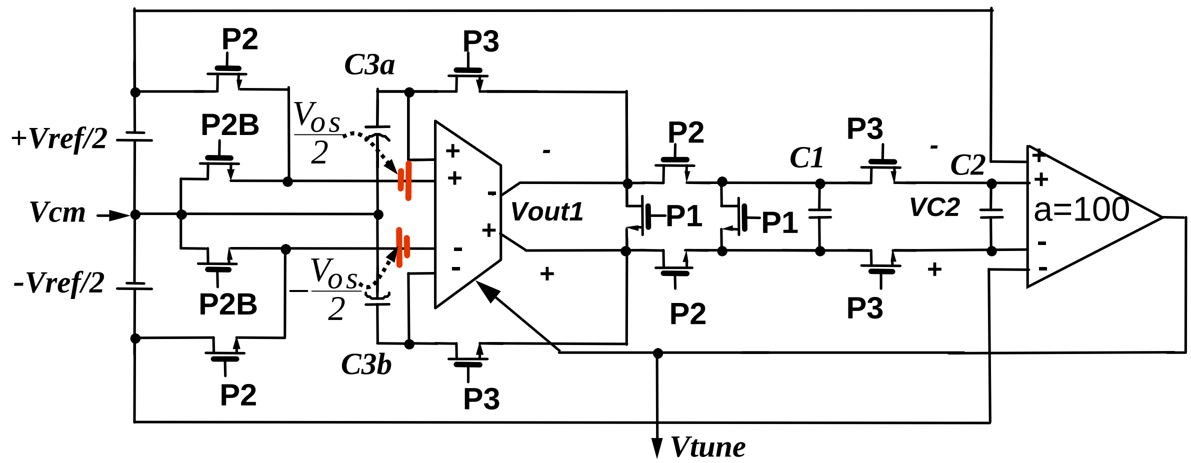
Solution to Homework #2 provided by Mr. Louis P. Alarcon



EE 247 Homework #2

Louis P. Alarcon
lalarcon@eecs.berkeley.edu

Problem #1:

Given parameters:

$C_2 := 10pF \quad C_1 := \frac{C_2}{2} \quad C_1 = 5\,pF \quad V_{ref} := 0.5V$

$V_{os} := 10mV \quad T_2 := 100ns$

$$G'_m(V_{tune}) := 40\frac{\mu A}{V} + V_{tune} \cdot 20\frac{\mu A}{V^2}$$

$$G'_{m\_aux}(V_{tune}) := \frac{1}{8} \cdot G'_m(V_{tune})$$

$a := 100$

$$V'_{tune}(V_{in}) := \begin{cases} 0V & \text{if } V_{in} < 0V \\ 1V & \text{if } V_{in} > \frac{1V}{a} \\ a \cdot V_{in} & \text{otherwise} \end{cases}$$

For P3=1 (and P2B=1):

$$V_{c3} = -V_{os} \cdot \frac{G_m}{G_{m\_aux}} = -8 \cdot V_{os} \qquad V_{c3} := -8 \cdot V_{os} \qquad V_{c3} = -80\,mV$$

$$V_{c2}^{new} = \frac{V_{c2}^{prev} \cdot C_2 + V_{c1} \cdot C_1}{C_1 + C_2}$$

For P1=1 (and P2B=1):

$V_{c1} = 0$

for P2=1 (and P2B=0):

$$V_{c1}(s) = \frac{1}{s \cdot C_1} \cdot \left[G_m \cdot (V_{ref} + V_{os}) + G_{m\_aux} \cdot \left(-V_{os} \cdot \frac{G_m}{G_{m\_aux}} \right) \right] = \frac{G_m \cdot V_{ref}}{s \cdot C_1}$$

$$V_{c1} = \frac{G_m}{C_1} \cdot \int_0^{T_2} V_{ref}\,dt = \frac{G_m}{C_1} \cdot V_{ref} \cdot T_2$$

For the first cycle::

$V_{c2_0} := 0V$ $\qquad V_{c2_0} = 0\,V$

$V_{tune_0} := V'_{tune}(V_{ref} - V_{c2_0})$ $\qquad V_{tune_0} = 1\,V$

$G_{m_0} := G'_m(V_{tune_0})$ $\qquad G_{m_0} = 60\,\frac{\mu A}{V}$

$V_{c1_0} := \frac{G_{m_0}}{C_1} \cdot V_{ref} \cdot T_2$ $\qquad V_{c1_0} = 0.6\,V$

For the second cycle:

