EE247 - Lecture 2  
Filters

- Material covered today:
  - Nomenclature
  - Filter specifications
    - Quality factor
    - Frequency characteristics
    - Group delay
  - Filter types
    - Butterworth
    - Chebyshev I
    - Chebyshev II
    - Elliptic
    - Bessel
  - Group delay comparison example

 Administrative

- Office hours for H.K. changed to:  
  Tues.- Thurs. 2:30-3:30 @ 463 Cory Hall
  Extra office hours by appointment
Summary
Last Lecture

- Major success in CMOS technology scaling:
  - Inexpensive DSPs technology
  - Resulted in the need for high performance Analog/Digital interface circuitry
- Main Analog/Digital interface building blocks includes
  - Analog filters
  - D/A converters
  - A/D converters

Filters

Filters \( \rightarrow \) Provide frequency selectivity and/or phase shaping
Nomenclature

Filter Types

**Lowpass**

\[ |H(j\omega)| \]

**Highpass**

\[ |H(j\omega)| \]

**Bandpass**

\[ |H(j\omega)| \]

**Band-reject (Notch)**

\[ |H(j\omega)| \]

**All-pass**

\[ |H(j\omega)| \]

Provide frequency selectivity

Phase shaping or equalization

Filter Specifications

- Frequency characteristics (lowpass filter):
  - Passband ripple (\( R_{pass} \))
  - Cutoff frequency or -3dB frequency
  - Stopband rejection
  - Passband gain
- Phase characteristics:
  - Group delay
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Power/pole & Area/pole
Quality Factor \((Q)\)

- The term quality factor \((Q)\) has different definitions in different contexts:
  - Component quality factor (inductor & capacitor \(Q\))
  - Pole quality factor
  - Bandpass filter quality factor

- Next 3 slides clarifies each
Component Quality Factor \((Q)\)

- For any component with a transfer function:

\[
H(j\omega) = \frac{1}{R(\omega) + jX(\omega)}
\]

- Quality factor is defined as:

\[
Q = \frac{X(\omega)}{R(\omega)} \rightarrow \frac{\text{Energy Stored}}{\text{Average Power Dissipation}} \text{ per unit time}
\]

Inductor & Capacitor Quality Factor

- Inductor \(Q\):

\[
Y_L = \frac{1}{R_s + j\omega L} \quad Q_L = \frac{\omega L}{R_s}
\]

- Capacitor \(Q\):

\[
Z_C = \frac{1}{\frac{1}{R_p} + j\omega C} \quad Q_C = \omega C R_p
\]
Pole Quality Factor

\[ Q_{Pole} = \frac{\omega_x}{2\sigma_x} \]

Bandpass Filter Quality Factor \((Q)\)

\[ Q = \frac{f_{center}}{\Delta f} \]

\[ \Delta f = f_2 - f_1 \]
What is Group Delay?

- Consider a continuous time filter with s-domain transfer function $G(s)$:
  $$G(j\omega) \equiv |G(j\omega)|e^{j\theta(\omega)}$$

- Let us apply a signal to the filter input composed of sum of two sinewaves at slightly different frequencies ($\Delta\omega \ll \omega$):
  $$v_{in}(t) = A_1\sin(\omega t) + A_2\sin[(\omega + \Delta\omega) t]$$

- The filter output is:
  $$v_{out}(t) = A_1 |G(j\omega)| \sin[\omega t + \theta(\omega)] +$$
  $$A_2 |G[j(\omega + \Delta\omega)]| \sin[(\omega + \Delta\omega) t + \theta(\omega + \Delta\omega)]$$
What is Group Delay?
Signal Magnitude and Phase Impairment

\[ v_{out}(t) = A_1 |G(j\omega)| \sin \left\{ \omega \left[ t + \frac{\theta(\omega)}{\omega} \right] \right\} + \]
\[ + A_2 |G[j(\omega+\Delta\omega)]| \sin \left\{ (\omega+\Delta\omega) \left[ t + \frac{\theta(\omega)}{\omega} + \left( \frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} \right) \frac{\Delta\omega}{\omega} \right] \right\} \]

- If the second term in the phase of the 2nd sin wave is non-zero, then the filter’s output at frequency \( \omega+\Delta\omega \) is time-shifted differently than the filter’s output at frequency \( \omega \) → “Phase distortion”
- If the second term is zero, then the filter’s output at frequency \( \omega+\Delta\omega \) and the output at frequency \( \omega \) are each delayed in time by \( -\frac{\theta(\omega)}{\omega} \)
- \( \tau_{PD} = -\frac{\theta(\omega)}{\omega} \) is called the “phase delay” and has units of time

