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College of Engineering
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Midterm Exam
Thursday, October 27, 2005

EECS 247
FALL 2005

Name: _____ **Solution** _____

SID: _____

Score: **/60**

Problem 1 20point	Problem 2 20point	Problem 3 20point	Total Score 60point
			Class average 51

- **Closed books and notes, no calculators allowed**
- **Mark all results with a box around it.**
- Write solutions on the exam sheets. Add extra pages where needed.
- Simplify algebraic results as much as possible.
- Bring into standard form where applicable.
- Show your work as much as possible in order to get points for partial results.

Useful Expressions:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^x \approx 1 + x \quad \text{for small } x,$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2} \quad \text{for small } x$$

$$\frac{1}{1+x} \approx 1 - x \quad \text{for small } x$$

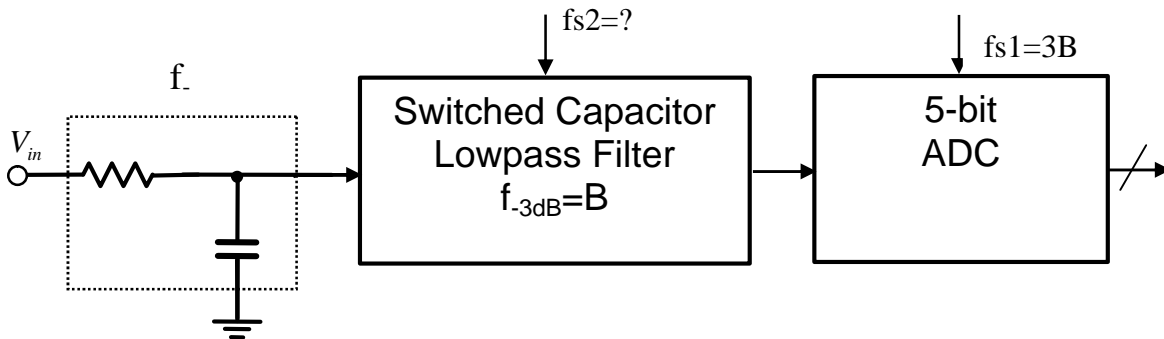
Problem 1:

The block diagram of a data acquisition system is shown below. The system characteristics are as follows:

- The incoming signal is expected to be wide-band with constant signal magnitude at all frequencies.

- The signal bandwidth of interest spans from DC to frequency B and the sampling frequency for the 5-bit ADC is $f_{s1}=3B$.
- The front-end RC filter is off-chip and has accurate -3dB frequency equal to $4B$.
- The filter preceding the ADC is of switched capacitor type with sampling frequency f_{s2} . This filter has a low-pass Butterworth shape with $f_{-3dB}=B$. The Butterworth lowpass filter provides 6dB-per-octave-per-pole (or 20dB-per-decade-per-pole) attenuation.

- Explain the purpose for having each of the two filters in the system.
- Find the minimum order for the switched capacitor filter such that all possible spurious signals are attenuated to -6dB with respect to the ADC dynamic range.
- Find the minimum sampling frequency (f_{s2}) for the switched-capacitor filter. Assume the same conditions as in part b.
- Find the overall magnitude response droop from 0 to $B/2$, assuming the ADC has a perfectly flat bandwidth within the frequency range of interest.



Solution:

- The SCF acts as an anti-aliasing filter for the ADC since the sampling function of the ADC aliases signals at $Nf_{s1} \pm B$ down to the band of interest. In addition, it may be used to reduce the out-of-band signal power in order to allocate the entire ADC dynamic range to the inband signal. The RC filter performs anti-aliasing for the SCF.
- The attenuation required at $[(f_{s1}-B)=2B]$ is $[(5\text{bits} \times 6\text{dB/bit}) + 6\text{dB}] = 36\text{dB}$. Since the SCF filter attenuation is 6dB-per-octave-per-order and the transition band is $2B/B=2$ or one octave then the filter order is found to be: $36/6 \rightarrow$ **Filter order = 6**
- The RC filter provides 6dB-per-octave attenuation. The attenuation required at $f_{s2}-B$ is 36dB. Since the filter is 1st order then to achieve 36dB attenuation 6 octave or $x2^6$ is required for $(f_{s2}-B)/4B=2^6 \rightarrow$ **$f_{s2}^{\min}=(2^8+1)B=257B$**

d) There are 4 sources of magnitude droop:

1- Droop due to the RC filter magnitude response. The filter roll-off is given by:

$$H(s) = \frac{1}{1 + j\frac{f}{4B}}$$

Magnitude droop at $B/2$:

$$H(s) = \frac{1}{1 + j\frac{1}{8}} \quad |H(s)| = \frac{1}{\sqrt{1 + \frac{1}{64}}} \approx \frac{1}{1 + \frac{1}{128}} \approx 1 - \frac{1}{128} \rightarrow \text{droop} = \frac{1}{128} < 1\%$$

2- Note that the SCF is of Butterworth type with maximally flat magnitude response and thus does not incur impairments in the magnitude response over the frequency range 0 to B/2.

