EE247
Lecture 3

• Active Filters
  – Active biquads
    • Sallen- Key & Tow-Thomas
    • Integrator-based filters
      – Signal flowgraph concept
      – First order integrator-based filter
      – Second order integrator-based filter & biquads
  – High order & high Q filters
    • Cascaded biquads
      – Cascaded biquad sensitivity to component mismatch
    • Ladder type filters

Correction & Clarification
From Last Lecture (2)

• Slide # 36- Pulse response for 4th order Chebyshev I was shown with an offset- error corrected and updated on lecture notes

• Slide # 43- There was a question in class regarding the frequency at which the Qs were measured- Inductors are measured at specific application frequencies, all above 1GHz:
  – Since $Q_L=(\omega L/R)$ thus Qs would be lower at our frequencies of interest
Filters

2nd Order Transfer Functions (Biquads)

- Biquadratic (2nd order) transfer function:

\[
H(s) = \frac{1}{s^2 + \frac{s}{Q_p} + \frac{1}{Q_p}}
\]

\[
|H(j\omega)| = \frac{1}{\sqrt{(1-\frac{\omega^2}{Q_p^2})^2 + \left(\frac{\omega}{Q_p}\right)^2}}
\]

\[
H(j\omega)_{\omega=0} = 1, \quad H(j\omega)_{\omega->\infty} = 0, \quad H(j\omega)_{\omega=\omega_p} = Q_p
\]

Biquad poles @: \( s = -\frac{\omega_p}{2Q_p}\sqrt{1-4Q_p^2} \)

Note: for \( Q_p \leq \frac{1}{2} \) poles are real, complex otherwise

Implementation of Biquads

- Passive RC: only real poles \( \rightarrow \) can’t implement complex conjugate poles

- Terminated LC
  - Low power, since it is passive
  - Only fundamental noise sources \( \rightarrow \) load and source resistance
  - As previously analyzed, not feasible in the monolithic form for \( f < \text{a few 100s of MHz} \)

- Active Biquads
  - Many topologies can be found in filter textbooks!
  - Widely used topologies:
    - Single-opamp biquad: Sallen-Key
    - Multi-opamp biquad: Tow-Thomas
    - Integrator based biquads
Active Biquad
Sallen-Key Low-Pass Filter

- Single gain element
- Can be implemented both in discrete & monolithic form
- "Parasitic sensitive"
- Versions for LPF, HPF, BP, ...
  - Advantage: Only one opamp used
  - Disadvantage: Sensitive to parasitic – all pole no zeros

\[
H(s) = \frac{G}{1 + \frac{s}{\omega_pQ_p} + \frac{s^2}{\omega_p^2}}
\]
\[
\omega_p = \frac{1}{\sqrt{R_1C_1R_2C_2}}
\]
\[
Q_p = \frac{\omega_p}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-G}{R_2C_2}}
\]

Addition of Imaginary Axis Zeros

- Sharpen transition band
- Can "notch out" interference
- High-pass filter (HPF)
- Band-reject filter

\[
H(s) = K \left( 1 + \frac{s}{\omega Z} \right)^2 \left( 1 + \frac{s}{\omega_pQ_p} + \frac{s^2}{\omega_p^2} \right)^2
\]
\[
|H(j\omega)|_{\omega->\infty} = K \left( \frac{\omega_p}{\omega Z} \right)^2
\]

Note: Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, and readily identifiable units.
Imaginary Zeros

- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequency

$$f_p = 100kHz$$
$$Q_p = 2$$
$$f_z = 3f_p$$

Moving the Zeros

$$f_p = 100kHz$$
$$Q_p = 2$$
$$f_z = f_p$$
Tow-Thomas Active Biquad


Frequency Response

\[
\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2a_i - b_i) s + (b_2a_0 - b_0)}{s^2 + a_1 s + a_0}
\]

\[
\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_i s + b_0}{s^2 + a_1 s + a_0}
\]

\[
\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1 \sqrt{a_0}} \frac{(b_0 - b_2a_0) s + (a_i b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}
\]

