EE247
Lecture 5

• Summary last lecture
• Continuous-time filters
  – Facts about monolithic Rs & Cs and its effect on integrated filter characteristics
  – Opamp MOSFET-C filters
  – Opamp MOSFET-RC filters
  – Gm-C filters
• Frequency tuning for continuous-time filters
  – Trimming
  – Automatic frequency tuning
    • Continuous tuning
    • Periodic tuning

Summary Last Lecture

• High Q high order filters
  – Transmission zero implementation
  – Example
• Various integrator topologies utilized in monolithic filters
  – Resistor + C based filters
  – Transconductance (gm) + C based filters
  – Switched-capacitor filters
• Effect of integrator non-idealities on filter behavior
Summary Last Lecture
Transmission zero Implementation for Integrator Based Ladder Filters

- Use KCL & KVL to derive:

\[ V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + \frac{C_a}{C_1 + C_a} \]

\[ V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + \frac{C_a}{C_3 + C_a} \]

Voltage Controlled Voltage Source!

- Replace shunt capacitor with voltage controlled voltage sources:

\[ V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a} \]

\[ V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a} \]
Summary Last Lecture
Integrator Based Ladder Filters
Transmission zeros

Transmission zeros implemented with coupling capacitors
Summary Last Lecture
Effect of Integrator Non-Idealities on Filter Performance

\[ V_i = -\frac{I}{sRC} = -\frac{s}{s} \]

\[ H(s) = \frac{-\omega_o}{s} \]

\[ H(s) = \frac{-\omega_o}{s + \frac{s}{\omega_o}} \]

\[ -90^\circ \]

\[ \omega_o \]

\[ -90^\circ \]

Effect of Integrator Finite DC Gain on Q

\[ \omega_o \]

\[ P1 = \frac{\omega}{a} \]

\[ \arctan \left( \frac{P1}{\omega_0} \right) \]

\[ -90^\circ \]

\[ \omega \]

\[ -\frac{\pi}{2} + \arctan \left( \frac{P1}{\omega_0} \right) \]

Example: \( P1/\omega_0 = 1/a = 1/100 \)
Effect of Integrator Finite DC Gain on Overall Filter Frequency Response

- Phase lead @ \( \omega_0 \)
  \( \rightarrow \) Droop in the passband

Effect of Integrator Non-Dominant Poles

Example: \( \omega_0 / P = 1 / 100 \)
Effect of Integrator Non-Dominant Poles on Overall Filter Frequency Response

- Phase lag @ $\omega_0$
  - Peaking in the passband
  - In extreme cases could result in oscillation!

Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q

$\angle = -\pi/2 + \arctan \frac{P_1}{\omega_0}$

Note that the two terms can cancel each other’s effect.
Summary

Effect of Integrator Non-Idealities on Q

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements.
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
  - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain frequency, the two terms can cancel each other out!

\[
\begin{align*}
Q_{\text{ideal}}^{\text{int}} &= \infty \\
Q_{\text{real}}^{\text{int}} &= \frac{1}{\frac{1}{\omega_0} \sum_{i=1}^{\infty} \frac{1}{\omega_i}} \\
\text{Phase lead @ } \omega_0 &\quad \text{Phase lag @ } \omega_0
\end{align*}
\]

Few Facts About Monolithic Rs & Cs & Gms

- Monolithic continuous-time filter critical frequency set by \( RxC \) or \( GmxC \)
- Absolute value of integrated \( Rs \) & \( Cs \) & \( Gms \) are quite variable
  - \( Rs \) vary due to doping and etching non-uniformities
    - Could vary by as much as \(-30\text{ to }40\%\) due to process & temperature variations
  - \( Cs \) vary because of oxide thickness variations and etching inaccuracies
    - Could vary \(-10\text{ to }15\%\)
  - \( Gms \) typically function of mobility, oxide thickness, current, device geometry …
    - Could vary \(>\text{ to }<40\%\) or more with process & temp. & supply voltage

→ Continuous-time filter critical frequency could vary by over \(+50\%\)
Few Facts About Monolithic Rs & Cs