Phase distortion is avoided only if:

\[ \frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} = 0 \]

- Clearly, if \( \theta(\omega) = k\omega \), \( k \) a constant, → no phase distortion
- This type of filter phase response is called “linear phase” → Phase shift varies linearly with frequency
- \( \tau_{GR} = -\frac{d\theta(\omega)}{d\omega} \) is called the “group delay” and also has units of time. For a linear phase filter \( \tau_{GR} = \tau_{PD} = k \)
  → \( \tau_{GR} = \tau_{PD} \) implies linear phase
- Note: Filters with \( \theta(\omega) = k\omega + c \) are also called linear phase filters, but they’re not free of phase distortion
What is Group Delay?

Signal Magnitude and Phase Impairment

- If $\tau_{GR} = \tau_{PD} \rightarrow$ No phase distortion

$$v_{OUT}(t) = A_1 \left| G(j\omega) \right| \sin \left[ \omega \left( t - \tau_{GR} \right) \right] +$$

$$+ A_2 \left| G[j(\omega+\Delta\omega)] \right| \sin \left[ (\omega+\Delta\omega) \left( t - \tau_{GR} \right) \right]$$

- If also $\left| G(j\omega) \right| = \left| G[j(\omega+\Delta\omega)] \right|$ for all input frequencies within the signal-band, $v_{OUT}$ is a scaled, time-shifted replica of the input, with no "signal magnitude distortion":

- In most cases neither of these conditions are realizable exactly

Summary

Group Delay

- Phase delay is defined as:
  $$\tau_{PD} = -\theta(\omega)/\omega \ [\text{time}]$$
- Group delay is defined as:
  $$\tau_{GR} = -d\theta(\omega)/d\omega \ [\text{time}]$$
- If $\theta(\omega) = k\omega$, $k$ a constant, $\rightarrow$ no phase distortion
- For a linear phase filter $\tau_{GR} = \tau_{PD} = k$
Lowpass Butterworth Filter

- All poles
- Poles located on the unit circle with equal angles

Example: 5th Order Butterworth filter
Filter Types
Chebyshev I Lowpass Filter

- Chebyshev I filter
  - Ripple in the passband
  - Sharper transition band compared to Butterworth
  - Poorer group delay
  - As more ripple is allowed in the passband:
    • Sharper transition band
    • Poorer phase response

Chebyshev I Lowpass Filter Characteristics

- All poles
- Poles located on an ellipse inside the unit circle
- Allowing more ripple in the passband:
  ⇒ Narrower transition band
  ⇒ Sharper cut-off
  ⇒ Higher pole Q
  ⇒ Poorer phase response

Example: 5th Order Chebyshev filter
Filter Types
Chebyshev II Lowpass

- Chebyshev II filter
  - Ripple in stopband
  - Sharper transition band compared to Butterworth
  - Passband phase more linear compared to Chebyshev I

Example: 5th Order Chebyshev II filter

Filter Types
Chebyshev II Lowpass

- Both poles & zeros
  - No. of poles $n$
  - No. of finite zeros $n-1$
- Poles located both inside & outside of the unit circle
- Zeros located on $j\omega$ axis
- Ripple in the stopband only
Filter Types
Elliptic Lowpass Filter

- Elliptic filter
  - Ripple in passband
  - Ripple in the stopband
  - Sharper transition band compared to Butterworth & both Chebyshevs
  - Poorest phase response

Magnitude (dB)

Example: 5th Order Elliptic Filter

Phase (degrees)

Normalized Frequency

Example: 5th Order Elliptic Filter

Filter Types
Elliptic Lowpass Filter

- Both poles & zeros
  - No. of poles \( n \)
  - No. of zeros \( n-1 \)
- Zeros located on \( j\omega \) axis
- Sharp cut-off
  - Narrower transition band
  - Pole Q higher compared to the previous filters

Example: 5th Order Elliptic Filter

s-plane
Filter Types
Bessel Lowpass Filter

- Bessel
  - All poles
  - Maximally flat group delay
  - Poor amplitude attenuation
  - Poles outside unit circle (s-plane)
  - Relatively low Q poles

Example: 5th Order Bessel filter

Magnitude Response of a Bessel Filter as a Function of Filter Order (n)
Filter Types
Comparison of Various Type LPF Magnitude Response

All 5th order filters with same corner freq.

