EE247
Lecture 3

- Last week's summary
- Active Filters
  - Active biquads
    - Sallen- Key & Tow-Thomas
    - Integrator based filters
      - Signal flowgraph concept
      - First order integrator based filter
      - Second order integrator based filter & biquads
  - High order & high Q filters
    - Cascaded biquads
      - Cascaded biquad sensitivity
    - Ladder type filters

Summary
Last Week

- Major success in CMOS technology scaling:
  → Inexpensive DSPs technology
  → Resulted in the need for high performance Analog/Digital interface circuitry
- Main Analog/Digital interface building blocks includes
  - Analog filters
  - D/A converters
  - A/D converters

Monolithic Filters

- Monolithic inductor in CMOS tech.
  - Integrated L<10nH with Q<10 combined with max. cap. 10pF

  → *LC filters in the monolithic form feasible: freq>500MHz*

- Analog/Digital interface circuitry require fully integrated filters with critical frequencies << 500MHz
- Good alternative:

  ⇨Active filters built without the need for inductors

2^{nd} Order Transfer Functions (Biquads)

- Biquadratic (2^{nd} order) transfer function:

$$H(s) = \frac{1}{1 + \frac{s}{w_P Q_P} + \frac{s^2}{w_P^2}}$$

$$|H(jw)|_{w=0} = 1$$
$$|H(jw)|_{w \to \infty} = 0$$
$$|H(jw)|_{w=w_P} = Q_P$$

Biquad poles @: $s = -\frac{w_P}{2Q_P}\left(1 \pm \sqrt{1 - 4Q_P^2}\right)$

for $Q_P \leq \frac{1}{2}$ *poles are real, complex otherwise*

Biquad Complex Poles

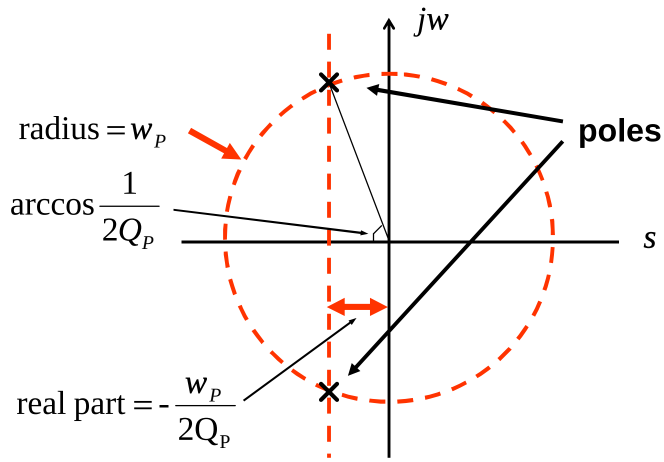
$$Q_P > \tfrac{1}{2}$$

$$s = -\frac{w_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Distance from origin in s-plane:

$$d^2 = \left(\frac{w_P}{2Q_P}\right)^2 \left(1 + 4Q_P^2 - 1\right)$$
$$= w_P^2$$

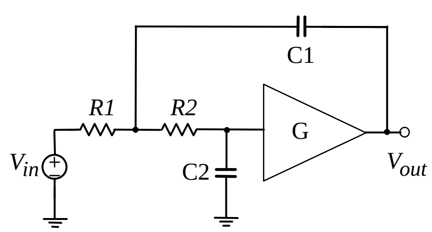
s-Plane



Implementation of Biquads

- Passive RC: only real poles- can't implement complex conjugate poles
- Terminated LC
  - Low power, since it is passive
  - Only noise source → load and source resistance
  - As previously analyzed, not feasible in the monolithic form for $f < 500MHz$
- Active Biquads
  - Many topologies can be found in filter textbooks!
  - Widely used topologies:
    - Single-opamp biquad: Sallen-Key
    - Multi-opamp biquad: Tow-Thomas
    - Integrator based biquads

Active Biquad
Sallen-Key Low-Pass Filter



$$H(s) = \frac{G}{1 + \frac{s}{w_P Q_P} + \frac{s^2}{w_P^2}}$$

$$w_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q_P = \frac{w_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

- Single gain element
- Can be implemented both in discrete & monolithic form
- "Parasitic sensitive"
- Versions for LPF, HPF, BP, …

  → Advantage: Only one opamp used

  → Disadvantage: Sensitive to parasitics

Imaginary Axis Zeros

- Sharpen transition band
- "notch out" interference
- High-pass filter (HPF)
- Band-reject filter

$$H(s) = K\frac{1+\left(\frac{s}{w_Z}\right)^2}{1+\frac{s}{w_P Q_P}+\left(\frac{s}{w_P}\right)^2}$$

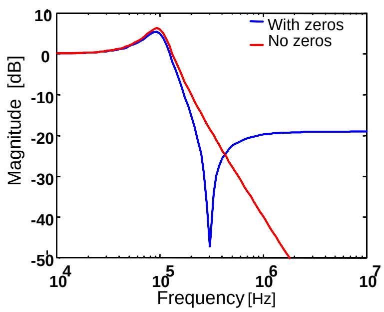
$$\left|H(jw)\right|_{w\to\infty} = K\left(\frac{w_P}{w_Z}\right)^2$$

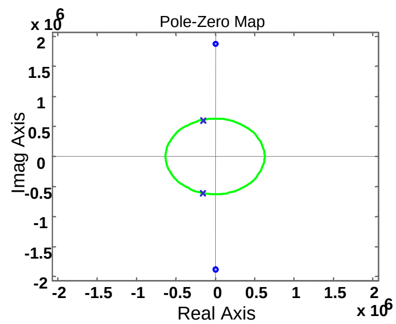
Note: Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, reasonable magnitude, and easily checkable unit.

Imaginary Zeros

- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequency

$$f_P = 100kHz$$
$$Q_P = 2$$
$$f_Z = 3f_P$$



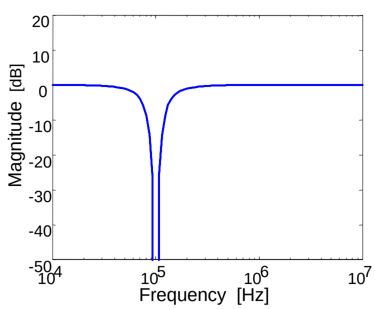


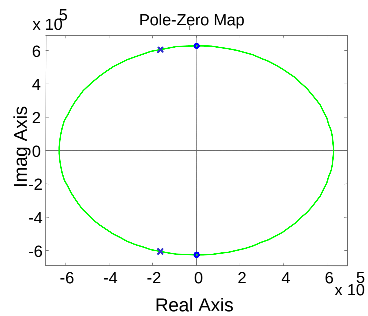
Moving the Zeros

$f_P = 100kHz$

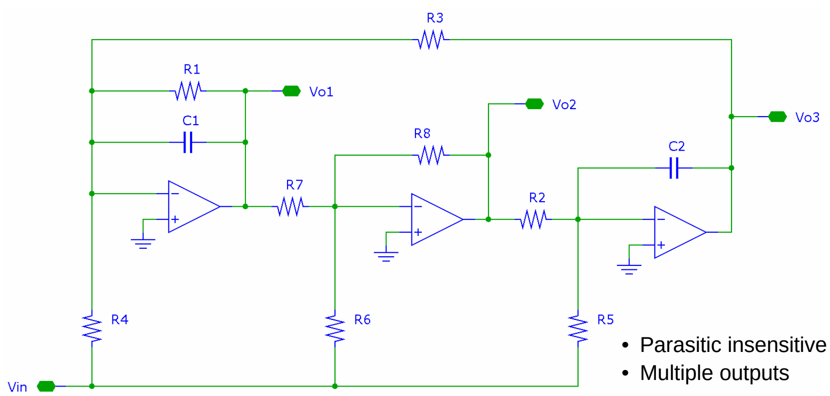
$Q_P = 2$

$\underline{f_Z = f_P}$





Tow-Thomas Active Biquad



- Parasitic insensitive
- Multiple outputs

Ref: P. E. Fleischer and J. Tow, "Design Formulas for biquad active filters using three operational amplifiers," Proc. IEEE, vol. 61, pp. 662-3, May 1973.