- While absolute value of monolithic Rs & Cs and gms are quite variable, with special attention paid to layout, C & R & gms quite well-matched
  - Ratios very accurate and stable over time and temperature
- With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries...):
  - Capacitor matching <<0.1%
  - Resistor matching <0.1%
  - Gm matching <0.5%

Impact of Process Variations on Filter Characteristics

Facts about RLC filters

- $\omega_{-3dB}$ determined by absolute value Ls & Cs

\[
C_{RLC}^{Norm} = C \times C_{1}^{Norm} = \frac{C_{1}^{Norm}}{R \times \omega_{-3dB}}
\]

\[
L_{2}^{RLC} = L_{2} \times L_{2}^{Norm} = \frac{L_{2}^{Norm} \times R^*}{\omega_{-3dB}}
\]

- Shape of filter depends on ratios of normalized Ls & Cs
Effect of Monolithic R & C Variations on Filter Characteristics

- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth, etc) is a function of ratio of normalized $L_s$ & $C_s$ in RLC filters.
- Critical frequency (e.g. $\omega_{-3dB}$) function of absolute value of $L_s$ & $C_s$.
- Absolute value of integrated $R_s$ & $C_s$ & $G_{ms}$ are quite variable.
- Ratios very accurate and stable over time and temperature.

→ What is the effect of on-chip component variations on monolithic filter frequency characteristics?

**Impact of Process Variations on Filter Characteristics**

\[ \tau_1 = \frac{c_{RLC}}{R_s} = \frac{c_{Norm}}{\omega_{-3dB}} \]
\[ \tau_2 = \frac{L_{2RLC}}{R_s} = \frac{L_{2Norm}}{\omega_{-3dB}} \]
\[ \tau_1 = \frac{c_{1Norm}}{l_{1Norm}} \]
\[ \tau_2 = \frac{L_{2Norm}}{L_{2Norm}} \]
Impact of Process Variations on Filter Characteristics

\[ \tau_1^{\text{intg}} = C_{11} R_1 = \frac{C_{11}^{\text{Norm}}}{\omega_{-3dB}} \]
\[ \tau_2^{\text{intg}} = C_{12} R_2 = \frac{C_{12}^{\text{Norm}}}{\omega_{-3dB}} \]
\[ \tau_1^{\text{intg}} = C_{11} R_1 = \frac{C_{11}^{\text{Norm}}}{\omega_{-3dB}} \]
\[ \tau_2^{\text{intg}} = C_{12} R_2 = \frac{C_{12}^{\text{Norm}}}{\omega_{-3dB}} \]

Variation in absolute value of integrated
\[ \tau_1^{\text{intg}} \]
\[ \tau_2^{\text{intg}} \]
\[ \text{Rs & Cs causes change in critical freq. } (\omega_{-3dB}) \]

Since Ratios of Rs & Cs very accurate
\[ \Rightarrow \text{Continuous time monolithic filters fully retain their shape} \]

Example: LPF Worst Case Corner Frequency Variations

- While absolute value of on-chip RC (gm-C) time-constants vary by as much as 100% (process & temp.)
- With proper precautions, excellent matching can be achieved:
  \[ \Rightarrow \text{Well-preserved relative amplitude & phase vs freq. characteristics} \]
  \[ \Rightarrow \text{Need to adjust (tune) continuous-time filter critical frequencies only} \]
Tunable Opamp-RC Filters

• Example: A 1st order Opamp-RC filter is designed to have a corner frequency of 1.6MHz.
• Assuming process variations of:
  • \( C \) varies by ±10%
  • \( R \) varies by ±25%
• Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.