3- Droop due to the $\sin x/x$ response of the SCF. Since $f_s \gg B/2$ this is negligible

4- Droop due to $\sin x/x$ associated with the ADC:

$$H(f) = \frac{\sin(\mathbf{p} f / f_s)}{\mathbf{p} f / f_s} = \frac{\sin(\mathbf{p} \times 0.5B / 3B)}{\mathbf{p} \times 0.5B / 3B} = \frac{\sin(\mathbf{p} / 6)}{\mathbf{p} / 6} = 0.955 \rightarrow \text{droop } 4.5\%$$

Considering droop associated with 1) & 4)

Overall droop $\rightarrow 5.2\%$ (not expected to calculate the final #!)

Problem 2:

The end point DNL for a 3-bit ADC is measured to be:

DNL: 0 -0.5 0 +0.5 -1 +0.5 +0.5 0 (all numbers in terms of LSB)

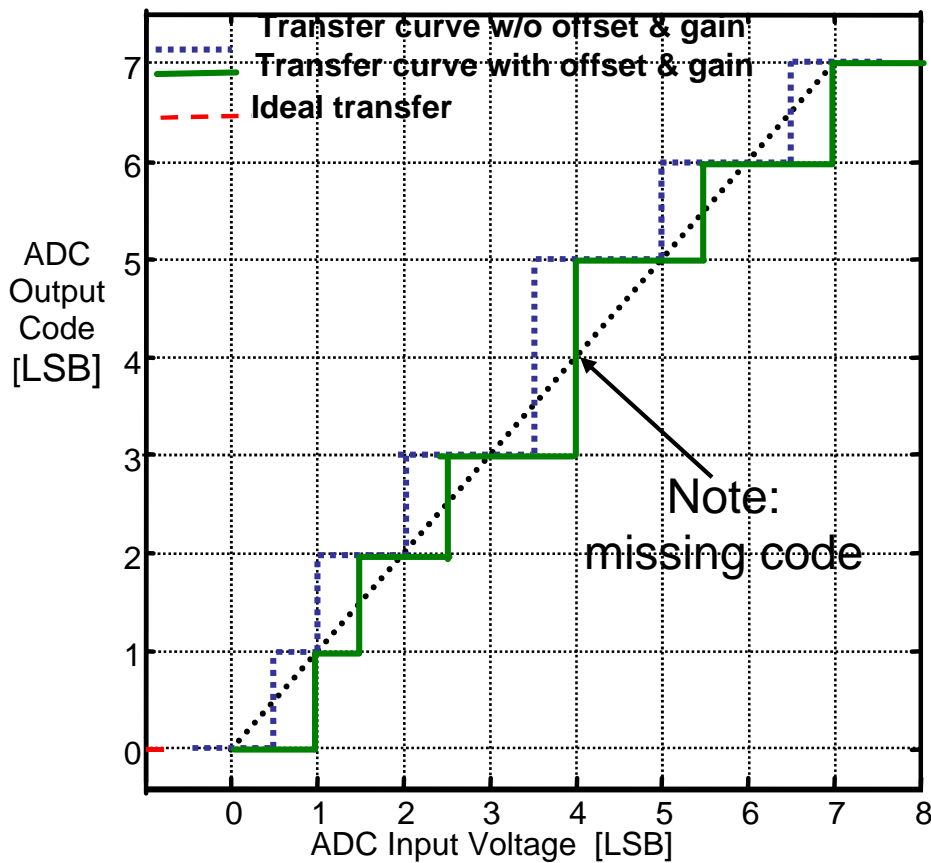
The ADC exhibits offset and full-scale error:

Offset error = +0.5LSB

Full scale error = +0.5LSB

- a) Find the INL for this ADC
- b) Draw the transfer curve on the graph below.