- \(V_{o2}\) implements a general biquad section with arbitrary poles and zeros
- \(V_{o1}\) and \(V_{o3}\) realize the same poles but are limited to at most one finite zero
### Component Values

\[
\begin{align*}
  b_i &= \frac{R_i}{R_i R R C_1 C_2} \\
  b_i &= \frac{1}{R C_1} \left( \frac{R_i}{R_i} - \frac{R R_i}{R_i R_i} \right) \\
  b_i &= \frac{R_i}{R_i} \\
  a_i &= \frac{R_i}{R_i R R C_1 C_2} \\
  a_i &= \frac{1}{R C_1} \\
  k_i &= \sqrt[2]{\frac{R_i R C_2}{R_i R C_1}} \\
  k_i &= \frac{R_i}{R_i}
\end{align*}
\]

given \(a_i, b_i, k_i, C_1, C_2\) and \(R_i\)

\[
\begin{align*}
  R_i &= \frac{1}{a_i C_1} \\
  R_i &= \frac{k_i}{\sqrt{a_i C_1}} \\
  R_i &= \frac{1}{k_i} \frac{1}{\sqrt{a_i C_1}} \\
  R_i &= \frac{1}{k_i} \frac{1}{\sqrt{a_i C_1}} \\
  R_i &= \frac{k_i}{\sqrt{a_i C_1}} \\
  R_i &= \frac{R_i}{R_i} \\
  R_i &= k_i R_i
\end{align*}
\]

it follows that

\[
\omega_c = \frac{R_i}{R_i R R C_1 C_2} \\
Q_c = \omega_c R_i C_i
\]

### Higher-Order Filters in the Integrated Form

- One way of building higher-order filters (\(n>2\)) is via cascade of 2\textsuperscript{nd} order biquads, e.g. Sallen-Key, or Tow-Thomas

\[
\begin{array}{ccc}
  \text{2nd order Filter} & \rightarrow & \text{2nd order Filter} \\
  1 & \rightarrow & 2 \\
  \rightarrow & \cdots & \cdots \\
  \rightarrow & \text{2nd order Filter} & \rightarrow \\
  N & \rightarrow & \text{Filter order: } n=2N
\end{array}
\]

Cascade of 2\textsuperscript{nd} order biquads:

- Easy to implement
- Highly sensitive to component mismatch good for low Q filters only
- \(\rightarrow\) Good alternative: Integrator-based ladder type filters
Integrator Based Filters

- Main building block for this category of filters
  - Integrator
- By using signal flowgraph techniques
  - Conventional RLC filter topologies can be converted to integrator based type filters

- Next few pages:
  - Introduction to signal flowgraph techniques
  - 1st order integrator based filter
  - 2nd order integrator based filter
  - High order and high Q filters

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What is a Signal Flowgraph (SFG)?

- SFG → Topological network representation consisting of nodes & branches- used to convert one form of network to a more suitable form (e.g. passive RLC filters to integrator based filters)
- Any network described by a set of linear differential equations can be expressed in SFG form
- For a given network, many different SFGs exists
- Choice of a particular SFG is based on practical considerations such as type of available components

What is a Signal Flowgraph (SFG)?

- Signal flowgraph consist of nodes & branches:
  - Nodes represent variables (V & i in our case)
  - Branches represent transfer functions (we will call the transfer function branch multiplication factor or BMF)

- To convert a network to its SFG form, KCL & KVL is used to derive state space description

- Simple example:

\[ I_{in} \times Z = V_o \]

Signal Flowgraph (SFG) Examples

- Circuit: \[ I_{in} \rightarrow V_o \]
  - State-space description: \[ I_{in} \times R = V_o \]
  - SFG: \[ I_{in} \rightarrow V_o \]

- Circuit: \[ V_{in} \rightarrow I_o \]
  - State-space description: \[ V_{in} \times \frac{1}{SL} = I_o \]
  - SFG: \[ V_{in} \rightarrow I_o \]

- Circuit: \[ I_{in} \rightarrow V_o \]
  - State-space description: \[ I_{in} \times \frac{1}{SC} = V_o \]
  - SFG: \[ I_{in} \rightarrow V_o \]
Useful Signal Flowgraph (SFG) Rules

- Two parallel branches can be replaced by a single branch with overall BMF equal to sum of two BMFs

- A node with only one incoming branch & one outgoing branch can be eliminated & replaced by a single branch with BMF equal to the product of the two BMFs

- An intermediate node can be multiplied by a factor $(k)$. BMFs for incoming branches have to be multiplied by $k$ and outgoing branches divided by $k$
Useful Signal Flowgraph (SFG) Rules

- Simplifications can often be achieved by shifting or eliminating nodes

\[
\begin{align*}
V_i & \quad V_2 & \quad V_3 & \quad V_o \\
\downarrow & \quad a & \quad -b & \quad 1 \\
& \quad b & \quad 1 & \quad V_i \\
& \quad 1 & \quad -b & \quad V_2 \\
& \quad 1 & \quad a & \quad V_3 \\
& \quad 1 & \quad b & \quad V_o
\end{align*}
\]