![Diagram of a 1st order Opamp-RC filter]

Nominal \( R \) & \( C \) values for 1.6MHz corner frequency

Tunable Resistor

• Make provisions for either \( R \) or \( C \) to be adjustable (example adjustable \( R \))
• Monolithic Rs can only be made adjustable in discrete steps (not continuous)
• Assuming expected process variations of:
  • Maximum \( C \) variations by ±10% \( \Rightarrow C_{\text{min}} = 9\text{pF}, C_{\text{max}} = 11\text{pF} \)
  • Maximum \( R \) variations by ±25% \( \Rightarrow R_{\text{min}} = 7.5\text{K}, R_{\text{max}} = 12.5\text{K} \)
  \( \Rightarrow \) Corner frequency varies by ±35%
• Assuming \( n = 3 \) bit (0 or 1) control signal for adjustment
  
  \[
  \begin{align*}
  R_{\text{max}} &= R_{\text{nom}}(1+35%) = 13.5\text{KOHM} \\
  R_{\text{min}} &= R_{\text{nom}}(1-35%) = 6.5\text{KOHM} \\
  R_1 &= R_{\text{min}} \\
  R_2 &= (R_{\text{max}}-R_{\text{min}})4/7 = 4\text{K} \Rightarrow (2^{n-1} / (2^n - 1)) \\
  R_3 &= (R_{\text{max}}-R_{\text{min}})2/7 = 2\text{K} \Rightarrow (2^{n-2} / (2^n - 1)) \\
  R_4 &= (R_{\text{max}}-R_{\text{min}})1/7 = 1\text{K} \Rightarrow (2^{n-3} / (2^n - 1)) \\
  \text{Tuning resolution 10%} & \Rightarrow (1\text{K}/10\text{K}) \\
  \text{If finer resolution needed add more bits & Rs}
  \end{align*}
  \]

If finer resolution needed add more bits & Rs

![Diagram of a tunable resistor with MOSFETs as switches]
Tunable Opamp-RC Filter

<table>
<thead>
<tr>
<th>D2</th>
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<th>R_{total}</th>
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<tbody>
<tr>
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<tr>
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<td>1</td>
<td>8.5K</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13.5K</td>
</tr>
</tbody>
</table>

Post manufacturing:
- Set all D_x
- Measure -3dB frequency
  - If frequency too high, decrement D to D-1
  - If frequency too low, increment D to D+1
  - If frequency within 10% of the desired corner freq., stop

For higher order filters, all filter integrators tuned simultaneously

Tunable Opamp-RC Filters

Summary
- Program C_s and/or R_s to freq. tune the filter
- All filter integrators tuned simultaneously
- Tuning in discrete steps & not continuous
- Tuning resolution limited
- Switch parasitic C & series R can affect the freq. response of the filter
Example: Tunable Low-Pass Opamp-RC Filter
Adjustable Capacitors

Opamp RC Filters

- Advantages
  - Since resistors are quite linear, linearity only a function of opamp linearity
    → good linearity

- Disadvantages
  - Opamps have to drive resistive load, low output impedance is required
    → High power consumption
  - Continuous tuning not possible
  - Tuning requires programmable Rs and/or Cs
Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC

\[ \int \]

\[ \frac{v_0}{v_{in}} = \frac{-\alpha_0}{s} \text{ where } \alpha_0 = \frac{I}{R_{eq}C} \]

Use of MOSFETs as Resistors

R replaced by MOSFET

\rightarrow \text{Continuously variable resistor:}

MOSFET IV characteristic:
Use of MOSFETs as Resistors

Single-Ended Integrator

\[ I_D = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) V_{ds} \left( \frac{V_{ds}}{2} \right) \]

\[ I_D = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \left( V_g - \frac{V_i}{2} \right) \]

\[ G = \frac{\partial I_D}{\partial V_i} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \]

\[ \Rightarrow \text{Tunable by varying } V_G. \]

Problem: Single-ended MOSFET-C Integrator → Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors

Differential Integrator

\[ I_{D1} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \left( V_{gs} - V_{th} - \frac{V_i}{2} \right) \]

\[ I_{D2} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} + \frac{V_i}{2} \right) \]

\[ I_{D1} - I_{D2} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) V_i \]

\[ G = \frac{\partial (I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} \left( V_{gs} - V_{th} \right) \]

- Non-linear term cancelled!
- Admittance independent of \( V_i \)

Problem: Threshold voltage dependence
MOSFET-C Integrator

- For the Opamp-RC integrator, opamp input stays at 0V (virtual gnd.)