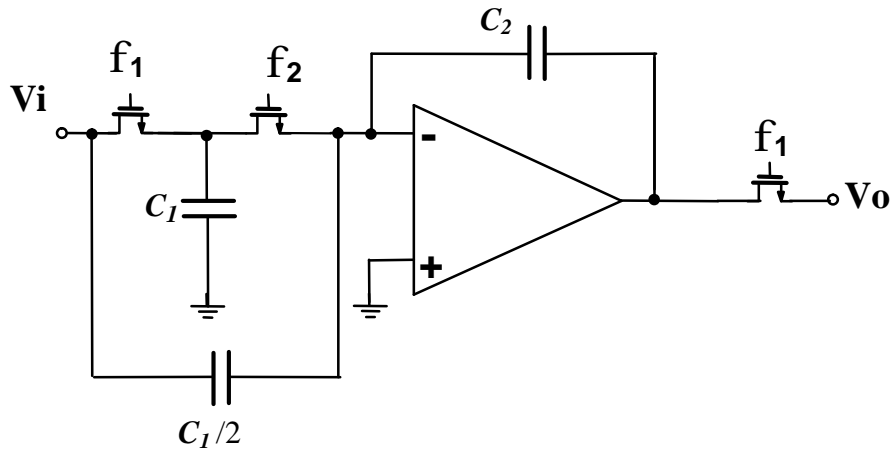
INL: 0 -0.5 -0.5 0 -1 -0.5 0 0



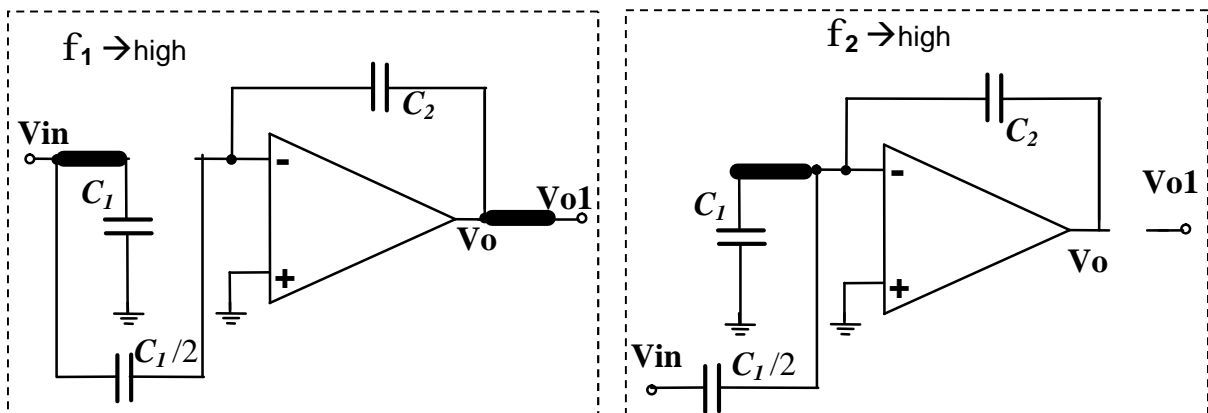
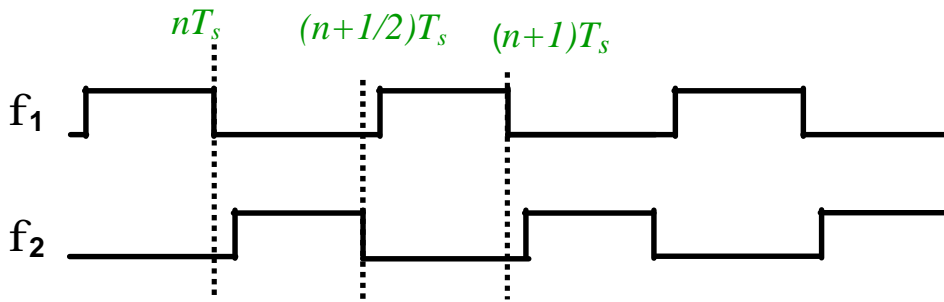
Problem 3:

For the switched capacitor network shown:

- Find the z-domain transfer function $H(z)$.
- Identify the type of circuit.
- Set $z=e^{j\omega T}$ and find an approximate expression for the magnitude response. Assume $f_{\text{signal}} \ll f_{\text{clk}}$.



The clock is a 2-phase non-overlapping signal as shown below



$$f1 \rightarrow Vo1[n] = Vo[n] = Vo1[(n-1/2)] - \frac{C1}{2C2} \{Vin[n] - Vin[n-1/2]\}$$

$$f2 \rightarrow Vo1[n+1/2] = Vo[n],$$

$$Vo[n+1/2] = Vo[n] - \frac{C1}{C2} Vin[n] - \frac{C1}{2C2} \{Vin[n+1/2] - Vin[n]\}$$

$$f1 \rightarrow Vo1[n+1] = Vo[n+1] = Vo[(n+1/2)] - \frac{C1}{2C2} \{Vin[n+1] - Vin[n+1/2]\}$$

Substituting for $Vo[(n+1/2)]$ from above:

$$Vo1[n+1] = Vo[n+1] = Vo[n] - \frac{C1}{C2} Vin[n] - \frac{C1}{2C2} \{Vin[n+1] - Vin[n]\}$$

Z-Transform:

$$Z Vo1 = Vo1 - \frac{C1}{C2} Vin - \frac{C1}{2C2} Vin[Z-1]$$

Re-arranging:

$$\boxed{\frac{V_o}{V_{in}}(Z) = -\frac{C1}{2C2} \times \frac{1+Z^{-1}}{1-Z^{-1}} \rightarrow \text{Bilinear integrator}}$$

Notice that the capacitor $C1/2$ acts as a voltage divider and it does not donate charge to $C2$ while $C1$ has its charge transferred to $C2$ and hence the integrator function.