- A self-loop branch with BMF $y$ can be eliminated by multiplying the BMF of incoming branches by $1/(1-y)$

\[
\begin{align*}
V_i & \quad V_2 & \quad V_3 & \quad V_o \\
\downarrow & \quad a & \quad -b & \quad 1 \\
& \quad b & \quad 1 & \quad V_i \\
& \quad 1 & \quad -b & \quad V_2 \\
& \quad 1 & \quad a & \quad V_3 \\
& \quad 1 & \quad b & \quad V_o
\end{align*}
\]

Integrator Based Filters

1st Order LPF

- Conversion of simple lowpass RC filter to integrator-based type by using signal flowgraph techniques

\[
\frac{V_o}{V_{in}} = \frac{1}{1 + sRC}
\]
What is an Integrator?

Example: Single-Ended Opamp-RC Integrator

\[ V_o = -V_{in} \int \frac{I}{sRC} \, ds, \quad V_o = -\frac{1}{RC} \int V_{in} \, dt \]

Note: Practical integrator in CMOS technology has input & output both in the form of voltage and not current \(\rightarrow\) Consideration for SFG derivation

Integrator Based Filters

1st Order LPF

1. Start from circuit prototype-
   Name voltages & currents for all components

2. Use KCL & KVL to derive state space description in such a way to have BMFs in the integrator form:
   - Capacitor voltage expressed as function of its current \(V_{cap} = f(I_{cap})\)
   - Inductor current as a function of its voltage \(I_{ind} = f(V_{ind})\)

3. Use state space description to draw signal flowgraph (SFG) (see next page)
Integrator Based Filters
First Order LPF

\[ V_1 = V_{in} - V_C \]
\[ V_C = I_2 \frac{1}{sC} \]
\[ V_o = V_C \]
\[ I_1 = V_1 \frac{1}{R_s} \]
\[ I_2 = I_1 \]

- All voltages & currents \( \rightarrow \) nodes of SFG
- Voltage nodes on top, corresponding current nodes below each voltage node

Normalize
- Since integrators are the main building blocks \( \rightarrow \) require in & out signals in the form of voltage (not current)
  - Convert all currents to voltages by multiplying current nodes by a scaling resistance \( R^* \)
  - Corresponding BMFs should then be scaled accordingly

\[ V_I = V_{in} - V_o \]
\[ I_1 = \frac{V_I}{R_s} \]
\[ V_o = \frac{I_2}{sC} \]
\[ I_2 = I_1 \]

\[ V_I' = \frac{R^*}{R_s} V_I \]
\[ I_1' = \frac{R^*}{sC R} I_1 \]
\[ I_2' = I_1' \]
1\textsuperscript{st} Order Lowpass Filter SGF

Normalize

\[ V_{\text{in}} \rightarrow V_1 \rightarrow \frac{1}{R_s} \rightarrow l \rightarrow V_0 \]

\[ \frac{1}{sC} \]

\[ I_1 \rightarrow 1 \rightarrow I_2 \]

\[ \frac{1}{sC} \]

\[ V_{\text{in}} \rightarrow V_1 \rightarrow \frac{1}{R_s} \rightarrow l \rightarrow V_0 \]

\[ \frac{1}{sC} \]

\[ I_1 \times R \]

\[ I_2 \times R \]

\[ l \]

\[ V_1 \rightarrow 1 \rightarrow V_2' \]

\[ \frac{1}{sC R^2} \]

1\textsuperscript{st} Order Lowpass Filter SGF

Synthesis

\[ V_{\text{in}} \rightarrow V_1 \rightarrow -l \rightarrow V_0 \]

\[ \frac{R^*}{R_s} \]

\[ \frac{l}{sC R^2} \]

\[ V_1 \rightarrow -l \rightarrow V_2' \]

\[ \frac{R^*}{R_s} \]

\[ \frac{l}{sC R^2} \]

Consolidate two branches

\[ R^* = R_s \quad \tau = R^* \times C \]
First Order Integrator Based Filter

\[
\begin{align*}
V_{in} & \quad + & 1 & V_{I} & \quad -l & V_{o} \\
\int & & \frac{1}{\tau s} & & 1 & \\
\end{align*}
\]

\[H(s) = \frac{1}{\tau s}\]

