EE247
Lecture 8

• Continuous-time filters
  – Bandpass filters
  • Example: Gm-C BP filter using simple diff. pair
    – Linearity & noise issues
  – Various Gm-C Filter implementations
  – Comparison of continuous-time filter topologies
• Switched-capacitor filters

Bandpass Filters

• Bandpass Filters:
  – \( Q < 5 \) → Combination of lowpass & highpass
  ![Diagram of Lowpass and Highpass Filters]
  \[ |\tilde{v}(\infty)| \]
  \[ \omega \]
  \[ |\tilde{v}(\infty)| \]
  \[ \omega \]

  – \( Q > 5 \) → Direct implementation
  ![Diagram of Direct Implementation]
  \[ |\tilde{v}(\infty)| \]
  \[ \omega \]
  \[ Q>5 \]
Lowpass to Bandpass Transformation Table

<table>
<thead>
<tr>
<th>LP</th>
<th>BP</th>
<th>BP Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Lowpass filter structures &amp; tables" /></td>
<td><img src="image2" alt="Bandpass filter structures" /></td>
<td>$C = QC' \times \frac{1}{R \omega_0}$</td>
</tr>
<tr>
<td>$L = \frac{1}{QC'} \times \frac{R}{\omega_0}$</td>
<td>$C' = \frac{1}{QC' \times \frac{R}{\omega_0}}$</td>
<td></td>
</tr>
</tbody>
</table>

$L'$ & $C'$ are normalized LP values


Lowpass to Bandpass Transformation

$C_1 = QC_1 \times \frac{1}{R \omega_0}$
$L_1 = \frac{1}{QC_1} \times \frac{R}{\omega_0}$
$C_2 = QC_2 \times \frac{1}{R \omega_0}$
$L_2 = QC_2 \times \frac{R}{\omega_0}$
$C_3 = QC_3 \times \frac{1}{R \omega_0}$
$L_3 = \frac{1}{QC_3} \times \frac{R}{\omega_0}$

Where:
- $C_1'$, $L_1'$, $C_2'$, $L_2'$, $C_3'$, $L_3'$ are normalized lowpass values
- $Q$ → bandpass filter quality factor & $Q$
- $\omega_0$ → filter center frequency
Signal Flowgraph
6th Order Bandpass Filter

Note each C & L in the original lowpass prototype replaced by a resonator
Substituting the bandpass L1, C1,... by their normalized lowpass equivalent previous page
The resulting SFG is:

*Note the integrators have different time constants
• Ratio of time constants for each resonator $\sim 1/Q^2$
  ⇒ typically, requires high component ratios
  ⇒ poor matching
•Desirable to convert SFG so that all integrators have equal time constants for optimum matching.
•Scale nodes to obtain equal integrator time constant
Signal Flowgraph
6th Order Bandpass Filter

Note: Three resonators
All integrator time-constants are equal
Let us try to build this bandpass filter using the simple Gm-C structure

Second Order Gm-C Filter
Using Simple Source-Couple Pair Gm-Cell

- Center frequency:
  \[ \omega_c = \frac{g_m M_{1,2}}{2 \times C_{intg}} \]
- Q function of:
  \[ Q = \frac{g_m M_{1,2}}{S_m M_{1,2}} \]

To use this structure it is more power efficient to couple resonators through capacitive coupling
Modified signal flowgraph to have equal coupling between resonators

- In most filter cases $C_1' = C_3'$
- Example: For a butterworth lowpass filter $C_1' = C_3' = 1$ & $L_2' = 2$
- Assume desired overall bandpass filter $Q = 10$

Sixth Order Bandpass Filter Signal Flowgraph

- Where for a Butterworth shape $\gamma = \frac{1}{Q\sqrt{2}}$
- Since $Q = 10$ then: $\gamma = \frac{1}{14}$
Sixth Order Bandpass Filter Signal Flowgraph

- Coupling paths (γ) between resonators can be implemented with extra differential input pairs → additional power dissipation
- Or modify SFG as shown in next page:
Sixth Order Bandpass Filter Signal Flowgraph
SFG Modification

For narrow band filters (high Q) where frequencies within the passband are close to $\omega_0$, narrow-band approximation can be used:

$$\left(\frac{\omega_0}{\omega}\right)^2 = 1$$

$$-\gamma \times \left(\frac{\omega_0}{s}\right)^2 = -\gamma \times \left(\frac{\omega_0}{\omega}\right)^2 = -\gamma$$

The resulting SFG:

Bidirectional coupling paths, can easily be implemented with coupling capacitors $\rightarrow$ no extra power dissipation
Sixth Order Gm-C Bandpass Filter
Utilizing Simple Source-Coupled Pair Gm-Cell

\[
\gamma = \frac{C_k}{2 \times C_{\text{int}} g}
\]

\[
\gamma = 1/14
\]

\[
C_k = \frac{1}{7 \times C_{\text{int}} g}
\]

Parasitic C at integrator output, if unaccounted for, will result in inaccuracy in \( \gamma \)

Sixth Order Gm-C Bandpass Filter
Frequency Response Simulation
Simplest Form of CMOS Gm-Cell
Nonidealities

- DC gain (integrator $Q$)
  \[
  a = \frac{M_{1,2}}{s_0 + s_{\text{load}}}
  \]
  \[
  a = \frac{2L}{\theta(V_{gs} - V_{th})_{M_{1,2}}}
  \]

  Where $a$ denotes DC gain & $\theta$ is related to channel length modulation by:
  \[
  \lambda = \frac{\theta}{L}
  \]

  • Seems no extra poles!

CMOS Gm-Cell High-Frequency Poles

- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles
CMOS Gm-Cell High-Frequency Poles

- Distributed nature of gate capacitance & channel resistance results in an effective pole at 2.5 times input device cut-off frequency

\[ f_{\text{effective}} = \frac{1}{\sum_{i=2}^{\infty} \frac{1}{P_i}} \]

\[ f_{\text{effective}} = 2.5 \omega_{M1,2} \]

\[ \omega_{M1,2} = \frac{g_{m}}{C_{ox} L} = \frac{3}{2} \frac{\mu (V_{gs} - V_{th})_{M1,2}}{L^2} \]

- Distributed nature of gate capacitance & channel resistance results in an effective pole at 2.5 times input device cut-off frequency

CMOS Gm-Cell Quality Factor

\[ a = \frac{2L}{\theta (V_{gs} - V_{th})_{M1,2}} \]

\[ f_{\text{effective}} = \frac{15}{4} \frac{\mu (V_{gs} - V_{th})_{M1,2}}{L^2} \]

\[ Q_{\text{real}}^{\text{int}} = \frac{1}{\frac{1}{a - \omega_0} \sum_{i=2}^{\infty} \frac{1}{f_i}} \]

\[ Q_{\text{real}}^{\text{int}} = \frac{1}{2L} \frac{\theta (V_{gs} - V_{th})_{M1,2}}{\omega_0 L^2} - \frac{4}{15} \frac{\omega_0 L^2}{\mu (V_{gs} - V_{th})_{M1,2}} \]

- Note that the phase lead associated with DC gain is inversely prop. to L
- The phase lag due to high-freq. poles directly prop. to L

\[ \rightarrow \text{For a given } \omega_0 \text{ there exists an optimum } L \text{ which cancel the lead/lag phase error resulting in high integrator } Q \]
CMOS Gm Cell Channel Length for Optimum Integrator Quality Factor

\[ L_{opt.} \approx \left( \frac{15}{J} \frac{\theta \mu (V_{GS} - V_{TH})^2}{\alpha_0} \right)^{1/3} M_{1,2} \]

- Optimum channel length computed based on process parameters (could vary from process to process)

Source-Coupled Pair CMOS Gm-Cell Linearity

- Large signal \( G_m \) drops as input voltage increases
  - Gives rise to filter nonlinearity
Measure of Linearity

\[ V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \ldots \]

\[ HD3 = \frac{\text{amplitude 3rd harmonic dist. comp.}}{\text{amplitude fundamental}} = \frac{1}{4} \frac{\alpha_3}{\alpha_1} V_{in}^3 + \ldots \]

\[ IM_3 = \frac{\text{amplitude 3rd order IM comp.}}{\text{amplitude fundamental}} = \frac{3}{4} \frac{\alpha_3}{\alpha_1} V_{in}^3 + \frac{25}{8} \frac{\alpha_3}{\alpha_1} V_{in}^4 + \ldots \]

Source-Coupled Pair CMOS Gm-Cell Linearity

\[ \Delta I_d = I_{ss} \left[ \frac{\Delta V_l}{V_{gs} - V_{th}}_{M1,2} \right] \left[ J - J \left( \frac{\Delta V_l}{V_{gs} - V_{th}}_{M1,2} \right) \right]^{1/2} \]

\[ \Delta I_d = a_1 \Delta V_l + a_2 \Delta V_l^2 + a_3 \Delta V_l^3 + \ldots \]