Frequency Response

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1\sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

- V_{o2} implements a general biquad section with arbitrary poles and zeros
- V_{o1} and V_{o3} realize the same poles but are limited to at most one finite zero

Component Values

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right)$$

$$b_2 = \frac{R_8}{R_6}$$

$$a_0 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

$$a_1 = \frac{1}{R_1 C_1}$$

$$k_1 = \sqrt{\frac{R_2 R_8 C_2}{R_3 R_7 C_1}}$$

$$k_2 = \frac{R_7}{R_8}$$

given a_i, b_i, k_i, C_1, C_2 and R_8

$$R_1 = \frac{1}{a_1 C_1}$$

$$R_2 = \frac{k_1}{\sqrt{a_0} C_2}$$

$$R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2 - b_1} \frac{1}{C_1}$$

$$R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2}$$

$$R_6 = \frac{R_8}{b_2}$$

$$R_7 = k_2 R_8$$

it follows that

$$w_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$Q_P = w_P R_1 C_1$$

Integrator Based Filters

- Main building block for this category of filters → integrator
- By using signal flowgraph techniques → conventional filter topologies can be converted to integrator based type filters
- Next few pages:
  - Signal flowgraph techniques
  - 1st order integrator based filter
  - 2nd order integrator based filter
  - High order and high Q filters

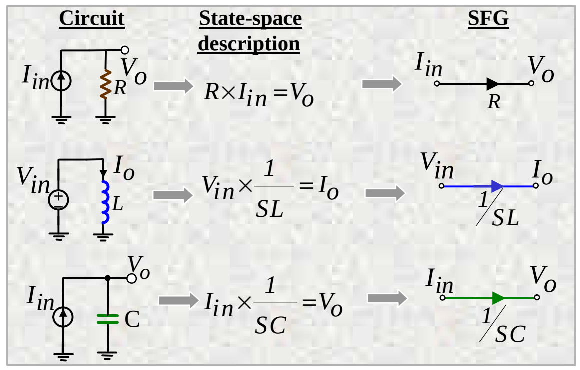
What is a Signal Flowgraph (SFG)?

- SFG → Topological network representation consisting of nodes & branches- used to convert one form of network to a more suitable form (e.g. passive RLC filters to integrator based filters)
- Any network described by a set of linear differential equations can be expressed in SFG form.
- For a given network, many different SFGs exists.
- Choice of a particular SFG is based on practical considerations such as type of available components.

*Ref: W.Heinlein & W. Holmes, "Active Filters for Integrated Circuits", Prentice Hall, Chap. 8, 1974.

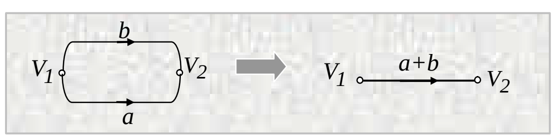
What is a Signal Flowgraph (SFG)?

- SFG nodes represent variables (*V & I* in our case), branches represent transfer functions (we will call these transfer functions branch multiplication factor *BMF*)
- To convert a network to its SFG form, *KCL* & *KVL* is used to derive state space description:
- Example:

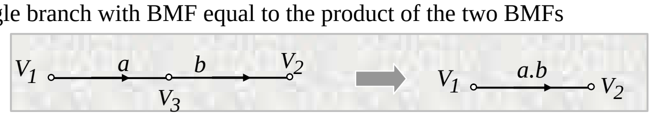


Signal Flowgraph (SFG) Rules

- Two parallel branches can be replaced by a single branch with overall BMF equal to sum of two BMFs



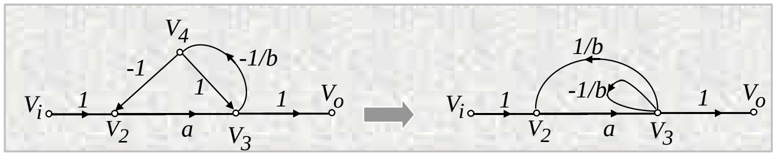
- A node with only one incoming branch & one outgoing branch can be replaced by a single branch with BMF equal to the product of the two BMFs



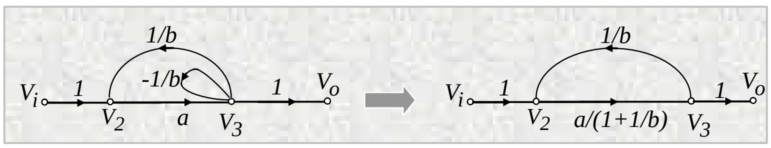
- An intermediate node can be multiplied by a factor (x). BMFs for incoming branches have to be multiplied by x and outgoing branches divided by x

Signal Flowgraph (SFG) Rules

- Simplifications can often be achieved by shifting or eliminating nodes

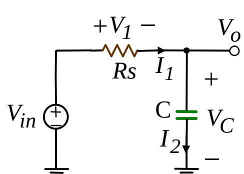


•A self-loop branch with BMF y can be eliminated by multiplying the BMF of incoming branches by 1/(1-y)



Integrator Based Filters
1st Order LPF

- Start from RC prototype
- Use KCL & KVL to derive state space description:



$$\frac{V_o}{V_{in}} = \frac{1}{1+sRC}$$

- Use state space description to draw signal flowgraph (SFG)

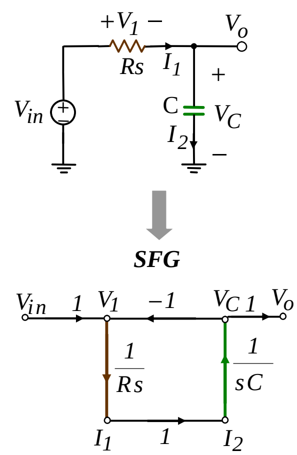
Integrator Based Filters

First Order LPF

•KCL & KVL to derive state space description:

$$V_C = \frac{I_2}{sC}$$
$$V_1 = V_{in} - V_C$$
$$I_1 = \frac{V_1}{Rs}$$
$$I_2 = I_1$$
$$V_o = V_C$$

• Use state space description to draw signal flowgraph (SFG)



Normalize

- Since integrators - the main building blocks- require in & out signals in the voltage form (not current)

  → Convert all currents to voltages by multiplying current nodes by a scaling resistance R^*

