EE247 - Lecture 2
Filters

- Material covered today:
  - Nomenclature
  - Filter specifications
    - Quality factor
    - Frequency characteristics
    - Group delay
  - Filter types
    - Butterworth
    - Chebyshev I
    - Chebyshev II
    - Elliptic
    - Bessel
  - Group delay comparison example

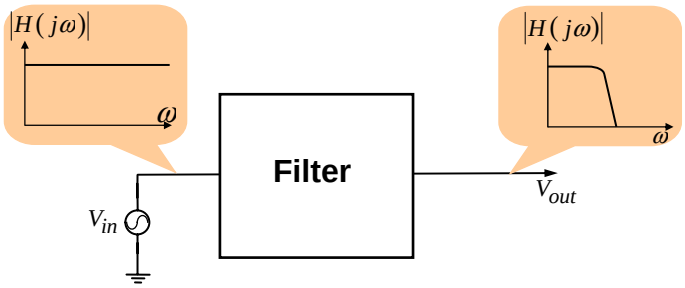
Administrative

- Office hours for H.K. changed to:
  Tues.- Thurs. 2:30-3:30 @ 463 Cory Hall
  Extra office hours by appointment

Summary
Last Lecture

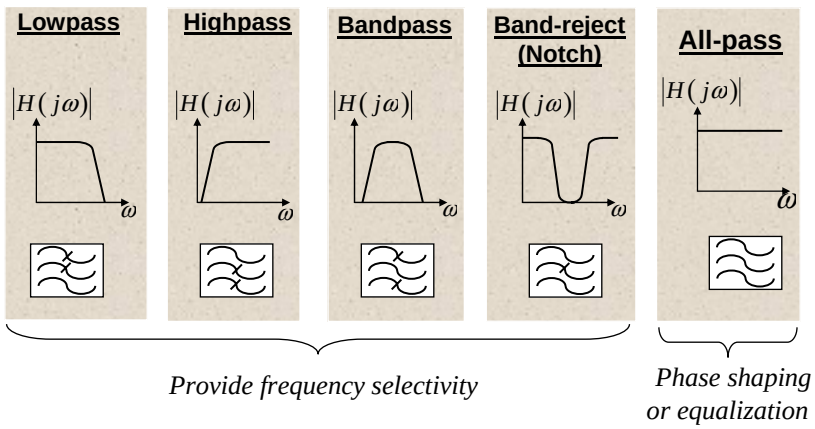
- Major success in CMOS technology scaling:
  - → Inexpensive DSPs technology
  - → Resulted in the need for high performance Analog/Digital interface circuitry
- Main Analog/Digital interface building blocks includes
  - – Analog filters
  - – D/A converters
  - – A/D converters

Filters



Filters → Provide frequency selectivity and/or phase shaping

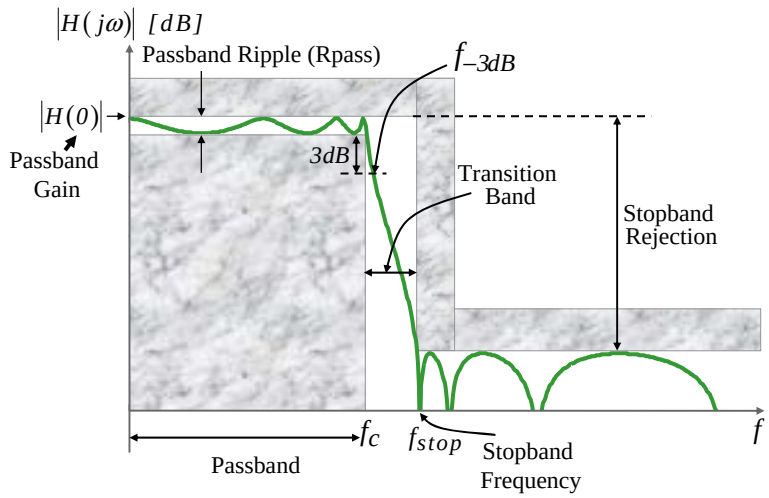
Nomenclature
Filter Types



Filter Specifications

- Frequency characteristics (lowpass filter):
  - Passband ripple (Rpass)
  - Cutoff frequency or *-3dB* frequency
  - Stopband rejection
  - Passband gain
- Phase characteristics:
  - Group delay
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Power/pole & Area/pole

Lowpass Filter Frequency Characteristics



Quality Factor (Q)

- The term quality factor (Q) has different definitions in different contexts:
  - Component quality factor (inductor & capacitor Q)
  - Pole quality factor
  - Bandpass filter quality factor
- Next 3 slides clarifies each

Component Quality Factor (Q)

- For any component with a transfer function:

$$H(j\omega) = \frac{1}{R(\omega) + jX(\omega)}$$

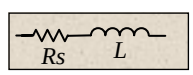
- Quality factor is defined as:

$$Q = \frac{X(\omega)}{R(\omega)} \rightarrow \frac{Energy\ Stored}{Average\ Power\ Dissipation}\ per\ unit\ time$$