Series expansion used in (1)

\[ a_1 = \frac{I_{ss}}{V_{gs} - V_{th}}_{M1,2} \quad \text{and} \quad a_2 = 0 \]

\[ a_3 = -\frac{8 \left( V_{gs} - V_{th} \right)}{V_{gs} - V_{th}}_{M1,2} \quad \text{and} \quad a_4 = 0 \]

\[ a_5 = -\frac{128 \left( V_{gs} - V_{th} \right)}{V_{gs} - V_{th}}_{M1,2} \quad \text{and} \quad a_6 = 0 \]
Linearity of the Source-Coupled Pair CMOS Gm-Cell

\[ IM_3 = \frac{3a_3}{4a_1}V_i^2 + \frac{25a_5}{8a_1}V_i^4 \]

Substituting for \( a_1, a_2, \ldots \)

\[ IM_3 = \frac{3}{32} \left( \frac{V_i}{(V_{GS} - V_{th})} \right)^2 + \frac{25}{1024} \left( \frac{V_i}{(V_{GS} - V_{th})} \right)^4 \]

\[ \dot{V}_{i_{\text{max}}} = 4(V_{GS} - V_{th}) \times \frac{1}{3} \times IM_3 \]

\[ IM_3 = 1\% \ & (V_{GS} - V_{th}) = JV \Rightarrow \dot{V}_{i_{\text{max}}} \approx 230mV \]

- Key point: Max. signal handling capability function of gate-overdrive

Simplest Form of CMOS Gm Cell

Disadvantages

- Max. signal handling capability function of gate-overdrive
  \[ IM_3 \propto (V_{GS} - V_{th})^{-2} \]
- Critical freq. function of gate-overdrive too
  \[ \omega_0 = \frac{g_{m_{1,2}}}{2 \times C_{\text{intg}}} \]
  since \( g_m = \mu C_{ox} \frac{W}{L}(V_{GS} - V_{th}) \)
  then \( \omega_0 \propto (V_{GS} - V_{th}) \)

\[ \Rightarrow \text{Filter tuning affects max. signal handling capability!} \]
Simplest Form of CMOS Gm Cell
Removing Dependence of Maximum Signal Handling Capability on Tuning

- Can overcome problem of max. signal handling capability being a function of tuning by providing tuning through:
  - Coarse tuning via switching in/out binary-weighted cross-coupled pairs. Try to keep gate overdrive voltage constant
  - Fine tuning through varying current sources

→ Dynamic range dependence on tuning removed (to 1st order)


Dynamic Range for Source-Coupled Pair Based Filter

\[ IM_1 = 1\% \quad \text{and} \quad (V_{GS} - V_A) = 1V \quad \Rightarrow \quad V_{rms} = 230mV \]

- Minimum detectable signal determined by total noise voltage
- It can be shown for the 6th order Butterworth bandpass filter noise is given by:

\[ \sqrt{V_{no}} = \sqrt{\frac{3}{Q} \frac{kT}{C_{intg}}} \]

Assuming \( Q = 10 \) \( \quad C_{intg} = 5pF \)

\[ V_{rms_{noise}} = 160\mu V \quad \text{since} \quad v_{max} = 230mV \]

Dynamic Range = 6.3dB
Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm-Cell

- 2nd source-coupled pair added to subtract current proportional to nonlinear component associated with the main SCP

\[
\frac{I_{ss1}}{I_{ss3}} = b \quad \text{and thus} \quad \frac{V_{gs} - V_{th}}{V_{gs} - V_{th}} = \frac{I_{ss1}}{I_{ss3}} = a
\]

Improving the Max. Signal Handling Capability of the Source-Coupled Pair Gm

- Improves maximum signal handling capability by about 12dB
  \[ \Delta V_1 \left| \frac{1}{(V_{GS}-V_{TH})_{M1,2}} \right| \]
- Dynamic range theoretically improved to \(63+12=75\)dB

Simplest Form of CMOS Gm-Cell

- Pros
  - Capable of very high frequency performance (highest?)
  - Simple design
- Cons
  - Tuning affects power dissipation
  - Tuning affects max. signal handling capability (can overcome)
  - Limited linearity (possible to improve)