  → Corresponding BMFs should then be scaled accordingly

$$V_1 = V_{in} - V_o \qquad I_1 = \frac{V_1}{Rs} \qquad V_o = \frac{I_2}{sC} \qquad I_2 = I_1$$

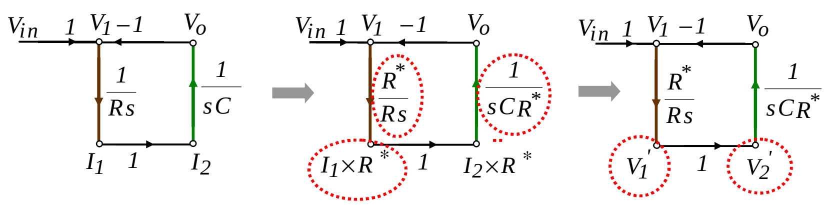
$$\longrightarrow \boxed{I_x R^* = V_x'} \longrightarrow$$

$$V_1 = V_{in} - V_o \qquad I_1 R^* = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{I_2 R^*}{sC\ R^*} \qquad I_2 R^* = I_1 R^*$$

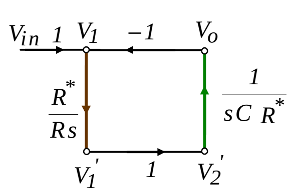
$$\Rightarrow$$

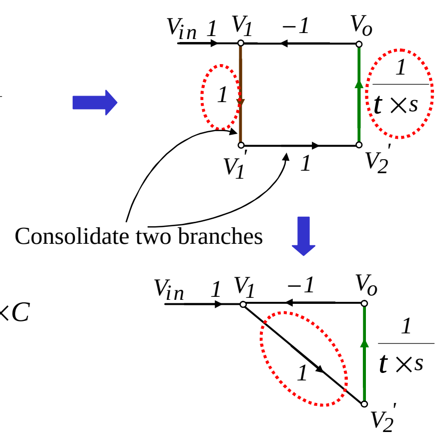
$$V_1 = V_{in} - V_o \qquad V_1' = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{V_2'}{sC\ R^*} \qquad V_2' = V_1'$$

Normalize



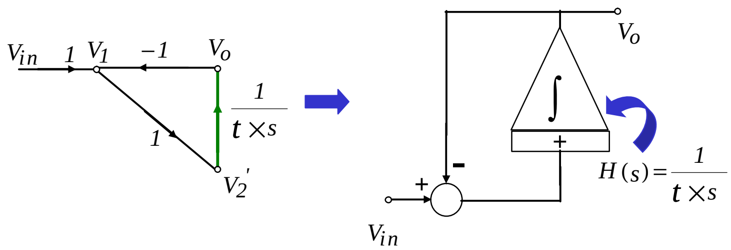
Synthesis



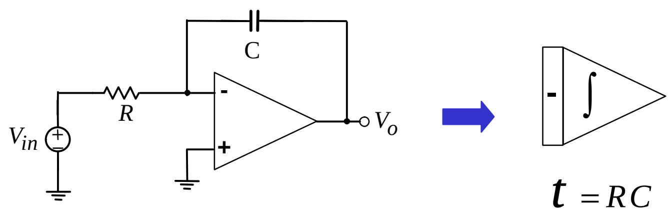


$$R^* = Rs \quad , \quad t = R^* \times C$$

First Order Integrator Based Filter



Opamp-RC Single-Ended Integrator

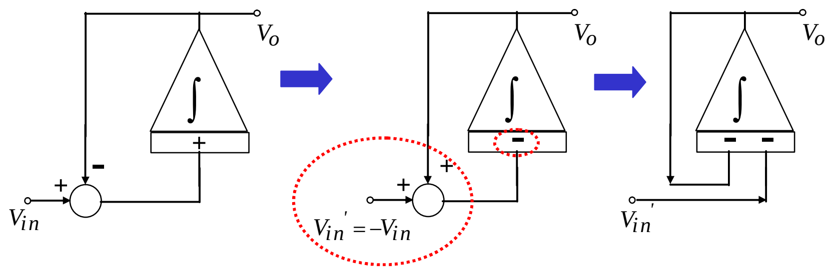


$$V_o = -\frac{1}{RC}\int V_{in}\,dt \quad , \quad \frac{V_o}{V_{in}} = -\frac{1}{sRC}$$

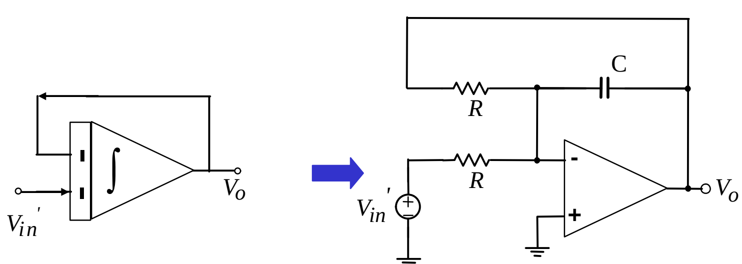
1st Order Filter Built with Opamp-RC Integrator

- Single-ended Opamp-RC integrator has a sign inversion from input to output

  → Convert SFG accordingly by modifying BMF



1st Order Filter Built with Opamp-RC Integrator (continued)



$$\frac{V_o}{V_{in}'} = -\frac{1}{1+sRC}$$

Opamp-RC 1st Order Filter Noise

Identify noise sources (here it is resistors & opamp)
Find transfer function from each noise source
to the output (opamp noise next page)

$$\overline{v_o^2} = \sum_{m=1}^{n} \int_0^{\infty} |H_m(f)|^2 S_i(f)\, df$$

$S_i(f) \rightarrow Input\ referred\ noise\ spectral\ density$

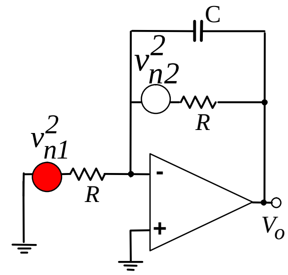
$$|H_1(f)|^2 = |H_2(f)|^2 = \frac{1}{1+(2p\, fRC)^2}$$

$$v_{n1}^2 = v_{n2}^2 = 4KTR\Delta f$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2\frac{k\,T}{C}}$$

$a = 2$

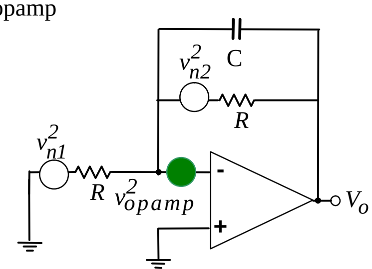
α



Typically, α increases as filter order increases

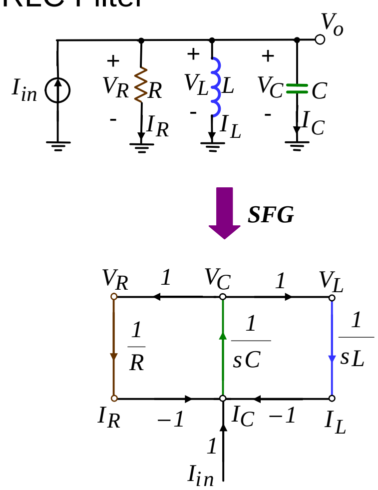
Opamp-RC Filter Noise Opamp Contribution

