# PROBLEM SET #7

*Issued: Friday, Apr. 25, 2014*

*Due (at 9 a.m.): Friday, May 9, 2014*, in the EE C247B HW box near 125 Cory.

Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc.) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), cell phones, and game controllers (e.g., the Wii). Of these applications, the last four already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best explained via example.

This problem involves the MEMS-based micro-gyroscope as shown in Fig. PS7.1. In this device, momentum is generated by driving the proof mass into resonance vibration using the capacitive comb fingers along the *x*-axis. When the device is rotated about the *z*-axis (indicated in Fig. PS7.1), the vibrating mass will attempt to preserve its momentum in the original *x*-direction, which will then make the mass appear to deflect in the *y*-direction. This *y*-directed motion is then sensed by parallel-plate capacitors to determine rotation rate. In quantitative terms, the angular velocity $\Omega$ about *z*-axis generates a *Coriolis Force* ($F = 2m_s\dot{x}_d \times \Omega$ where $m_s$ is the equivalent mass in the sense mode, and $\dot{x}_d$ is the structure velocity in the drive mode) along the *y*-direction that drives the proof mass into *y*-directed vibration. The amplitude of the vibration is then picked up by the varying gap capacitances.

The suspended movable structure of the gyroscope is symmetric along both *x*-axis and *y*-axis. Figs. PS7.1-7.3 identify different parts of the structure, indicate which portions are freely suspended and which are anchored, and provide key dimensions. Fig. PS7.4 presents the drive and sense circuits of the gyroscope. The thicknesses of the structures are all 20 μm and the suspension beams are all 2 μm wide. The structure is constructed via polysilicon material with a density $\rho = 2300\ \mathrm{kg/m^3}$ and Young's Modulus $E = 150\ \mathrm{GPa}$. The movable structure is DC biased relative to all electrodes at $V_P$ = 20 V.

1. Calculate the *x*- and *y*-directed resonance frequencies of the gyroscope structure when all the ports are grounded. Ignore the suspension beam masses in these calculations.

2. Determine the capacitance and change in capacitance per unit displacement for (i) one of the drive electrodes and (ii) the positive sense electrodes (+).

3. Calculate the *y*-directed resonance frequency of the gyroscope structure when the structure is biased at 20 V (i.e. $V_P$ = 20 V).

**4.** Suppose the measured quality factors of the structure the in *x*- and *y*-directions are 150 and 100, respectively. Draw and specify (numerically) all element values in the equivalent circuits (transformer + *LCR*) modeling the (i) drive mode and (ii) sense mode when $V_P = 20$ V.

**5.** Code the equivalent circuits of problem 4 into SPICE netlists, add the necessary elements (e.g., a voltage source, a resistor, a capacitor, or an inductor) to drive at one end and detect velocity of the vibrating proof mass for both drive mode and sense mode, and simulate Bode plots for the voltage-to-velocity transfer functions.

**6.** Assume that an ac voltage $v_i$ with an amplitude of 1.5 V and a frequency equal to the resonance frequency of the drive mode is applied to the drive electrodes. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.

**7.** If the device is subjected to a rotation rate of $\Omega = 1$ rad/s along the *z*-axis indicated in Fig. PS7.1, what is the magnitude of the output voltage $V_{out}$ if $R_1$ is set to 0 Ω and $R_2$ is set to 10 kΩ. Assume the differential amplifier employs an ideal op-amp.