1st Order Filter
Built with Opamp-RC Integrator

- Single-ended Opamp-RC integrator has a sign inversion from input to output
  - Convert SFG accordingly by modifying BMF

\[V_{in} = -V_{in}'\]
1st Order Filter
Built with Opamp-RC Integrator

- To avoid requiring an additional opamp to perform summation at the input node:

\[ V_{in}' = -V_{in} \]

1st Order Filter
Built with Opamp-RC Integrator (continued)

\[ \frac{V_O}{V_{in}} = \frac{1}{1 + sRC} \]
Opamp-RC 1st Order Filter Noise

Identify noise sources (here it is resistors & opamp)
Find transfer function from each noise source to the output (opamp noise next page)

\[ v_n = k \sum_{m=1}^{\infty} H_m(f) \frac{d}{df} S_m(f) \]

\[ S_m(f) \rightarrow \text{Noise spectral density of } m^{th} \text{ noise source} \]

\[ V_{n1}^2 = 4KTR\alpha^2 \]

\[ V_{n2}^2 = 4KTR\Delta f \]

Typically, \( \alpha \) increases as filter order increases

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Opamp-RC Filter Noise Opamp Contribution

- So far only the fundamental noise sources are considered
- In reality, noise associated with the opamp increases the overall noise
- For a well-designed filter opamp is designed such that noise contribution of opamp is negligible compared to other noise sources
- The bandwidth of the opamp affects the opamp noise contribution to the total noise
Integrator Based Filter

2nd Order RLC Filter

- State space description:
  \[ V_R = V_L = V_C = V_o \]
  \[ V_C = \frac{I_C}{sC} \]
  \[ I_R = \frac{V_R}{R} \]
  \[ I_L = \frac{V_L}{sL} \]
  \[ I_C = I_{in} - I_R - I_L \]

- Draw signal flowgraph (SFG)

---

\[ I_{in} \quad \begin{array}{c}
+ \quad V_R \\
R \\
- \quad I_{in} \\
\end{array} \quad \begin{array}{c}
+ \quad V_C \\
C \\
- \quad V_L \\
\end{array} \quad \begin{array}{c}
+ \quad V_o \\
O \\
- \quad I_{in} \\
\end{array} \]

---

2nd Order RLC Filter SGF Normalize

- Convert currents to voltages by multiplying all current nodes by the scaling resistance \( R^* \)

\[ I_{in} \quad \begin{array}{c}
\frac{1}{R} \\
\frac{1}{sC} \\
- \quad I_{in} \\
\end{array} \quad \begin{array}{c}
\frac{1}{sL} \\
\end{array} \quad \begin{array}{c}
I_{in} \quad \begin{array}{c}
\frac{1}{R} \\
- \quad I_{in} \\
\end{array} \quad \begin{array}{c}
\frac{1}{sC} \\
- \quad I_{in} \\
\end{array} \quad \begin{array}{c}
\frac{1}{sL} \\
\end{array} \quad \begin{array}{c}
V_1 \quad \begin{array}{c}
\frac{1}{R} \\
- \quad V_1 \\
\end{array} \quad \begin{array}{c}
\frac{1}{sC} \\
- \quad V_2 \\
\end{array} \quad \begin{array}{c}
\frac{1}{sL} \\
- \quad V_3 \\
\end{array} \quad \begin{array}{c}
V_0 \\
\end{array} \]

\[ I_{X}R^* = V_{X} \]
2nd Order RLC Filter SGF
Synthesis

\[ \tau_1 = \frac{R^*}{L} \quad \tau_2 = \frac{L}{R^*} \]

Second Order Integrator Based Filter
Second Order Integrator Based Filter

\[ \frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + \beta} \]
\[ \frac{V_{LP}}{V_{in}} = \frac{1}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + \beta} \]
\[ \frac{V_{HP}}{V_{in}} = \frac{\tau_1 s^2}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + \beta} \]

\[ \tau_1 = R^* C \quad \tau_2 = L / R^* \]
\[ \beta = \frac{R^*}{R} \]
\[ \omega_0 = \frac{1}{\sqrt{\tau_1 \tau_2}} \]
\[ Q = \frac{1}{\beta} \sqrt{\frac{\tau_1}{\tau_2}} \]

From matching point of view desirable:
\[ \tau_1 = \tau_2 \rightarrow Q = \frac{R^*}{R} \]

Second Order Bandpass Filter Noise

\[ \overline{v^2_0} = k \sum_{m=1}^{\infty} W_m(f) \alpha S_m(f) \, df \]