So far only the fundamental noise sources are considered.
In reality, noise associated with the opamp increases the overall noise.
The bandwidth of the opamp affects the opamp noise contribution to the total noise



Integrator Based Filter

2^{nd} Order RLC Filter



- State space description:

$$V_R = V_L = V_C = V_o$$

$$V_C = \frac{I_C}{sC}$$

$$I_R = \frac{V_R}{R}$$

$$I_L = \frac{V_L}{sL}$$

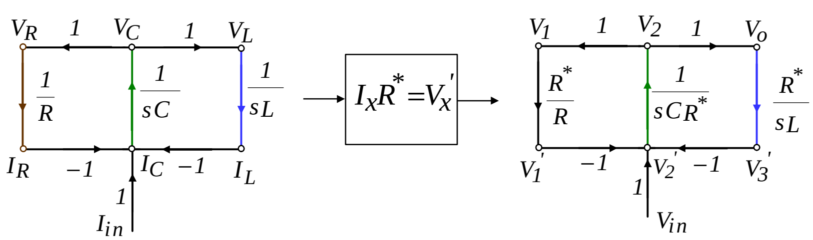
$$I_C = I_{in} - I_R - I_L$$

Integrator form

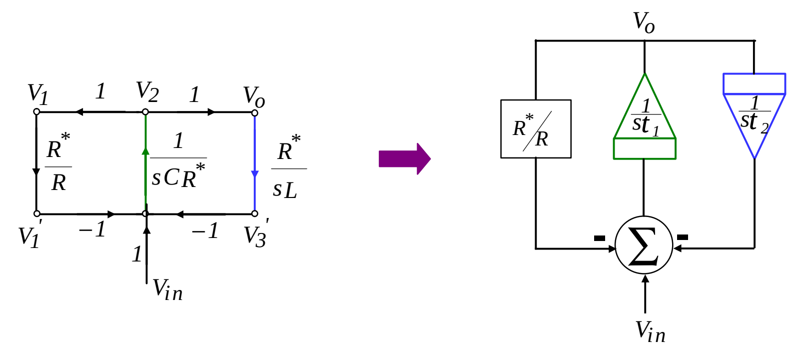
- Draw signal flowgraph (SFG)

Normalize

- Convert currents to voltages by multiplying all current nodes by the scaling resistance R^*

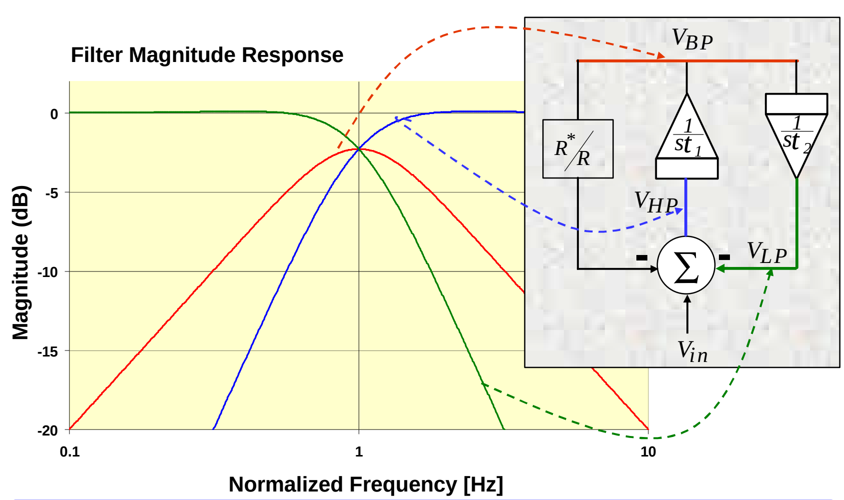


Synthesis



$t_1 = R^* \times C \quad t_2 = L / R^*$

Second Order Integrator Based Filter



Second Order Integrator Based Filter

$$\frac{V_{BP}}{V_{in}} = \frac{t_2 s}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

$$\frac{V_{LP}}{V_{in}} = \frac{1}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

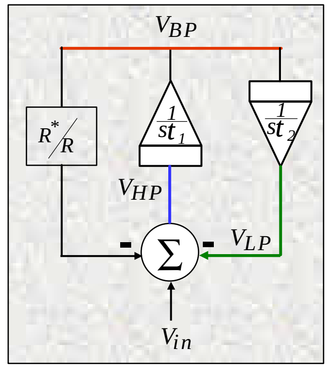
$$\frac{V_{HP}}{V_{in}} = \frac{t_1 t_2 s^2}{t_1 t_2 s^2 + b\, t_2 s + 1}$$

$$t_1 = R^* \times C \qquad t_2 = L \Big/ R^*$$

$$b = R^* \Big/ R$$

$$w_0 = 1 \Big/ \sqrt{t_1 t_2} = 1 \Big/ \sqrt{L\,C}$$

$$Q = 1/b \times \sqrt{t_1 \Big/ t_2}$$



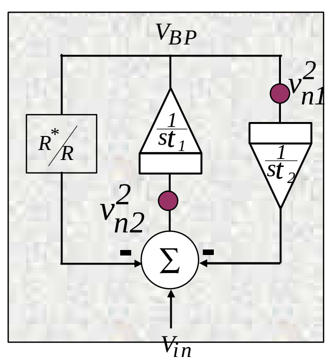
From matching point of view desirable:

$$t_1 = t_2 \rightarrow Q = R \Big/ R^*$$

Second Order Bandpass Filter Noise

$$\overline{v_o^2} = \sum_{m=1}^{n} \int_0^{\infty} |H_m(f)|^2 S_i(f)\, df$$

- Find transfer function of each noise source to the output
- Integrate contribution of all noise sources
- Here it is assumed that opamps are noise free (not usually the case!)



$$v_{n1}^2 = v_{n2}^2 = 4KTRdf$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2\,Q\,\frac{k\,T}{C}}$$

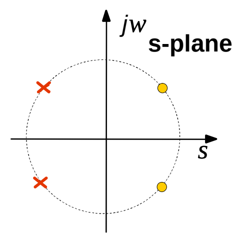
α

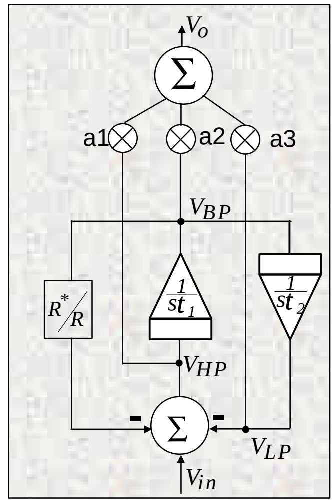
Typically, α increases as filter order increases
Note the noise power is directly proportion to Q

Second Order Integrator Based Filter Biquad

- By combining outputs can generate general biquad function:

$$\frac{V_0}{V_{in}} = \frac{a_1 t_1 t_2 s^2 + a_2 t_2 s + a_3}{t_1 t_2 s^2 + b\, t_2 s + 1}$$





Summary Integrator Based Monolithic Filters

- Signal flowgraph techniques utilized to convert RLC networks to integrator based active filters
- Each reactive element (L& C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor:

  – For lowpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a\,\frac{k\,T}{C}}$

  – Bandpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a\;Q\,\frac{k\,T}{C}}$

  where a is a function of filter order and topology

Higher Order Filters

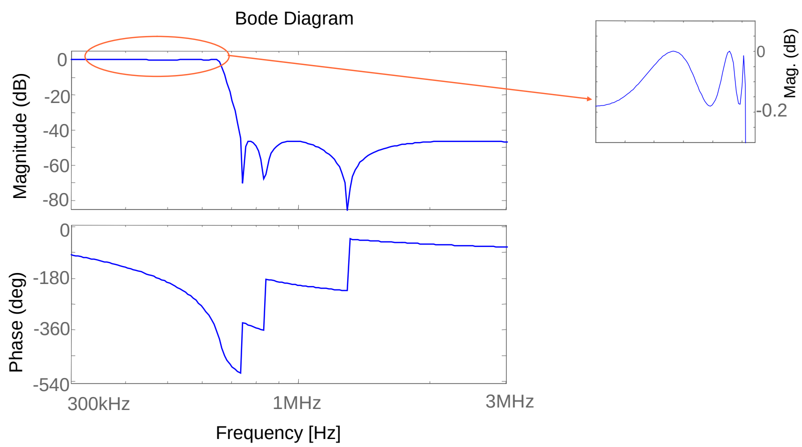
- How do we build higher order filters?
  - Cascade of biquads and 1st order sections
    - Each complex conjugate pole built with a biquad and real pole with 1st order section
    - Easy to implement
    - In the case of high order high Q filters → highly sensitive to component variations
  - Direct conversion of high order ladder type RLC filters
    - SFG techniques used to perform exact conversion of ladder type filters to integrator based filters
    - More complicated conversion process
    - Much less sensitive to component variations compared to cascade of biquads

Higher Order Filters
Cascade of Biquads

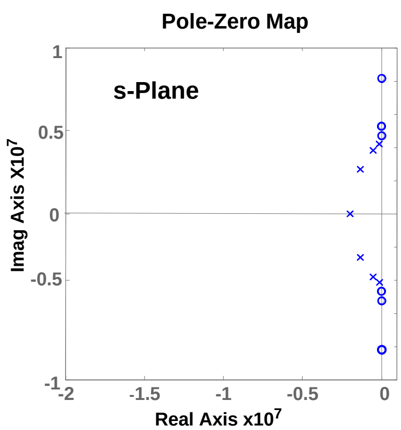
Example: LPF filter for CDMA baseband receiver

- LPF with
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 45 dB
  - Assumption: Can compensate for phase distortion in the digital domain
- 7th order Elliptic Filter
- Implementation with Biquads
  Goal: Maximize dynamic range
  - Pair poles and zeros
    highest Q poles with closest zeros is a good starting point,
    but not necessarily optimum
  - Ordering:
    Lowest Q poles first is a good start

Filter Frequency Response



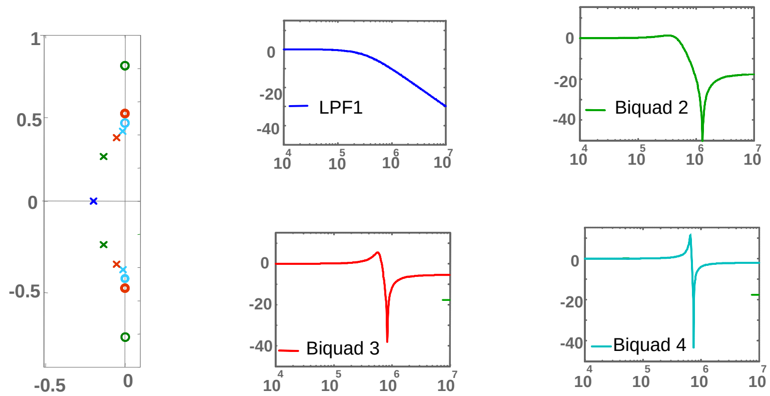
Pole-Zero Map



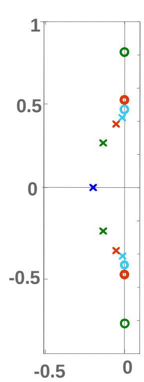
Q_{pole}	f_{pole} [kHz]
16.7902	659.496
3.6590	611.744
1.1026	473.643
	319.568

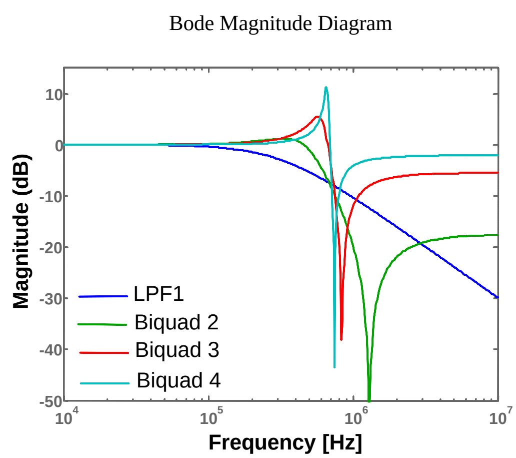
f_{zero} [kHz]
1297.5
836.6
744.0

Biquad Response

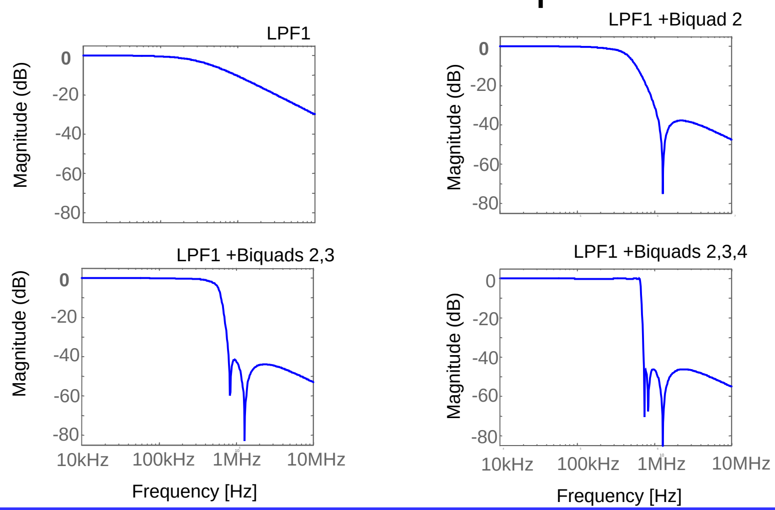


Biquad Response





Intermediate Outputs

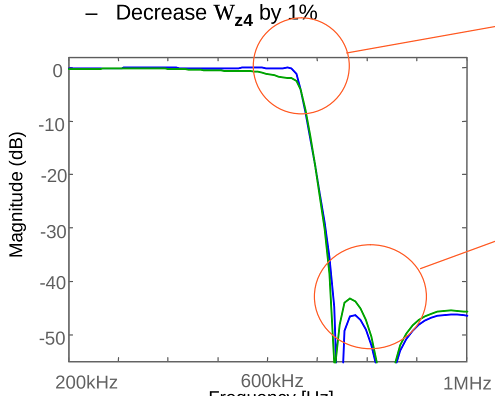


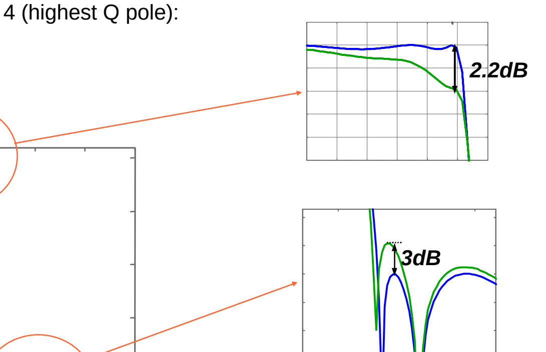
Sensitivity

Component variation in Biquad 4 (highest Q pole):