Gm-Cell
Source-Coupled Pair with Degeneration

\[ I_d = \frac{\mu C_{ox} W}{2} \left[ v_{gs} - v_{th} \right] v_{ds} - v_{th}^2 \]

\[ g_{ds} = \frac{\partial I_d}{\partial v_{ds}} = \frac{\mu C_{ox} W}{L} \left( v_{gs} - v_{th} \right) \left| v_{ds} \right| \text{ small} \]

\[ g_{eff} = \frac{1}{g_{ds}} \left( \frac{1}{g_{ds}^{M3}} + \frac{2}{g_{m}^{M1,2}} \right) \]

\[ g_{m}^{M1,2} \gg g_{ds}^{M3} \]

\[ g_{eff} = g_{ds}^{M3} \]

M3 operating in triode mode → source degeneration → determines overall gm

Pros
- Moderate linearity
- Continuous tuning provided by \( V_c \)
- Tuning does not affect power dissipation

Cons
- Extra poles associated with the source of M1,2
- Low frequency applications only

BiCMOS Gm-Cell

- MOSFET in triode mode:

\[
I_d = \frac{\mu C_{ox} W}{2L} \left[ 2(V_{gs} - V_{th})V_{ds} - V_{ds}^2 \right]
\]

- Note that if \( V_{ds} \) is kept constant:

\[
s_m = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} V_{ds}
\]

- Linearity performance \( \Rightarrow \) keep \( gm \) constant \( \Rightarrow \) function of how constant \( V_{ds} \) can be held
  - Gain @ Node X must be minimized

\[
A_X = \frac{s_m}{s_B}
\]

- Since for a given current, \( gm \) of BJT is larger compared to MOS preferable to use BJT
- Extra pole at node X

Alternative Fully CMOS Gm-Cell

- BJT replaced by a MOS transistor with boosted \( gm \)

- Lower frequency of operation compared to the BiCMOS version due to more parasitic capacitance at node A & B
BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt
- Freq. tuned by varying Vb

- Design tradeoffs:
  - Extra poles at the input device drain junctions
  - Input devices have to be small to minimize parasitic poles
  - Results in high input-referred offset voltage could drive ckt into non-linear region
  - Small devices → high 1/f noise

7th Order Elliptic Gm-C LPF
For CDMA RX Baseband Application

- Gm-Cell in previous page used to build a 7th order elliptic filter for CDMA baseband applications (650kHz corner frequency)
- In-band dynamic range of <50dB achieved
Comparison of 7th Order Gm-C versus Opamp-RC LPF

• Gm-C filter requires 4 times less integr. cap. area compared to Opamp-RC
  → For low-noise applications where filter area is dominated by cap. area could make a significant difference in the total area
• Opamp-RC linearity superior compared to Gm-C
• Power dissipation tends to be lower for Gm-C since output is high impedance and thus no need for buffering

BiCMOS Gm-OTA-C Integrator

• Used to build filter for disk-drive applications
• Since high frequency of operation, time-constant sensitivity to parasitic caps significant.
  → Opamp used
• M2 & M3 added to compensate for phase lag (provides phase lead)

6th Order BiCMOS Continuous-time Filter & Second Order Equalizer for Disk Drive Read Channels

- Gm-C-opamp of the previous page used to build a 6th order filter for Disk Drive
- Filter consists of 3 Biquad with max. Q of 2 each
- Performance in the order of 40dB SNDR achieved for up to 20MHz corner frequency


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Gm-Cell
Source-Coupled Pair with Degeneration

- Gm-cell intended for low Q disk drive filter

Gm-Cell
Source-Coupled Pair with Degeneration

- M7,8 operating in triode mode determine the gm of the cell
- Feedback provided by M5,6 maintains the gate-source voltage of M1,2 constant
  by forcing their current to be constant → helps linearize $r_{ds}$ of M7,8
- Current mirrored to the output via M9,10 with a factor of $k$
- Performance level of about 50dB SNDR at fcorner of 25MHz achieved

BiCMOS Gm-C Integrator

- Needs higher supply voltage compared to the previous design since quite a few devices are stacked vertically
- M1,2 → triode mode
- Q1,2 → hold $V_{ds}$ of M1,2 constant
- Current ID used to tune filter critical frequency by varying $V_{ds}$ of M1,2 and thus $gm$ of M1,2
- M3, M4 operate in triode mode and added to provide CMFB

• M5 & M6 configured as capacitors added to compensate for RHP zero due to Cgd of M1,2 (moves it to LHP) size of M5,6 → 1/3 of M1,2