$V_{tune_1} := V'_{tune}(V_{ref} - V_{c2_0})$ $\qquad V_{tune_1} = 1\,V$

$G_{m_1} := G'_m(V_{tune_1})$ $\qquad G_{m_1} = 60\,\frac{\mu A}{V}$

$V_{c1_1} := \frac{G_{m_1}}{C_1} \cdot V_{ref} \cdot T_2$ $\qquad V_{c1_1} = 0.6\,V$

$V_{c2_1} := \frac{V_{c2_0} \cdot C_2 + V_{c1_1} \cdot C_1}{C_1 + C_2}$ $\qquad V_{c2_1} = 0.2\,V$

For the third cycle:

$V_{tune_2} := V'_{tune}(V_{ref} - V_{c2_1})$ $\qquad V_{tune_2} = 1\,V$

$G_{m_2} := G'_m(V_{tune_2})$ $\qquad G_{m_2} = 60\,\frac{\mu A}{V}$

$V_{c1_2} := \frac{G_{m_2}}{C_1} \cdot V_{ref} \cdot T_2$ $\qquad V_{c1_2} = 0.6\,V$

$V_{c2_2} := \frac{V_{c2_1} \cdot C_2 + V_{c1_2} \cdot C_1}{C_1 + C_2}$ $\qquad V_{c2_2} = 0.333\,V$

For the fourth cycle:

$$V_{tune_3} := V'_{tune}\left(V_{ref} - V_{c2_2}\right) \qquad V_{tune_3} = 1\,V$$

$$G_{m_3} := G'_m\left(V_{tune_3}\right) \qquad G_{m_3} = 60\,\frac{\mu A}{V}$$

$$V_{c1_3} := \frac{G_{m_3}}{C_1} \cdot V_{ref} \cdot T_2 \qquad V_{c1_3} = 0.6\,V$$

$$V_{c2_3} := \frac{V_{c2_2} \cdot C_2 + V_{c1_3} \cdot C_1}{C_1 + C_2} \qquad V_{c2_3} = 0.422\,V$$

During steady-state:

$$V_{c1} = V_{c2} = \frac{40 \cdot \frac{\mu A}{V} + V_{tune\_ss} \cdot 20 \cdot \frac{\mu A}{V^2}}{C_1} \cdot V_{ref} \cdot T_2 = V_{ref} - \frac{V_{tune\_ss}}{a}$$