– Increase W_{p4} by 1%

– Decrease W_{z4} by 1%





High Q poles → High sensitivity in Biquad realizations

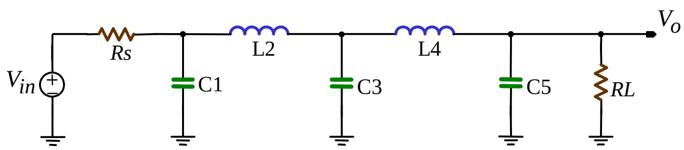
High Q & High Order Filters

- Cascade of biquads
  - Highly sensitive to component variations → not suitable for implementation of high Q & high order filters
  - Cascade of biquads only used in cases where required Q for all biquads <4 (e.g. filters for disk drives)
- LC ladder filters more appropriate for high Q & high order filters (next topic)
  - Less sensitive to component variations

Ladder Type Filters

- For simplicity, will start with all pole ladder type filters
  - Convert to integrator based form
  - Example shown
- Then will attend to high order ladder type filters incorporating zeros
  - Implement the same 7^{th} order elliptic filter in the form of ladder type
    - Find level of sensitivity to component variations
    - Compare with cascade of biquads
  - Convert to integrator based form utilizing SFG techniques
  - Example shown

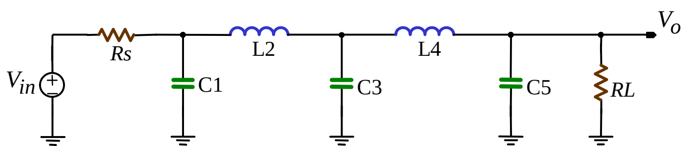
LC Ladder Filters



- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated (with or w/o R_L)

Doubly terminated LC ladder filters ⇨ Lowest sensitivity to component variations

LC Ladder Filters



- Design:
  - CAD tools
    - Matlab
    - Spice
  - Filter tables
    - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
    - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3rd edition, McGraw-Hill, 1995.

LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

f-3dB = 10MHz, fstop = 20MHz

Rs >27dB

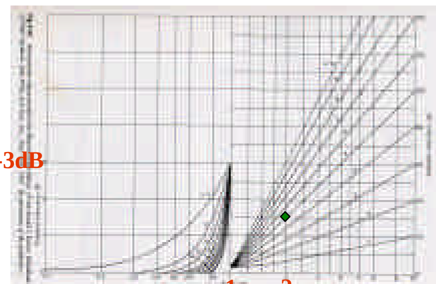
- Maximally flat passband ⇨ Butterworth
- Determine minimum filter order :
  - -Use of Matlab
  - -or Tables
- Here tables used

fstop / f-3dB = 2

Rs >27dB

Minimum Filter Order

⇨5th order Butterworth



From: Williams and Taylor, p. 2-37

LC Ladder Filter Design Example

Find values for L & C from Table: →

Note L &C values normalized to

$w_{-3dB} = 1$

Denormalization:

Multiply all L_{Norm}, C_{Norm} by:

$L_r = R/w_{-3dB}$

$C_r = 1/(R X w_{-3dB})$

R is the value of the source and termination resistor (choose both 1Ω for now)

Then: $L = L_r \times L_{Norm}$

$C = C_r \times C_{Norm}$

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

From: Williams and Taylor, p. 11.3

LC Ladder Filter Design Example

Find values for L & C from Table:

Normalized values:

$C1_{Norm} = C5_{Norm} = 0.618$

$C3_{Norm} = 2.0$

$L2_{Norm} = L4_{Norm} = 1.618$

Denormalization:

Since $w_{-3dB} = 2\pi X10MHz$

$L_r = R/w_{-3dB} = 15.9$ nH

$C_r = 1/(RXw_{-3dB}) = 15.9$ nF

$R = 1$

⇨C1=C5=9.836nF, C3=31.83nF

⇨L2=L4=25.75nH

NORMALIZED FILTER DESIGN TABLES 11.3

TABLE 11-2 Butterworth *LC* Element Values (*Continued*)

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
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	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
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	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
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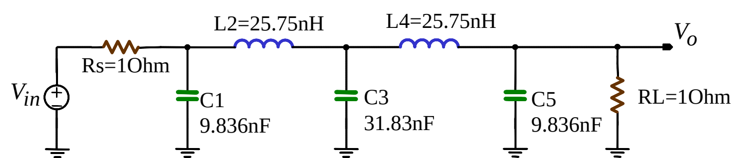
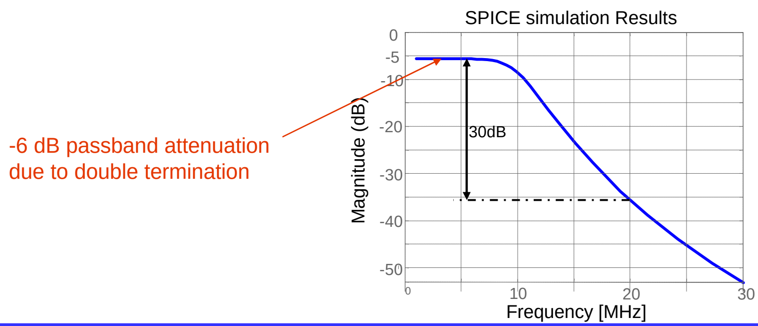
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

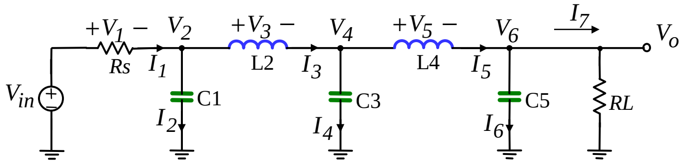
From: Williams and Taylor, p. 11.3

Magnitude Response Simulation





LC Ladder Filter
Conversion to Integrator Based Active Filter



- Use KCL & KVL to derive equations:

$V_1 = V_{in} - V_2$, $V_2 = \frac{I_2}{sC_1}$, $V_3 = V_2 - V_4$

$V_4 = \frac{I_4}{sC_3}$, $V_5 = V_4 - V_6$, $V_6 = \frac{I_6}{sC_5}$ $V_o = V_6$

$I_1 = \frac{V_1}{Rs}$, $I_2 = I_1 - I_3$, $I_3 = \frac{V_3}{sL_2}$

$I_4 = I_3 - I_5$, $I_5 = \frac{V_5}{sL_4}$, $I_6 = I_5 - I_7$, $I_7 = \frac{V_6}{RL}$