- For the MOSFET-C integrator, opamp input stays at the voltage $V_x$ which is a function of 2nd order MOSFET non-linearities

  $\rightarrow$ Common-mode voltage sensitivity

Use of MOSFET as Resistor Issues

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles $\rightarrow$ excess phase
- Filter performance mandates well-matched MOSFETs $\rightarrow$ long channel devices
- Excess phase increases with $L^2$
  $\rightarrow$ Tradeoff between matching and integrator $Q$
  $\rightarrow$ This type of filter limited to low frequencies
Example: Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning

5th Order Elliptic MOSFET-C LPF with 4kHz Bandwidth


Improved MOSFET-C Integrator

\[ I_D = \mu C_{ox} \frac{W}{L} \left( V_{gs1} - V_{th1} - \frac{V_{ds1}}{2} \right) \]
\[ I_{D1} = \mu C_{ox} \frac{W}{L} \left( V_{gs1} - V_{th1} - \frac{V_{ds1}}{2} \right) \]
\[ I_{D2} = -\mu C_{ox} \frac{W}{L} \left( V_{gs2} - V_{th2} - \frac{V_{ds2}}{2} \right) \]
\[ I_{X1} = I_{D1} + I_{D3} - \mu C_{ox} \frac{W}{L} \left( V_{gs1} - V_{th1} - \frac{V_{ds1}}{2} \right) \]
\[ I_{X2} = -\mu C_{ox} \frac{W}{L} \left( V_{gs2} - V_{th2} - \frac{V_{ds2}}{2} \right) \]
\[ I_{X1} - I_{X2} = \mu C_{ox} \frac{W}{L} \left( V_{gs1} - V_{gs2} \right) V_i \]
\[ G = \frac{\partial (I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} \left( V_{gs1} - V_{gs2} \right) \]

No threshold dependence
First order Common-mode non-linearity cancelled
Linearity achieved in the order of 60-70dB

R-MOSFET-C Integrator

Improvement over MOSFET-C by adding resistor in series with MOSFET
Voltage drop primarily across resistor $\rightarrow$ small MOSFET $V_{ds}$ $\rightarrow$ improved linearity
Linearity in the order of 90dB possible
Generally low frequency applications


R-MOSFET-C Lossy Integrator

Negative feedback around the non-linear MOSFETs improves linearity
Reduced frequency response accuracy

Example:
Opamp MOSFET-RC Filter

![Opamp MOSFET-RC Filter circuit diagram]

- Suitable for low frequency applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning


Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

<table>
<thead>
<tr>
<th><strong>Opamp</strong></th>
<th><strong>OTA</strong></th>
</tr>
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<tbody>
<tr>
<td>Voltage controlled voltage source</td>
<td>Voltage controlled current source</td>
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</table>

- Low output impedance
- Output in the form of voltage
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation
- High output impedance
- In the context of filter design called gm-cells
- Output in the form of current
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp → higher freq. potential
- Typically lower power
Integrator Implementation
Gm-C & Opamp-Gm-C

\[
\frac{V_o}{V_{in}} = -\frac{\alpha_o}{s} \quad \text{where} \quad \alpha_o = \frac{G_m}{C}
\]

Gm-C Filters
Simplest Form of CMOS Gm-C Integrator

- MOSFET in saturation region:
  \[
  I_d = \frac{\mu C_{ox} W}{2 L} (V_{gs} - V_{th})^2
  \]

- Gm is given by:
  \[
  g_m = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th})
  \]
  \[
  = 2 \frac{I_d}{V_{gs} - V_{th}}
  \]
  \[
  = 2 \left( \frac{I_d}{2} \mu C_{ox} \frac{W}{L} \right)^{1/2}
  \]
  \[\text{Id varied via } V_{control}\]
  \[\Rightarrow \text{gm tunable via } V_{control}\]
Gm-C Filters
Simplest Form of CMOS Gm

- Integrator behavior:
  \[
  \frac{V_o}{V_{in}} = -\omega_o \frac{s}{s}
  \]
  \[\omega_o = \frac{g_{m M_{1.2}}}{2 \times C_{int} g}\]

- Critical frequency continuously tunable via \(V_{control}\)