- Find transfer function of each noise source to the output
- Integrate contribution of all noise sources
- Here it is assumed that opamps are noise free (not usually the case!)

\[ \overline{v^2_{n1}} = \overline{v^2_{n2}} = 4KTRdf \]

\[ \sqrt{\overline{v^2_0}} = \sqrt{\frac{4 K T R}{Q \frac{kT}{C}}} \]

Typically, \( \alpha \) increases as filter order increases
Note the noise power is directly proportion to \( Q \)
Second Order Integrator Based Filter Biquad

- By combining outputs can generate general biquad function:

\[
\frac{V_0}{V_{in}} = \frac{a_1 \tau_1 \tau_2 s^2 + a_2 \tau_2 s + a_3}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}
\]

s-plane

Summary
Integrator Based Monolithic Filters

- Signal flowgraph techniques utilized to convert RLC networks to integrator based active filters
- Each reactive element (L & C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor value:
  - For lowpass filter: \( \sqrt{V_0} = \sqrt{\alpha \frac{k T}{C}} \)
  - Bandpass filter: \( \sqrt{V_0} = \sqrt{\alpha Q \frac{k T}{C}} \)

where \( \alpha \) is a function of filter order and topology
Higher Order Filters

- How do we build higher order filters?
  - Cascade of biquads and 1st order sections
    - Each complex conjugate pole built with a biquad and real pole with 1st order section
    - Easy to implement
    - In the case of high order high Q filters → highly sensitive to component mismatch
  - Direct conversion of high order ladder type RLC filters
    - SFG techniques used to perform exact conversion of ladder type filters to integrator based filters
    - More complicated conversion process
    - Much less sensitive to component mismatch compared to cascade of biquads

Higher Order Filters
Cascade of Biquads

Example: LPF filter for CDMA baseband receiver

- LPF with
  - fpass = 650 kHz    Rpass = 0.2 dB
  - fstop = 750 kHz    Rstop = 45 dB
  - Assumption: Can compensate for phase distortion in the digital domain

- 7th order Elliptic Filter
- Implementation with cascaded Biquads
  Goal: Maximize dynamic range
  - Pair poles and zeros
  - In the cascade chain place lowest Q poles first and progress to higher Q poles moving towards the output node
Overall Filter Frequency Response

Bode Diagram

Phase (deg) Magnitude (dB)
-80 -60 -40 -20 0

Frequency [Hz] 300kHz 1MHz 3MHz

Mag. (dB)
-0.2 0

Pole-Zero Map

Qpole fpole [kHz]
16.7902 659.496
3.6590 611.744
1.1026 473.643
319.568

fzero [kHz]
1297.5
836.6
744.0
CDMA Filter
Built with Cascade of 1\textsuperscript{st} and 2\textsuperscript{nd} Order Sections

- 1\textsuperscript{st} order filter implements the single real pole
- Each biquad implements a pair of complex conjugate poles and a pair of imaginary axis zeros

Biquad Response

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Biquad Response

Intermediate Outputs
Sensitivity to Relative Component Mismatch

Component variation in Biquad 4 relative to the rest (highest Q poles):
- Increase $\omega_{p4}$ by 1%
- Decrease $\omega_{z4}$ by 1%

High Q poles $\rightarrow$ High sensitivity in Biquad realizations

High Q & High Order Filters

• Cascade of biquads
  - Highly sensitive to component mismatch $\rightarrow$ not suitable for implementation of high Q & high order filters
  - Cascade of biquads only used in cases where required Q for all biquads <4 (e.g. filters for disk drives)

• LC ladder filters more appropriate for high Q & high order filters (next topic)
  - Will show later $\rightarrow$ Less sensitive to component mismatch
Ladder Type Filters

- For simplicity, will start with all pole ladder type filters
  - Convert to integrator based form
  - Example shown

- Next will attend to high order ladder type filters incorporating zeros
  - Implement the same 7th order elliptic filter in the form of ladder type
    - Find level of sensitivity to component mismatch
    - Compare with cascade of biquads
  - Convert to integrator based form utilizing SFG techniques
  - Example shown

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LC Ladder Filters

- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated (with or w/o \( R_L \))

\textit{Doubly terminated LC ladder filters} \(\rightarrow\) \textit{Lowest sensitivity to component mismatch}
LC Ladder Filters

• Design:
  – Filter tables
  – CAD tools
    • Matlab
    • Spice

LC Ladder Filter Design Example

Design a LPF with maximally flat passband:
- $f_{-3dB} = 10MHz$, $f_{stop} = 20MHz$
- $Rs > 27dB$

