa} \end{cases}$

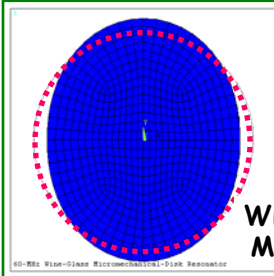
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Anisotropic Design Implications

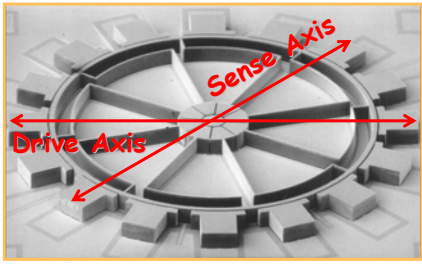
- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
 - ↳ Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
 - ↳ Mode matching is required, where frequencies along different axes of a ring must be the same
 - ↳ Not okay to ignore anisotropic variations, here



```
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Wine-Glass Mode Disk



Ring Gyroscope

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