Second Order Gm-C Filter

- Simple design
- Tunable
- \(Q\) function of device ratios:
  \[
  Q = \frac{g_{m M_{1.2}}}{g_{m M_{3.4}}}
  \]
Filter Frequency Tuning Techniques

- Component trimming

- Automatic on-chip filter tuning
  - Continuous tuning
    - Master-slave tuning
  - Periodic off-line tuning
    - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

Example: Tunable Opamp-RC Filter

Post manufacturing:

- Usually at wafer-sort tuning performed
- Measure -3dB frequency
  - If frequency too high, decrement D to D-1
  - If frequency too low, increment D to D+1
  - If frequency within 10% of the desired corner freq., stop

Not practical to require end-user to tune the filter
- Need to fix the adjustment at the factory
Trimming

• Component trimming
  - Build fuses on-chip,
    • Based on measurements @ wafer-sort blow fuses by applying high current to the fuse
      - Expensive
      - Fuse regrowth problems!
      - Does not account for temp. variations & aging
  - Laser trimming
    • Trim components or cut fuses by laser
      - Even more expensive
      - Does not account for temp. variations & aging

Example: Tunable/Trimmable Opamp-RC Filter

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Automatic Frequency Tuning

• By adding additional circuitry to the main filter circuit
  – Have the filter critical frequency automatically tuned
    → Expensive trimming avoided
    → Accounts for critical frequency variations due to temp. and voltage changes

Master-Slave Automatic Frequency Tuning

• Following facts used in this scheme:
  – Use a replica (master) of the main filter (called the slave) in the tuning circuitry
  – Place the replica in close proximity of the main filter
  – Use the tuning signal generated to tune the replica, to also tune the main filter
  – In the literature, this scheme is called master-slave tuning!
Master-Slave Frequency Tuning
Reference Filter (VCF)

- Use a biquad for master filter (VCF)
- Utilize the fact that @ the frequency $f_o$ the lowpass (or highpass) outputs are 90 degree out of phase wrt to input

$$\frac{V_{LP}}{V_{in}} = \frac{1}{s^2 + \frac{s}{\omega_0} + \frac{1}{Q\omega_0}} \quad @ \quad \omega = \omega_0, \quad \phi = -90^\circ$$

- Apply a sinusoid at the desired $f_o$
- Compare the LP output phase to the input
- Based on the phase difference
  - Increase or decrease filter critical freq.

Master-Slave Frequency Tuning
Reference Filter (VCF)

$$V_{tune} = -K \times V_{ref}^{rms} \times V_{LP}^{rms} \times \cos \phi$$
Master-Slave Frequency Tuning
Reference Filter (VCF)

- By closing the loop, feedback tends to drive the error voltage to zero.
  \( \rightarrow \) Locks \( f_0 \), the critical frequency of the filter to the accurate reference frequency
- Typically the reference frequency is provided by a crystal oscillator with accuracies in the order of few ppm

Master-Slave Frequency Tuning
Reference Filter (VCF)

• Issues to be aware of:
  – Input reference tuning signal needs to be sinusoid → disadvantage since clocks are usually available as square waveform
  – Reference signal feed-through to the output of the filter can limit filter dynamic range (reported levels or about 100μVrms)
  – Ref. signal feed-through is a function of:
    • Reference signal frequency wrt filter passband
    • Filter topology
    • Care in the layout
    • Fully differential topologies beneficial

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Master-Slave Frequency Tuning
Reference Voltage-Controlled-Oscillator (VCO)

• Instead of VCF a voltage-controlled-oscillator (VCO) is used
• VCO made or replica integrator used in main filter
• Tuning circuit operates exactly as a conventional phase-locked loop (PLL)
• Tuning signal used to tune main filter

Master-Slave Frequency Tuning
Reference Voltage-Controlled-Oscillator (VCO)

- Issues to be aware of:
  - Design of stable & repeatable oscillator challenging
  - VCO operation should be limited to the linear region or else the operation loses accuracy
  - Limiting the VCO signal range to the linear region not a trivial design issue
  - In the case of VCF based tuning ckt there was only ref. signal feedthrough. In this case, there is also the feedthrough of the VCO signal!!
  - Advantage over VCF based tuning → Reference input signal square wave (not sin.)