**8.** Repeat problem 7 with $R_1$ and $R_2$ set to 1 kΩ and 20 kΩ, respectively.

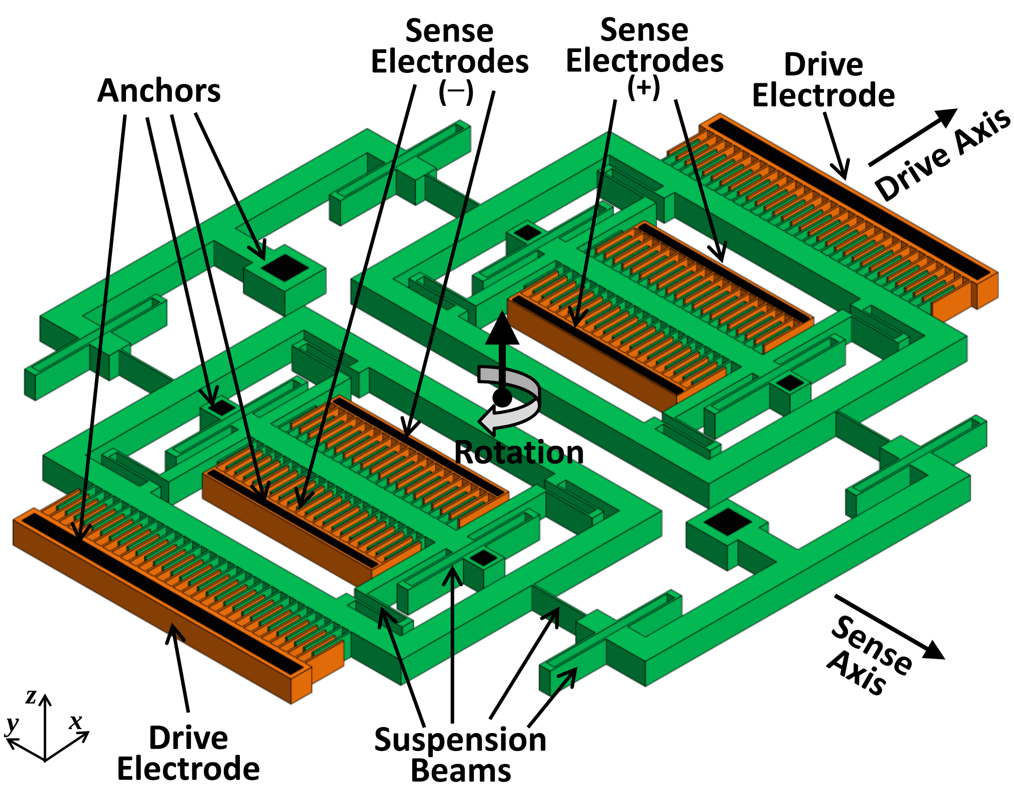


Figure PS7.1 – Perspective view of the gyroscope

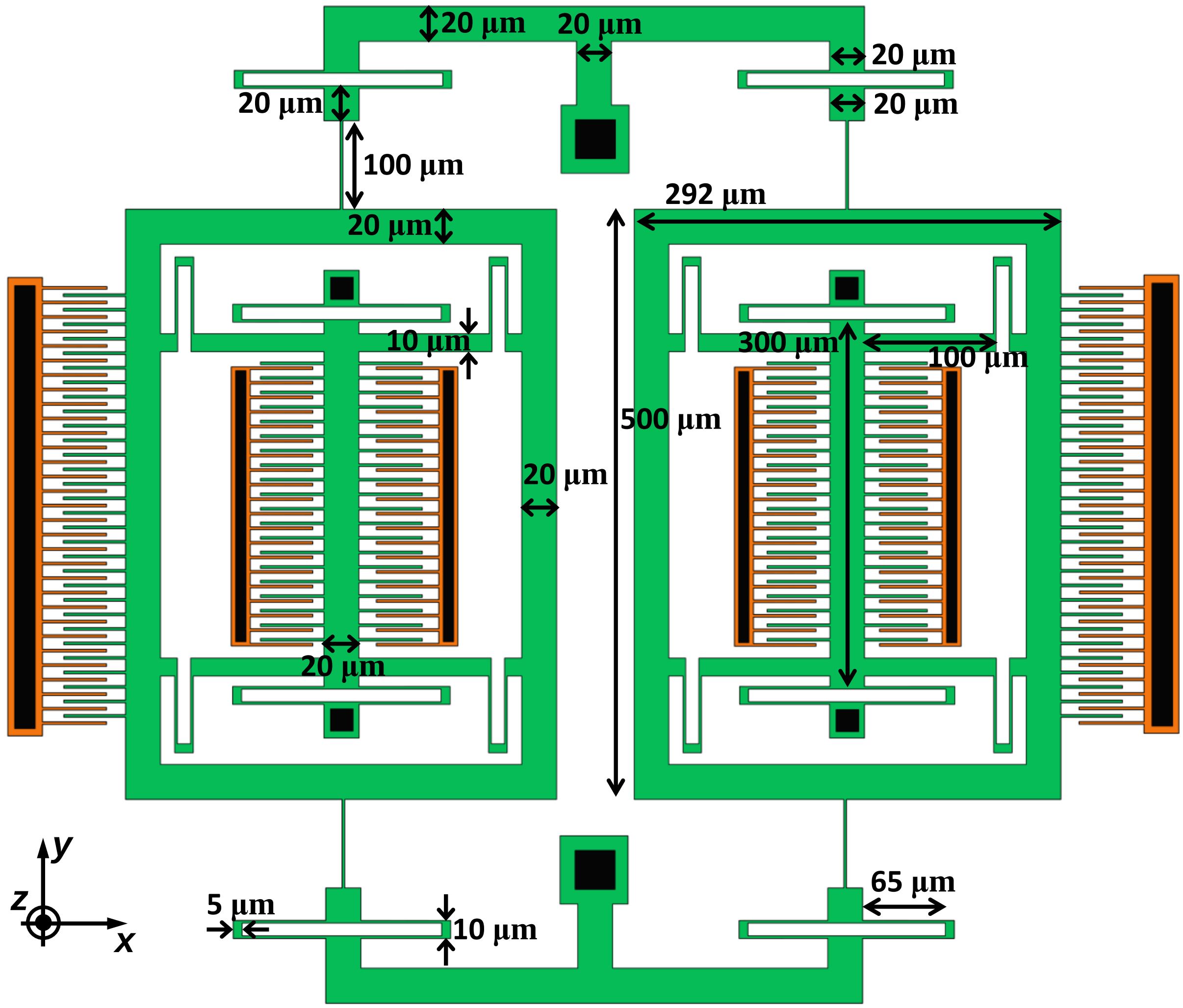


Figure PS7.2. – Top view of the gyroscope