$$V_{tune\_ss} := 0.5V$$

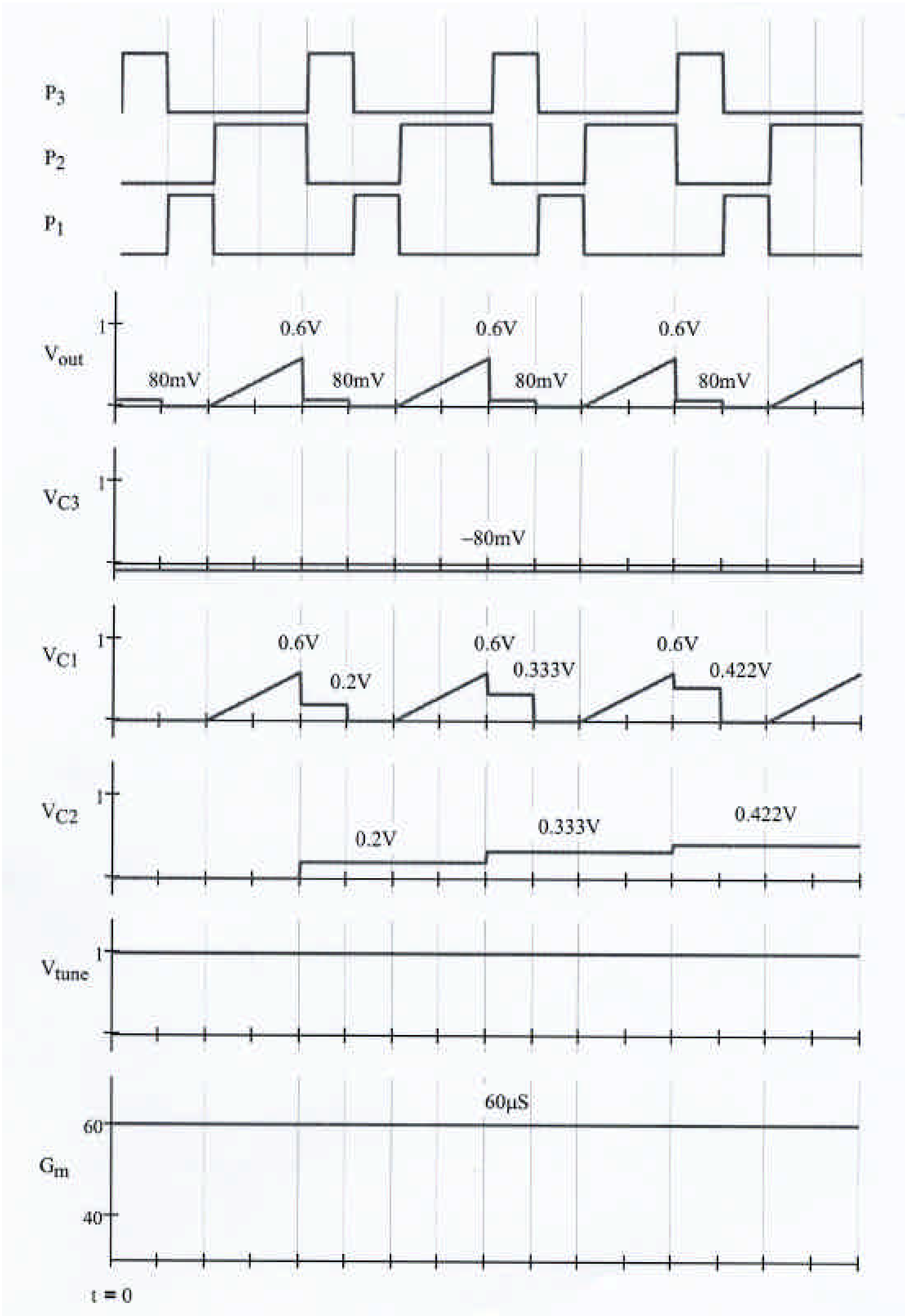
Given

$$\frac{40 \cdot \frac{\mu A}{V} + V_{tune\_ss} \cdot 20 \cdot \frac{\mu A}{V^2}}{C_1} \cdot V_{ref} \cdot T_2 = V_{ref} - \frac{V_{tune\_ss}}{a}$$

$$V_{tune\_ss} := \text{Find}\left(V_{tune\_ss}\right) \qquad V_{tune\_ss} = 476.19\,mV$$

$$V_{c2\_ss} := V_{ref} - \frac{V_{tune\_ss}}{a} \qquad V_{c2\_ss} = 495.238\,mV$$

$$V_{c1\_ss} := \frac{40 \cdot \frac{\mu A}{V} + V_{tune\_ss} \cdot 20 \cdot \frac{\mu A}{V^2}}{C_1} \cdot V_{ref} \cdot T_2 \qquad V_{c1\_ss} = 495.238\,mV$$



Comment for problem 1 by H.K.:

For simplicity, a lowpass filter typically used at the output of the Vtune generating amplifier was deleted. Thus, if you attempted simulating the circuit, it should be oscillating after a few cycles!
Also, in practice C2 is chosen to be much larger than C1. This helps generate a cleaner Vtune and also stabilizes the circuit.

Problem 2:

Problem #2:

Transistors Q3 and Q4 provide a means to control the drain-to-source voltages of Q1 and Q2. Since transistors Q1 and Q2 are in the triode region, their transconductances are dependent on their respective drain-to-source voltages.

The amplifiers are added to provide the proper gate bias on Q3 and Q4, using negative feedback, to force the drain-to-source voltages of Q1 and Q2 to be equal to Vc. Thus, the transconductances of Q1 and Q2 are dependent on the control voltage Vc.

Given parameters:

$k' := 100\frac{\mu A}{V^2}$ $\quad W := 10\mu m$ $\quad L := 1\mu m$ $\quad V_{th} := 0.5V$ $\quad V_{cm} := 1V$

For transistors in triode, we can find the transconductance as:

$$I_d = k' \cdot \frac{W}{L} \cdot \left[\left(V_{gs} - V_{th}\right) \cdot V_{ds} - \frac{V_{ds}^{\;2}}{2}\right]$$

$$g_m = \frac{\partial}{\partial V_{gs}} I_d = k' \cdot \frac{W}{L} V_{ds} \qquad g_m(V_c) := k' \cdot \frac{W}{L} \cdot V_c$$

$g_m(0.2V) = 0.2\,mS$ $\qquad g_m(0.4V) = 0.4\,mS$

The unity gain frequency of the integrator can be expressed as:

$C_{int} := 10pF$

$$\omega_u(V_c) := \frac{g_m(V_c)}{2 \cdot C_{int}} \qquad f_u(V_c) := \frac{\omega_u(V_c)}{2 \cdot \pi}$$

$f_u(0.2V) = 1.592\,MHz$ $\qquad f_u(0.4V) = 3.183\,MHz$

When Vi=0, we can express the drain currents of Q1 and Q2 as:

$$I_d(V_c) := k' \cdot \frac{W}{L} \cdot \left[\left(V_{cm} - V_{th}\right) \cdot V_c - \frac{V_c^{\;2}}{2}\right]$$

$I_d(0.2V) = 80\,\mu A$ $\qquad I_d(0.4V) = 120\,\mu A$

To exit the triode region, the drain-to-source voltage must be less than the gate overdrive voltages. Thus:

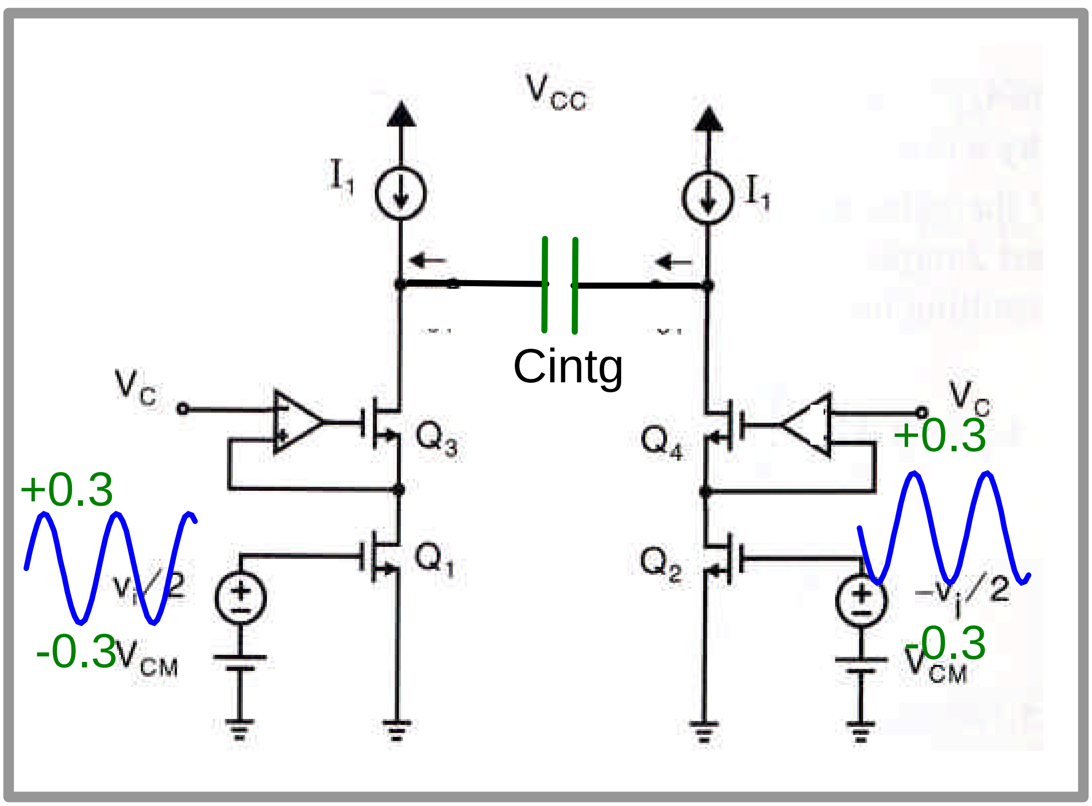
$$V_c < \frac{V_i}{2} + V_{cm} - V_{th}$$

$$V_{i\_max}(V_c) := \left|2 \cdot \left[V_c - \left(V_{cm} - V_{th}\right)\right]\right|$$

$V_{i\_max}(0.2V) = 600\,mV$ $\qquad$ $V_{i\_max}(0.4V) = 200\,mV$

Note that it is the large negative swings that will push Q1 and Q2 out of the triode region and into the active region.

Comment by H.K.



Notice that for example in case a) where to find the maximum voltage swing at the input of Q, need to find the voltage which drives Q1 to edge of saturation, the max voltage swing at the input of Q1 is 0.6V and the overall differentially measured peak-to-peak voltage is: $V_{ppd}{}^{max}$=<u>1.2V</u>. Simularly, in case b) $V_{ppd}{}^{max}$=<u>0.4V</u>