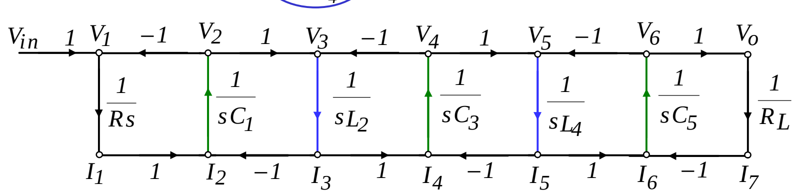
LC Ladder Filter
Signal Flowgraph

$V_1 = V_{in} - V_2$, $V_2 = \frac{I_2}{sC_1}$, $V_3 = V_2 - V_4$

$V_4 = \frac{I_4}{sC_3}$, $V_5 = V_4 - V_6$, $V_6 = \frac{I_6}{sC_5}$ $V_o = V_6$

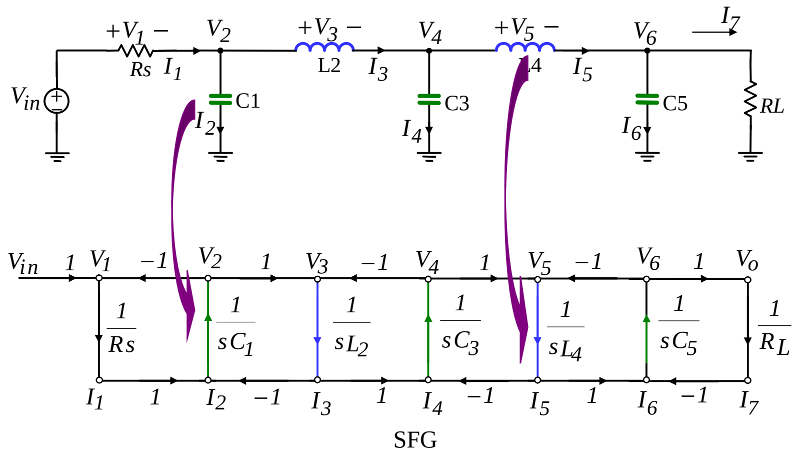
$I_1 = \frac{V_1}{Rs}$, $I_2 = I_1 - I_3$, $I_3 = \frac{V_3}{sL_2}$

$I_4 = I_3 - I_5$, $I_5 = \frac{V_5}{sL_4}$, $I_6 = I_5 - I_7$, $I_7 = \frac{V_6}{RL}$

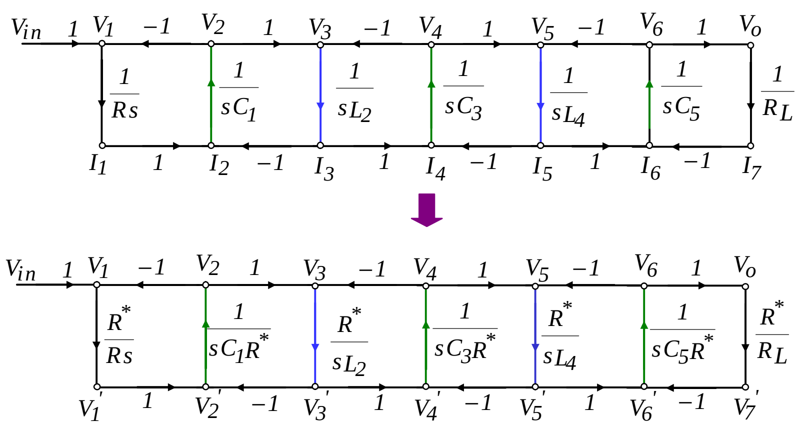


SFG

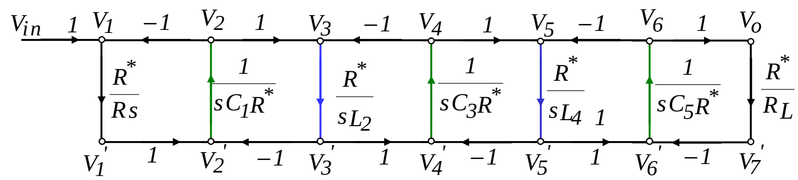
LC Ladder Filter
Signal Flowgraph

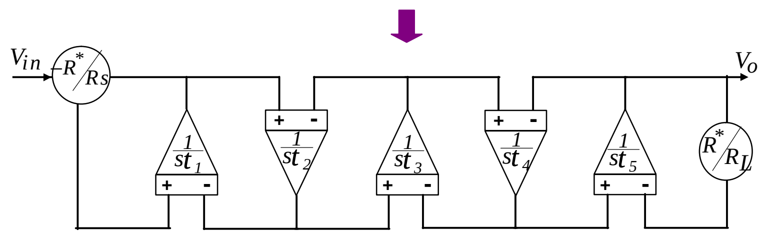


LC Ladder Filter
Normalize

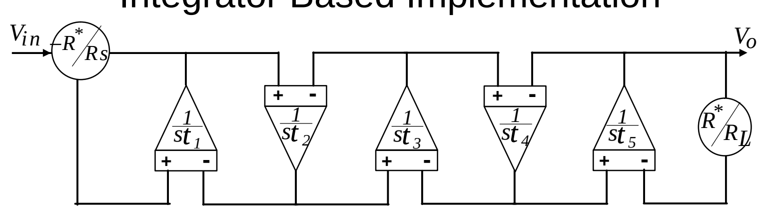


LC Ladder Filter
Synthesize



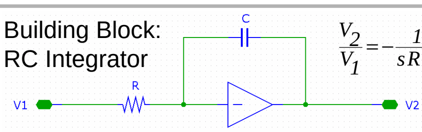


LC Ladder Filter
Integrator Based Implementation



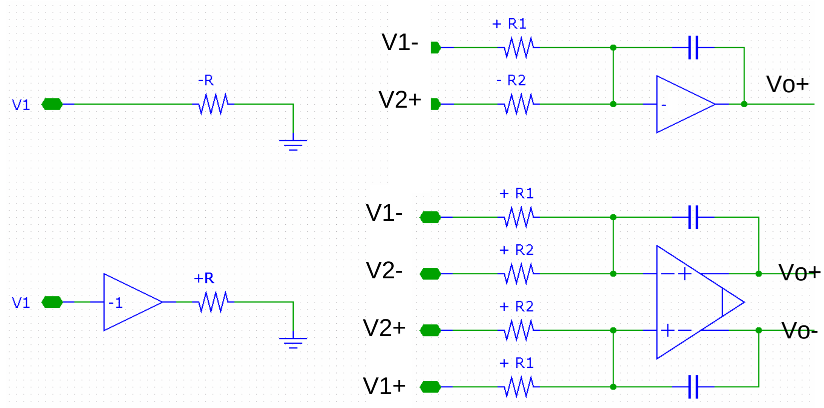
$$t_1 = C_1.R^* \quad , \quad t_2 = \frac{L_2}{R^*} = C_2.R^* \quad , \quad t_3 = C_3.R^* \quad , \quad t_4 = \frac{L4}{R^*} = C_4.R^* \quad , \quad t_5 = C_5.R^*$$