- Maximally flat passband $\Rightarrow$ Butterworth
  - Find minimum filter order:
    - Use of Matlab
    - or Tables
    - Here tables used

  $f_{stop} / f_{-3dB} = 2$
  $Rs > 27dB$

Minimum Filter Order $\Rightarrow$ 5th order Butterworth
Find values for L & C from Table:

Note L & C values normalized to $\omega_{-3dB} = 1$

**Denormalization:**

Multiply all $L_{Norm}$, $C_{Norm}$ by:

$L_r = \frac{R}{\omega_{-3dB}}$

$C_r = \frac{1}{RX \omega_{-3dB}}$

$R$ is the value of the source and termination resistor

(choose both 1Ω for now)

Then: $L = L_r \times L_{Norm}$

$C = C_r \times C_{Norm}$

**From:** Williams and Taylor, p. 11.3
Magnitude Response Simulation

-6 dB passband attenuation due to double termination

SPICE simulation Results

LC Ladder Filter
Conversion to Integrator Based Active Filter

\[
\begin{align*}
V_1 &= V_{in} - V_2, & I_2 &= -\frac{V_2}{sC_1} \\
V_3 &= V_2 - V_4, & V_4 &= \frac{I_4}{sC_3} \\
V_5 &= V_4 - V_6, & V_6 &= \frac{I_6}{sC_5} \\
V_o &= V_6 \\
I_1 &= \frac{V_1}{R_s}, & I_2 &= I_1 - I_3 \\
I_4 &= I_3 - I_5, & I_5 &= \frac{V_5}{sL_2} \\
I_6 &= I_5 - I_7, & I_7 &= \frac{V_6}{R_L}
\end{align*}
\]
LC Ladder Filter
Signal Flowgraph

\[ V_i = V_{in} - V_2 \]
\[ V_2 = \frac{I_2}{sC_1} \]
\[ V_3 = V_2 - V_4 \]
\[ V_4 = \frac{I_4}{sC_3} \]
\[ V_5 = V_4 - V_6 \]
\[ V_6 = \frac{I_6}{sC_4} \]
\[ V_o = V_6 \]

\[ I_i = \frac{V_i}{R_s} \]
\[ I_2 = I_1 - I_3 \]
\[ I_4 = I_3 - I_5 \]
\[ I_6 = I_5 - I_7 \]
\[ I_7 = \frac{V_o}{R_L} \]
LC Ladder Filter

**Normalize**

\[ \frac{V_1}{R_s} \]
\[ \frac{1}{sC_1} \]
\[ \frac{1}{sL_2} \]
\[ \frac{1}{sC_3} \]
\[ \frac{1}{sL_4} \]
\[ \frac{1}{sC_5} \]
\[ \frac{1}{R_L} \]

\[ V_{in} \]
\[ V_1 \]
\[ -1 \]
\[ V_2 \]
\[ 1 \]
\[ V_3 \]
\[ -1 \]
\[ V_4 \]
\[ 1 \]
\[ V_5 \]
\[ -1 \]
\[ V_6 \]
\[ 1 \]
\[ V_7 \]

\[ V_{in} \]
\[ V_1 \]
\[ -1 \]
\[ V_2 \]
\[ 1 \]
\[ V_3 \]
\[ -1 \]
\[ V_4 \]
\[ 1 \]
\[ V_5 \]
\[ -1 \]
\[ V_6 \]
\[ 1 \]
\[ V_7 \]

LC Ladder Filter

**Synthesize**

\[ \frac{R^*}{R_s} \]
\[ \frac{1}{sC_1R^*} \]
\[ \frac{R^*}{sL_2} \]
\[ \frac{1}{sC_3R^*} \]
\[ \frac{R^*}{sL_4} \]
\[ \frac{1}{sC_5R^*} \]
\[ \frac{R^*}{R_L} \]

\[ V_{in} \]
\[ V_1 \]
\[ -1 \]
\[ V_2 \]
\[ 1 \]
\[ V_3 \]
\[ -1 \]
\[ V_4 \]
\[ 1 \]
\[ V_5 \]
\[ -1 \]
\[ V_6 \]
\[ 1 \]
\[ V_7 \]

\[ V_{in} \]
\[ V_1 \]
\[ -1 \]
\[ V_2 \]
\[ 1 \]
\[ V_3 \]
\[ -1 \]
\[ V_4 \]
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\[ V_5 \]
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\[ V_6 \]
\[ 1 \]
\[ V_7 \]