Inductor & Capacitor Quality Factor

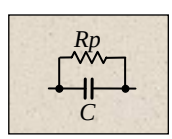
- Inductor Q :

$$Y_L = \frac{1}{Rs + j\omega L} \qquad Q_L = \frac{\omega L}{Rs}$$



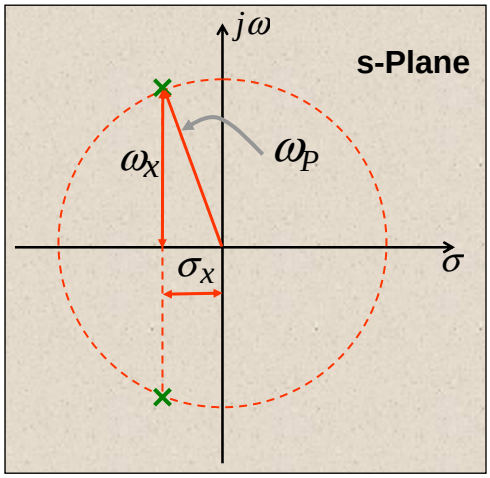
- Capacitor Q :

$$Z_C = \frac{1}{\frac{1}{Rp} + j\omega C} \qquad Q_C = \omega C Rp$$

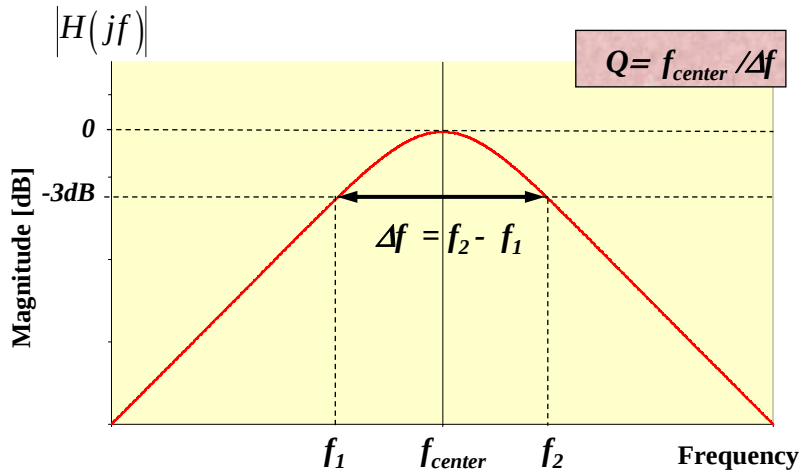


Pole Quality Factor

$$Q_{Pole} = \frac{\omega_X}{2\sigma_X}$$



Bandpass Filter Quality Factor (Q)



What is Group Delay?

- Consider a continuous time filter with s-domain transfer function G(s):

$$G(j\omega) \equiv |G(j\omega)|\, e^{j\theta(\omega)}$$

- Let us apply a signal to the filter input composed of sum of two sinewaves at slightly different frequencies ($\Delta\omega \ll \omega$):

$$v_{IN}(t) = A_1 \sin(\omega t) + A_2 \sin[(\omega+\Delta\omega)\, t]$$

- The filter output is:

$$v_{OUT}(t) = A_1\, |G(j\omega)|\, \sin[\omega t + \theta(\omega)] +$$

$$A_2\, |G[\, j(\omega+\Delta\omega)]|\, \sin[(\omega+\Delta\omega)t + \theta(\omega+\Delta\omega)]$$

What is Group Delay?

$$v_{OUT}(t) = A_1\, |G(j\omega)|\, \sin \left\{ \omega \left[t + \frac{\theta(\omega)}{\omega} \right] \right\} +$$

$$+ A_2\, |G[\, j(\omega+\Delta\omega)]|\, \sin \left\{ (\omega+\Delta\omega) \left[t + \frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \right] \right\}$$

Since $\frac{\Delta\omega}{\omega} \ll 1$ then $\left[\frac{\Delta\omega}{\omega}\right]^2 \rightarrow 0$

$$\frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \cong \left[\theta(\omega) + \frac{d\theta(\omega)}{d\omega} \Delta\omega \right] \left[\frac{1}{\omega} \left(1 - \frac{\Delta\omega}{\omega} \right) \right]$$

$$\cong \frac{\theta(\omega)}{\omega} + \left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} \right) \frac{\Delta\omega}{\omega}$$

What is Group Delay?
Signal Magnitude and Phase Impairment

$$v_{OUT}(t) = A_1 \left| G(j\omega) \right| \sin\left\{ \omega \left[t + \frac{\theta(\omega)}{\omega} \right] \right\} +$$

$$+ A_2 \left| G[j(\omega+\Delta\omega)] \right| \sin\left\{ (\omega+\Delta\omega) \left[t + \frac{\theta(\omega)}{\omega} + \underbrace{\left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} \right) \frac{\Delta\omega}{\omega}} \right] \right\}$$