Building Block:
RC Integrator

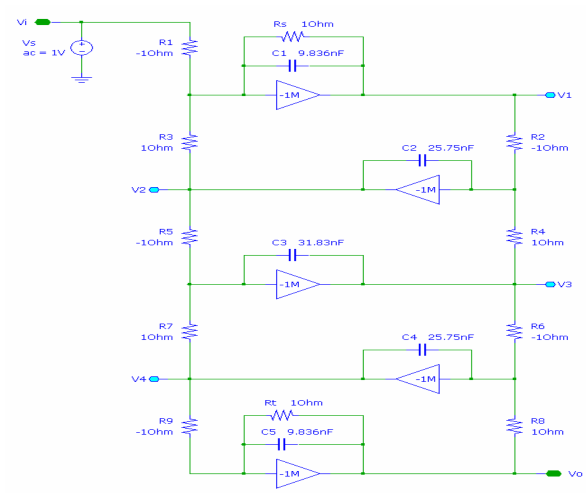


$$\frac{V_2}{V_1} = -\frac{1}{sRC}$$

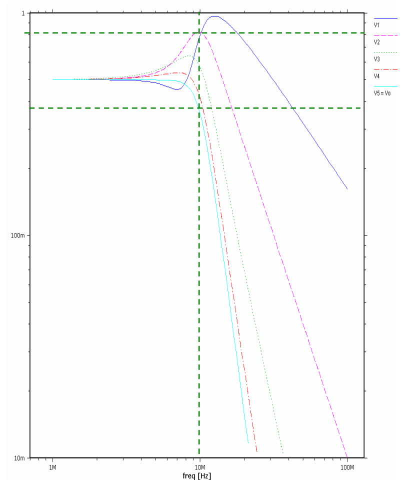
Negative Resistors

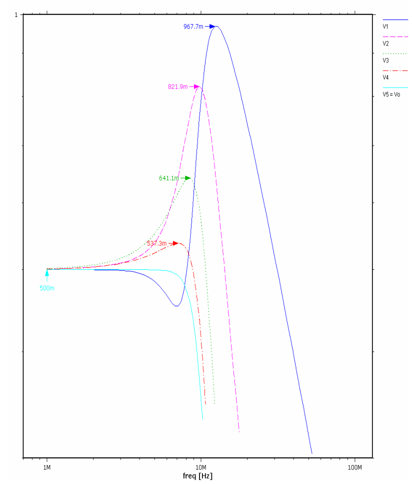


Synthesize



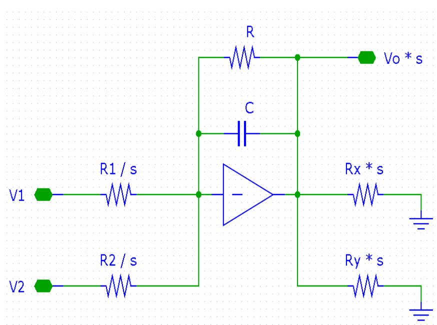
Frequency Response

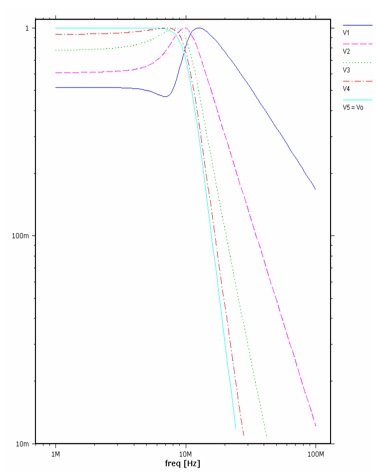




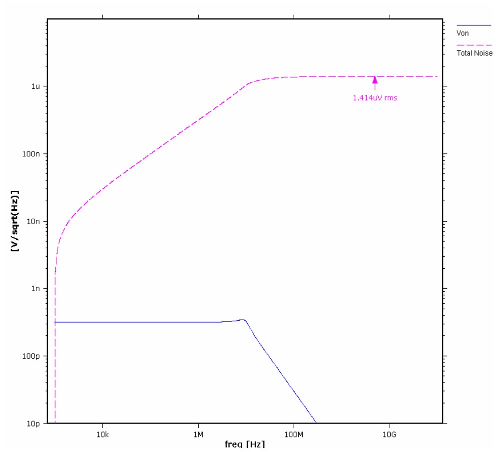
Scale Node Voltages

Scale V_o by factor "s"





Noise



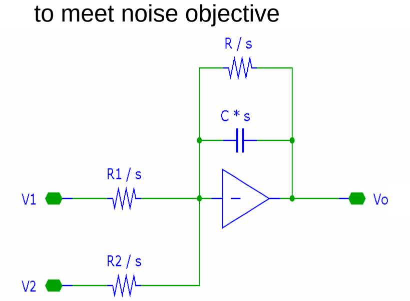
Total noise: 1.4 μV rms (noiseless opamps)

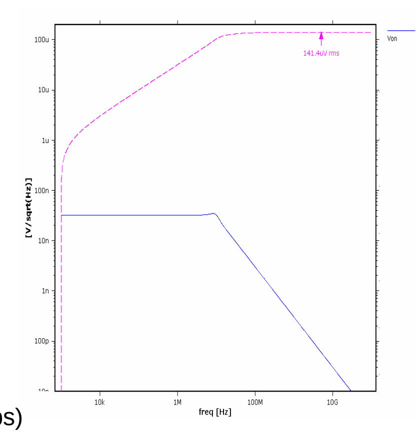
That's excellent, but the capacitors are very large (and the resistors small). Not possible to integrate.

Suppose our application allows higher noise in the order of 140 μV rms …

Scale to Meet Noise Target

Scale capacitors and resistors to meet noise objective

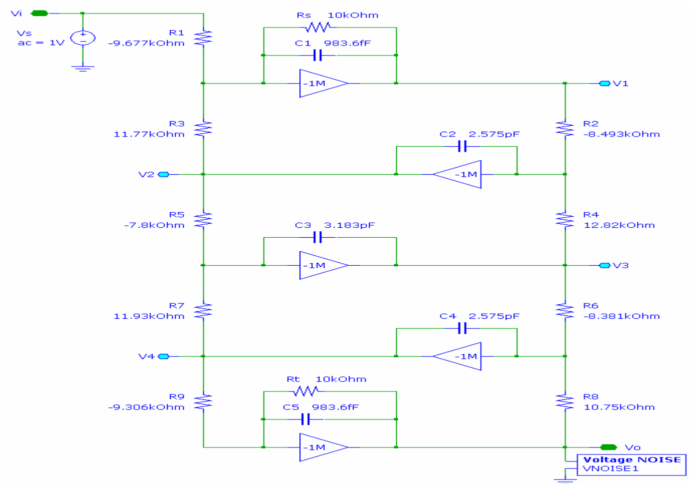




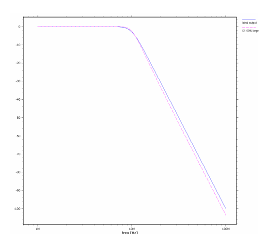
$s = 10^{-4}$

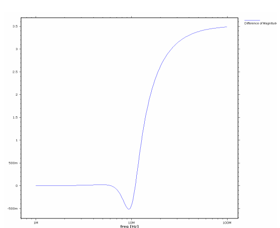
Noise: 141 μV rms (noiseless opamps)

Completed Design



Sensitivity





- C_1 made (arbitrarily) 50% (!) larger than its nominal value
- 0.5 dB error at band edge
- 3.5 dB error in stopband
- Looks like <u>very low sensitivity</u>