Filter Types
Comparison of Various LPF Singularities
Comparison of Various LPF Group Delay


Group Delay Comparison Example

- Lowpass filter with 100kHz corner frequency
- Chebyshev I versus Bessel
  - Both filters 4th order- same -3dB point
  - Passband ripple of 1dB allowed for Chebyshev I
Magnitude Response

Phase Response
Group Delay

Normalized Group Delay
Step Response

Intersymbol Interference (ISI)

ISI → Broadening of pulses resulting in interference between successive transmitted pulses
Example: Simple RC filter
Pulse Broadening
Bessel versus Chebyshev

Chebyshev filter incurs more severe pulse broadening compared to Bessel
→ More ISI

Response to Pseudo-Random Data
Chebyshev versus Bessel

Input Signal:
Symbol rate 1/130kHz
Eye Diagram

- Eye diagram is a useful graphical illustration for signal degradation
- Consists of many overlaid traces of a signal using an oscilloscope where the symbol timing serves as the scope trigger
- It is a visual summary of all possible intersymbol interference waveforms
  - The vertical opening $\rightarrow$ immunity to noise
  - Horizontal opening $\rightarrow$ timing jitter

Measure of Signal Degradation

Random data with symbol rates:
- 1/50kHz
- 1/100kHz
- 1/130kHz
Eye Diagram
Chebyshev versus Bessel

Input Signal
Pseudo-random data
Symbol rate: 1/ 130kHz

Eye Diagrams

Pseudo-random data with maximum signal power @ 50kHz
Eye Diagrams

Pseudo-random data maximum signal power @ 100kHz

Filter with constant group delay
- Less timing jitter & more open eye
- Lower BER (bit-error-rate)

Summary
Filter Types

- Filters with high signal attenuation per pole ⇒ poor phase response
- For a given signal attenuation requirement of preserving constant groupdelay ⇒ Higher order filter
  - In the case of passive filters ⇒ higher component count
  - Case of integrated active filters ⇒ higher chip area & power dissipation
- In cases where filter is followed by ADC and DSP
  - Possible to digitally correct for phase non-linearities incurred by the analog circuitry by using phase equalizers
RLC Filters

• Bandpass filter:

\[
\frac{V_o}{V_{in}} = \frac{s}{s^2 + \frac{\omega_b}{Q} s + \omega_b^2}
\]

\[
\omega_b = \frac{1}{\sqrt{LC}}
\]

\[
Q = \frac{\omega_b R C}{L \omega_b}
\]

Singularities: Pair of complex conjugate poles
Zeros @ \( f=0 \) & \( f=\infty \).

• Design a bandpass filter with:
  - Center frequency of 1kHz
  - Q of 20

  • First assume the inductor is ideal
  • Next consider the case where the inductor has series R resulting in an inductor Q of 40
  • What is the effect of finite inductor Q on the overall Q?
RLC Filters

Effect of Finite Component \( Q \)

\[
\frac{1}{Q_{\text{filt}}} = \frac{1}{Q_{\text{filt}}^{\text{ideal}}} + \frac{1}{Q_{\text{ind}}}.
\]

\( Q_{\text{filt}} = 20 \) (ideal \( L \))

\( Q_{\text{filt}} = 13.3 \) (\( Q_{\text{ind}} = 40 \))

Component \( Q \) must be much higher compared to desired filter \( Q \)

RLC Filters

Question:
Can RLC filters be integrated on-chip?
Monolithic Inductors
Feasible Quality Factor & Value

Feasible monolithic inductor in CMOS tech. <10nH with Q <7
Ref: "Radio Frequency Filters", Lawrence Larson; Mead workshop presentation 1999

Monolithic LC Filters

- Monolithic inductor in CMOS tech.
  - L<10nH with Q<7
- Max. capacitor size (based on realistic chip area)
  - C< 10pF

LC filters in the monolithic form feasible:
- Frequency >500MHz
- Only low quality factor filters

Learn more in EE242
Monolithic Filters

- Desirable to integrate filters with critical frequencies \(<500\text{MHz}\)
- Per previous slide LC filters not a practical option in the integrated form for non-RF frequencies
- Good alternative:
  - Active filters built without the need for inductors