- If the second term in the phase of the 2nd sin wave is non-zero, then the filter's output at frequency $\omega+\Delta\omega$ is time-shifted differently than the filter's output at frequency ω
  → "Phase distortion"
- If the second term is zero, then the filter's output at frequency $\omega+\Delta\omega$ and the output at frequency ω are each delayed in time by $-\theta(\omega)/\omega$
- $\tau_{PD} \equiv -\theta(\omega)/\omega$ is called the "phase delay" and has units of time

What is Group Delay?
Signal Magnitude and Phase Impairment

- Phase distortion is avoided only if:

$$\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} = 0$$

- Clearly, if $\theta(\omega)=k\omega$, k a constant, → no phase distortion
- This type of filter phase response is called "linear phase"
  →Phase shift varies linearly with frequency
- $\tau_{GR} \equiv -d\theta(\omega)/d\omega$ is called the "group delay" and also has units of time. For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = k$
  → $\tau_{GR} = \tau_{PD}$ implies linear phase
- Note: Filters with $\theta(\omega)=k\omega+c$ are also called linear phase filters, but they're not free of phase distortion

What is Group Delay?
Signal Magnitude and Phase Impairment

- If $\tau_{GR} = \tau_{PD}$ → No phase distortion

$$v_{OUT}(t) = A_1 \left| G(j\omega) \right| \sin\left[\omega\left(t - \tau_{GR}\right)\right] +$$
$$+ A_2 \left| G[\, j(\omega+\Delta\omega)] \right| \sin\left[(\omega+\Delta\omega)\left(t - \tau_{GR}\right)\right]$$

- If also $\left| G(\ \ j\omega) \right| = \left| G[\, j(\omega+\Delta\omega)] \right|$ for all input frequencies within the signal-band, v_{OUT} is a scaled, time-shifted replica of the input, with no "signal magnitude distortion" :
- In most cases neither of these conditions are realizable exactly

Summary
Group Delay

- Phase delay is defined as:

$$\tau_{PD} \equiv -\theta(\omega)/\omega \quad [\text{ time}]$$

- Group delay is defined as :

$$\tau_{GR} \equiv -d\theta(\omega)/d\omega \quad [\text{time}]$$

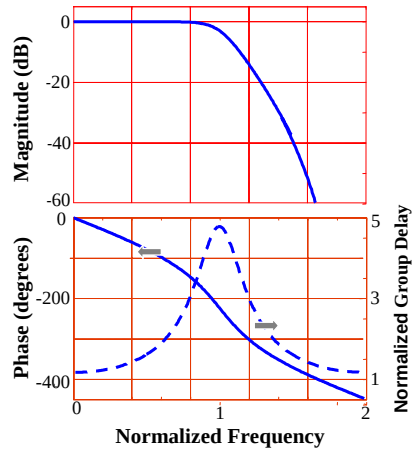
- If $\theta(\omega)=k\omega$, k a constant, → no phase distortion
- For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = k$

Filter Types
Lowpass Butterworth Filter

- Maximally flat amplitude within the filter passband

$$\left.\frac{d^N |H(j\omega)|}{d\omega}\right|_{\omega=0} = 0$$

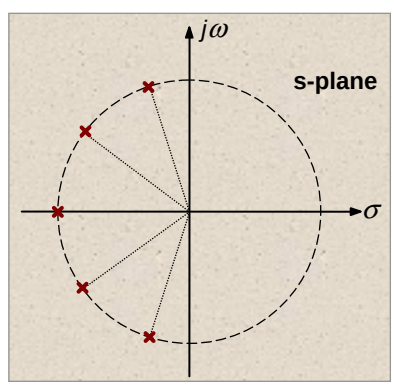
- Moderate phase distortion



Example: 5th Order Butterworth filter

Lowpass Butterworth Filter

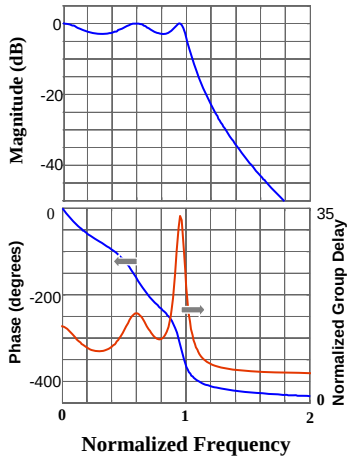
- All poles
- Poles located on the unit circle with equal angles



Example: 5th Order Butterworth Filter

Filter Types
Chebyshev I Lowpass Filter

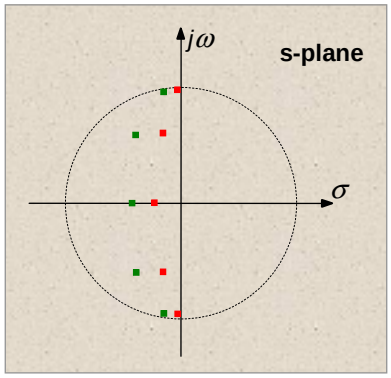
- Chebyshev I filter
  - Ripple in the passband
  - Sharper transition band compared to Butterworth
  - Poorer group delay
  - As more ripple is allowed in the passband:
    - Sharper transition band
    - Poorer phase response