BiCMOS Gm-C Integrator

BiCMOS Gm-C Filter For Disk-Drive Application

• Using the integrators shown in the previous page
• Biquad filter for disk drives
• \( gm1 = gm2 = gm4 = 2gm3 \)
• \( Q = 2 \)
• Tunable from 8MHz to 32MHz

Summary Continuous-Time Filters

- **Opamp RC filters**
  - Good linearity $\rightarrow$ High dynamic range (60-90dB)
  - Only discrete tuning possible
  - Medium usable signal bandwidth (<10MHz)

- **Opamp MOSFET-C**
  - Linearity compromised (typical dynamic range 40-60dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)

- **Opamp MOSFET-RC**
  - Improved linearity compared to Opamp MOSFET-C (D.R. 50-90dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)

- **Gm-C**
  - Highest frequency performance (at least an order of magnitude higher compared to the rest <100MHz)
  - Dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (40-70dB)

Switched-Capacitor Filters

Example: Codec Chip

Switched-Capacitor Resistor

- Capacitor C is the "switched capacitor"
- Non-overlapping clocks $\phi_1$ and $\phi_2$ control switches S1 and S2, respectively
- $v_{IN}$ is sampled at the falling edge of $\phi_1$
  - Sampling frequency $f_s$
- Next, $\phi_2$ rises and the voltage across C is transferred to $v_{OUT}$
- Why is this a resistor?

Switched-Capacitor Resistors

- Charge transferred from $v_{IN}$ to $v_{OUT}$ during each clock cycle is:

- Average current flowing from $v_{IN}$ to $v_{OUT}$ is:

  $Q = C(v_{IN} - v_{OUT})$

  $i = \frac{Q}{t} = Qf_s$

  $i = f_sC(v_{IN} - v_{OUT})$
Switched-Capacitor Resistors

\[ i = f S C(v_{IN} - v_{OUT}) \]

With the current through the switched capacitor resistor proportional to the voltage across it, the equivalent “switched capacitor resistance” is:

\[ R_{eq} = \frac{1}{f S C} \]

Example

\[ f = 1 \text{MHz}, C = 1 \text{pF} \]

\[ \rightarrow R_{eq} = 1 \text{Mega}\Omega \]

Switched-Capacitor Filter

- Let’s build a “SC” filter …
- We’ll start with a simple RC LPF
- Replace the physical resistor by an equivalent SC resistor
- 3-dB bandwidth:

\[ \omega_{-3dB} = \frac{1}{R_{eq} C_2} = f_S \times \frac{C_1}{C_2} \]

\[ f_{-3dB} = \frac{1}{2\pi} f_S \times \frac{C_1}{C_2} \]
Switched-Capacitor Filter Advantage versus Continuous-Time Filters

\[ f_{-3dB} = \frac{1}{2\pi} f_s \times \frac{C_1}{C_2} \]

- Corner freq. proportional to:
  - System clock (accurate to few ppm)
  - C ratio accurate \( \rightarrow < 0.1\% \)

\[ f_{-3dB} = \frac{l}{2\pi R_{eq} C_2} \]

- Corner freq. proportional to:
  - Absolute value of Rs & Cs
  - Poor accuracy \( \rightarrow 20 \text{ to } 50\% \)

Main advantage of SC filter inherent corner frequency accuracy

Typical Sampling Process
Continuous-Time (CT) \( \Rightarrow \) Sampled Data (SD)

- Continuous-Time Signal
- Sampled Data
- Sampled Data + ZOH
- Clock
Uniform Sampling

Nomenclature:
- Continuous time signal \( x(t) \)
- Sampling interval \( T \)
- Sampling frequency \( f_s = 1/T \)
- Sampled signal \( x(kT) = x(k) \)

- Problem: Multiple continuous time signals can yield exactly the same discrete time signal
- Let's look at samples taken at 1\( \mu s \) intervals of several sinusoidal waveforms …

Sampling Sine Waves

\[ v(t) = \sin [2\pi(101000)t] \]

\( T = 1\mu s \)
\( f_s = 1/T = 1MHz \)
\( f_{in} = 101kHz \)
Sampling Sine Waves

\[ v(t) = -\sin(2\pi(899000)t) \]

\[ T = 1\mu s \]
\[ f_s = 1\text{MHz} \]
\[ f_{in} = 899\text{kHz} \]

Sampling Sine Waves

\[ v(t) = \sin(2\pi(1101000)t) \]

\[ T = 1\mu s \]
\[ f_s = 1\text{MHz} \]
\[ f_{in} = 1101\text{kHz} \]