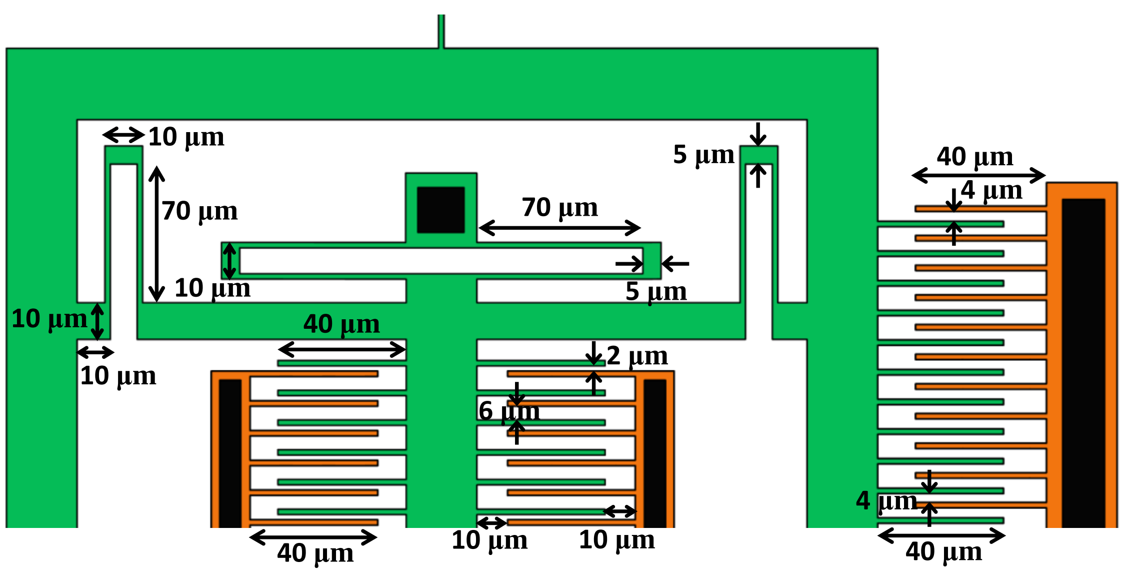


Figure PS7.3 – Partial zoom-in of the gyroscope

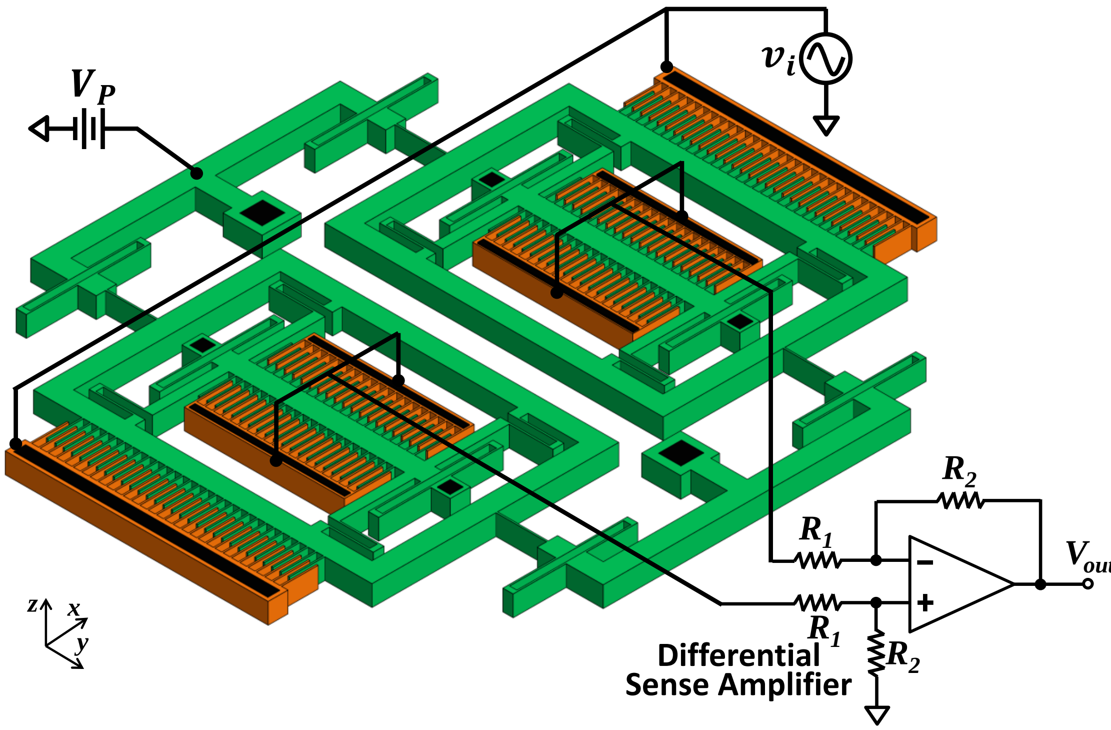


Figure PS7.4 – Perspective view of the gyroscope showing the actuating and sensing circuits