Example: 5th Order Chebyshev filter

Chebyshev I Lowpass Filter Characteristics

- All poles
- Poles located on an ellipse inside the unit circle
- Allowing more ripple in the passband:
  - ⇨Narrower transition band
  - ⇨Sharper cut-off
  - ⇨Higher pole Q
  - ⇨Poorer phase response

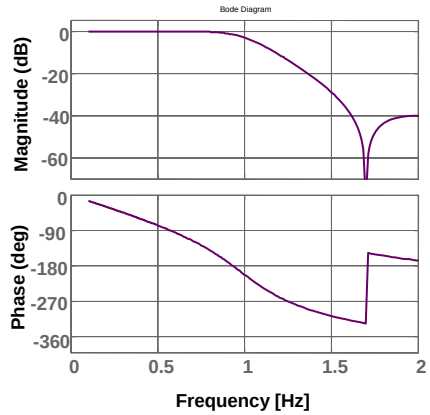


- Chebyshev I LPF 3dB passband ripple
- Chebyshev I LPF 0.1dB passband ripple

Example: 5th Order Chebyshev I Filter

Filter Types
Chebyshev II Lowpass

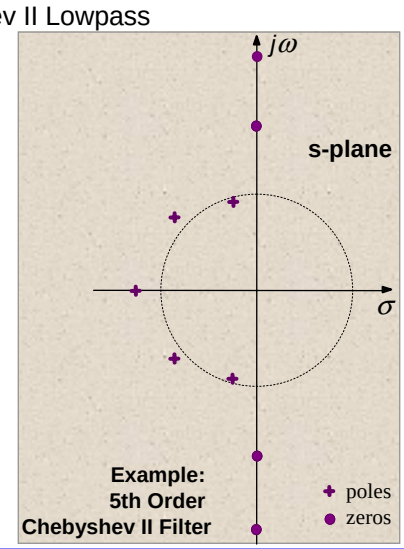
- Chebyshev II filter
  - Ripple in stopband
  - Sharper transition band compared to Butterworth
  - Passband phase more linear compared to Chebyshev I



Example: 5th Order Chebyshev II filter

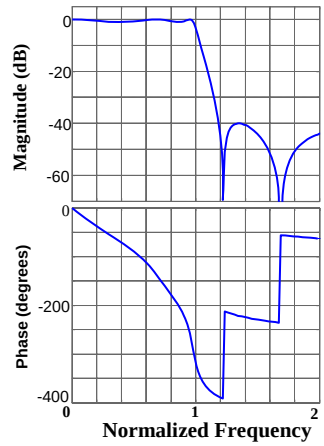
Filter Types
Chebyshev II Lowpass

- Both poles & zeros
  - No. of poles *n*
  - No. of finite zeros *n-1*
- Poles located both inside & outside of the unit circle
- Zeros located on $j\omega$ axis
- Ripple in the stopband only



Filter Types
Elliptic Lowpass Filter

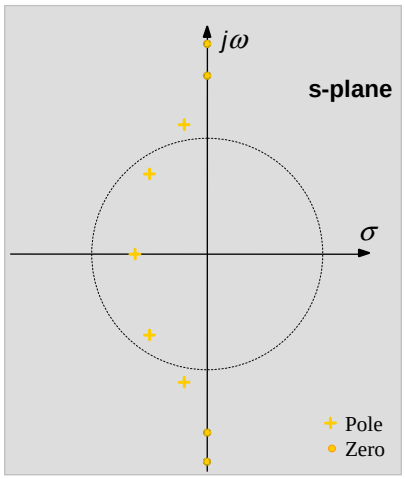
- Elliptic filter
  - Ripple in passband
  - Ripple in the stopband
  - Sharper transition band compared to Butterworth & both Chebyshevs
  - Poorest phase response



Example: 5th Order Elliptic filter

Filter Types
Elliptic Lowpass Filter

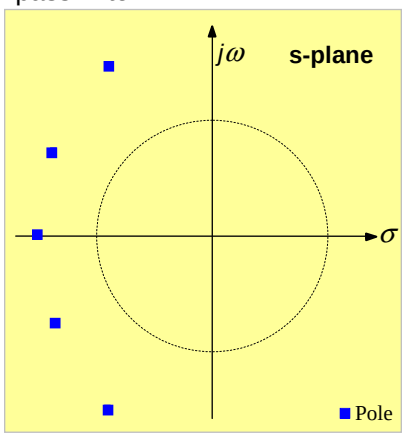
- Both poles & zeros
  - No. of poles *n*
  - No. of zeros *n-1*
- Zeros located on $j\omega$ axis
- Sharp cut-off
  - ⇨Narrower transition band
  - ⇨Pole Q higher compared to the previous filters



Example: 5th Order Elliptic Filter

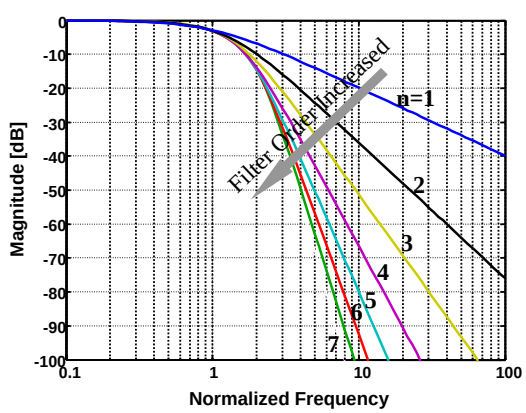
Filter Types
Bessel Lowpass Filter

- Bessel
  - All poles
  - Maximally flat group delay
  - Poor amplitude attenuation
  - Poles outside unit circle (s-plane)
  - Relatively low Q poles

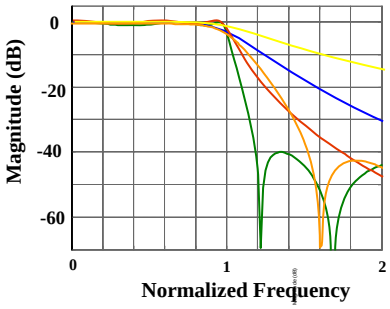


Example: 5th Order Bessel filter

Magnitude Response of a Bessel Filter as a Function of Filter Order (n)



Filter Types
Comparison of Various Type LPF Magnitude Response

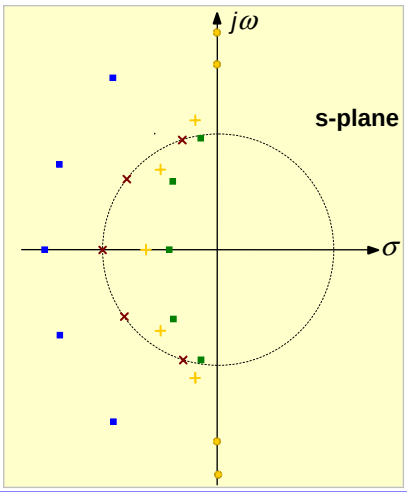


All 5th order filters with same corner freq.

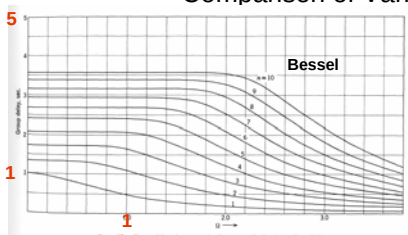
- Bessel
- Butterworth
- Chebyshev I
- Chebyshev II
- Elliptic

Filter Types
Comparison of Various LPF Singularities

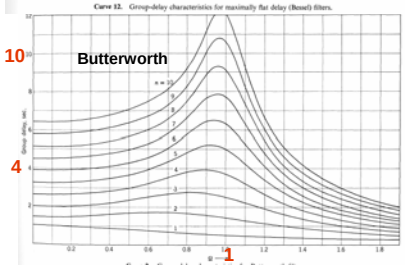
- Poles Bessel
- Poles Butterworth
- Poles Elliptic
- Zeros Elliptic
- Poles Chebyshev I 0.1dB

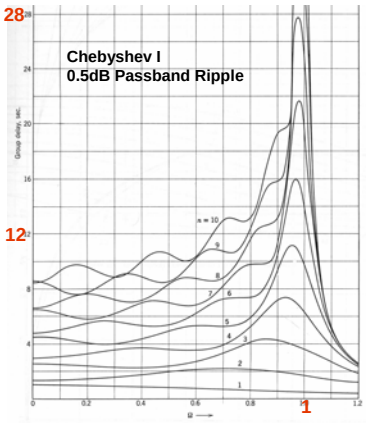


Comparison of Various LPF Groupdelay



Curve 12. Group-delay characteristics for maximally flat delay (Bessel) filters.





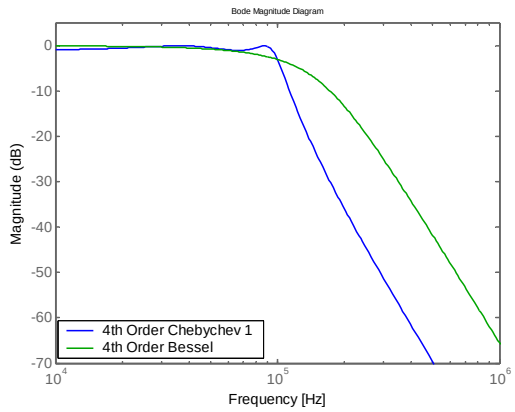
Curve 8. Group-delay characteristics for Chebyshev filter with 0.5 dB ripple.

Ref: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.

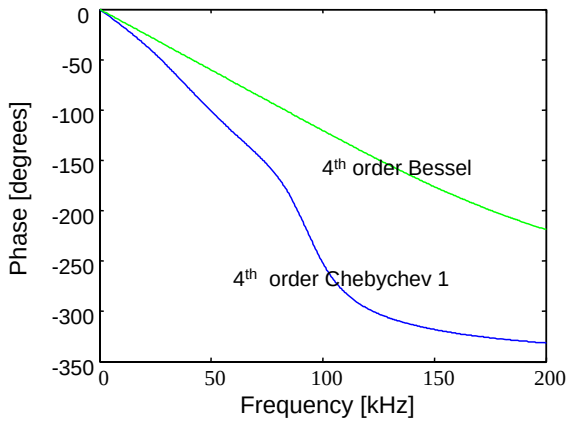
Group Delay Comparison Example

- Lowpass filter with 100kHz corner frequency
- Chebyshev I versus Bessel
  - Both filters 4th order- same ***-3dB*** point
  - Passband ripple of ***1dB*** allowed for Chebyshev I

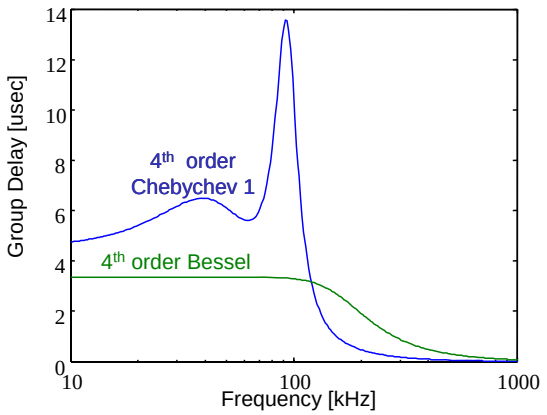
Magnitude Response



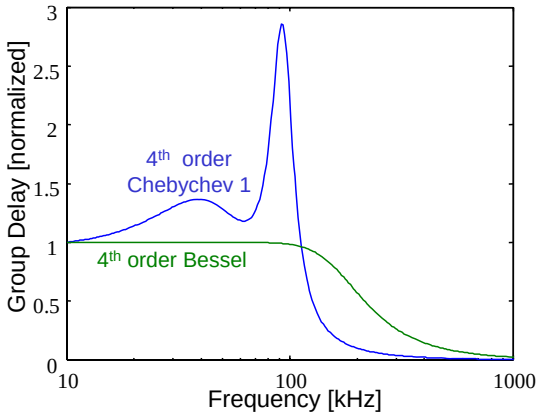
Phase Response



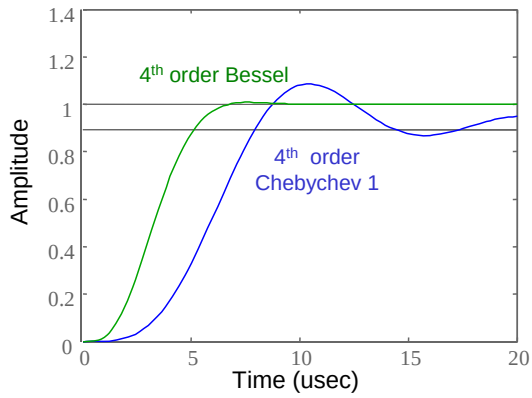
Group Delay



Normalized Group Delay



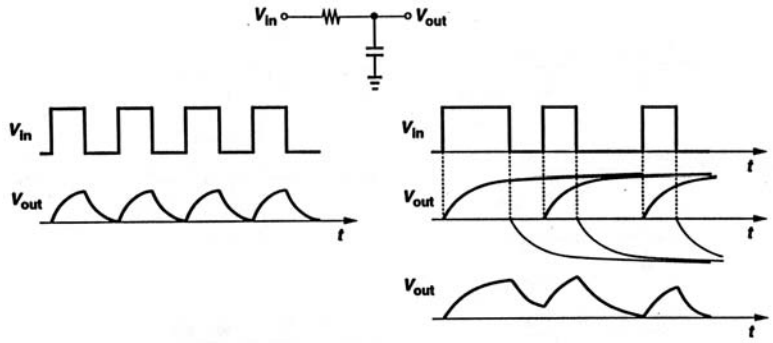
Step Response



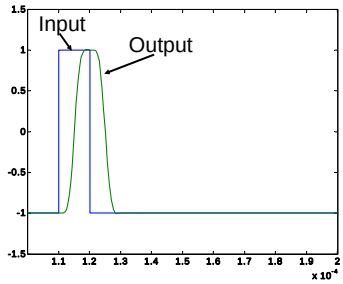
Intersymbol Interference (ISI)

ISI→ Broadening of pulses resulting in interference between successive transmitted pulses

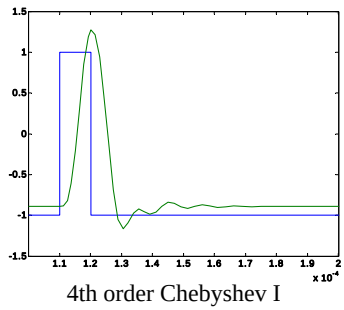
Example: Simple RC filter



Pulse Broadening
Bessel versus Chebyshev



8th order Bessel

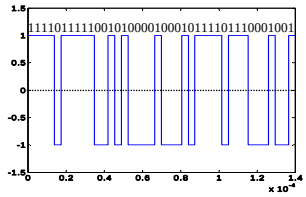


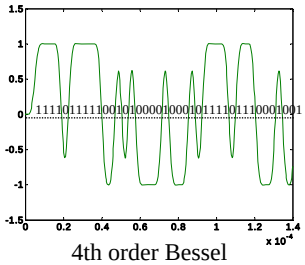
4th order Chebyshev I

Chebyshev filter incurrs more severe pulse broadening compared to Bessel
→ More ISI

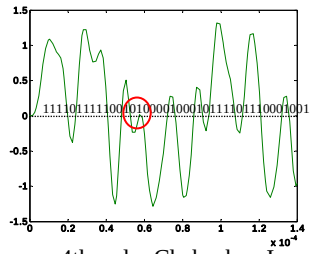
Response to Psuedo-Random Data
Chebyshev versus Bessel

Input Signal:
Symbol rate 1/130kHz ⇨



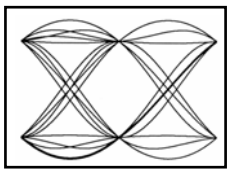


4th order Bessel



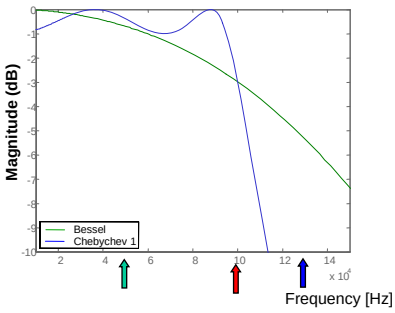
4th order Chebyshev I

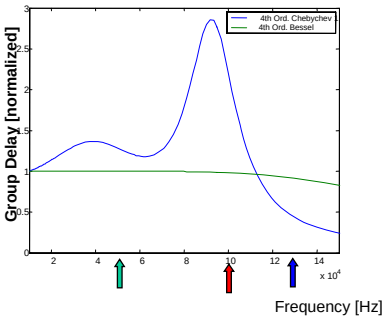
Measure of Signal Degradation
Eye Diagram



- Eye diagram is a useful graphical illustration for signal degradation
- Consists of many overlaid traces of a signal using an oscilloscope where the symbol timing serves as the scope trigger
- It is a visual summary of all possible intersymbol interference waveforms
  - The vertical opening → immunity to noise
  - Horizontal opening → timing jitter

Measure of Signal Degradation
Eye Diagram

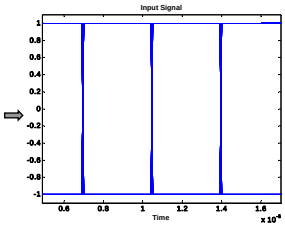


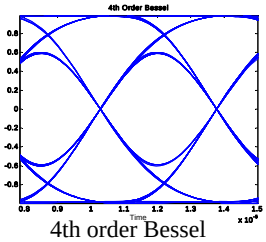


- Random data with symbol rates:
  - 1/50kHz
  - 1/100kHz
  - 1/130kHz

Eye Diagram
Chebyshev versus Bessel

Input Signal
Pseudo-random data
Symbol rate:1/ 130kHz

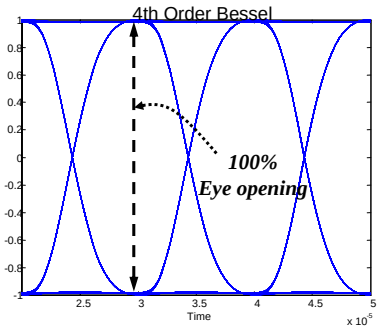


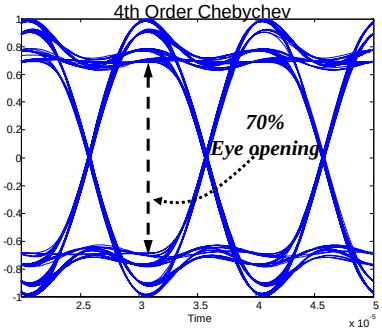


4th order Bessel

4th order Chebyshev I

Eye Diagrams

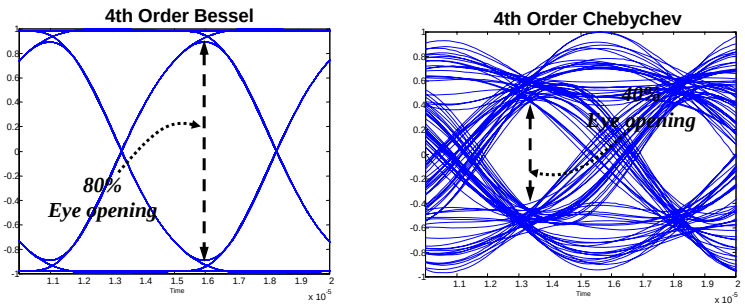




Pseudo-random data with maximum signal power @ 50kHz

Eye Diagrams

Pseudo-random data maximum signal power @ 100kHz



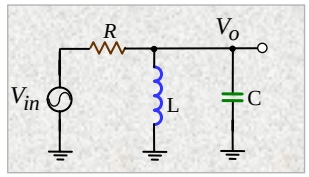
Filter with constant group delay → Less timing jitter & more open eye
→ Lower BER (bit-error-rate)

Summary
Filter Types

- Filters with high signal attenuation per pole ⇨ poor phase response
- For a given signal attenuation requirement of preserving constant groupdelay →Higher order filter
  - In the case of passive filters ⇨ higher component count
  - Case of integrated active filters ⇨ higher chip area & power dissipation
- In cases where filter is followed by ADC and DSP
  - Possible to digitally correct for phase non-linearities incurred by the analog circuitry by using phase equalizers

RLC Filters

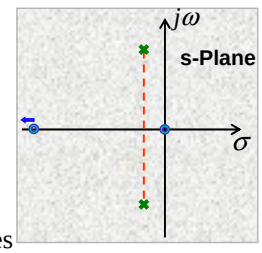
•Bandpass filter:



$$\frac{V_o}{V_{in}} = \frac{\frac{s}{RC}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0 = 1 \big/ \sqrt{LC}$$

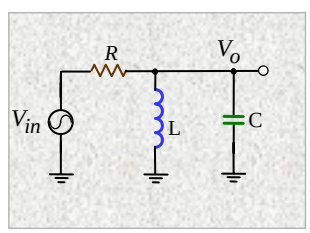
$$Q = \omega_0 RC = \frac{R}{L\omega_0}$$



Singularities: Pair of complex conjugate poles
Zeros @ *f=0* & *f=inf.*

RLC Filters

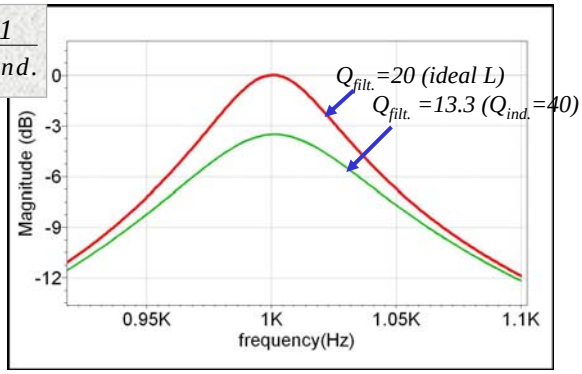
- Design a bandpass filter with:
  - ❑Center frequency of 1kHz
  - ❑Q of 20



- First assume the inductor is ideal
- Next consider the case where the inductor has series R resulting in an inductor Q of 40
- What is the effect of finite inductor Q on the overall Q?

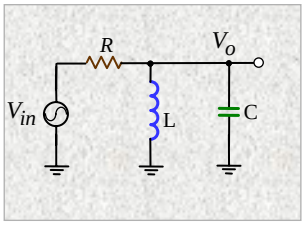
RLC Filters
Effect of Finite Component Q

$$\frac{1}{Q_{filt}} = \frac{1}{Q_{filt}^{ideal}} + \frac{1}{Q_{ind.}}$$



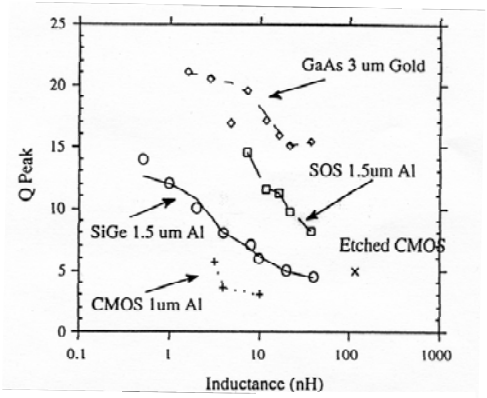
⇨Component Q must be much higher compared to desired filter Q

RLC Filters



Question:
Can RLC filters be integrated on-chip?

Monolithic Inductors
Feasible Quality Factor & Value



⇨ Feasible monolithic inductor in CMOS tech. <10nH with Q <7

❖Ref: "Radio Frequency Filters", Lawrence Larson; Mead workshop presentation 1999

Monolithic LC Filters

- Monolithic inductor in CMOS tech.
  - L<10nH with Q<7
- Max. capacitor size (based on realistic chip area)
  - C< 10pF

⇨*LC filters in the monolithic form feasible:*
- Frequency >500MHz
- Only low quality factor filters

Learn more in EE242

Monolithic Filters

- Desirable to integrate filters with critical frequencies << 500MHz
- Per previous slide LC filters not a practical option in the integrated form for non-RF frequencies
- Good alternative:

  ⇨Active filters built without the need for inductors