PROBLEM SET #1

Issued: Thursday, Jan. 30, 2014

Due (at 9 a.m.): Wednesday Feb. 12, 2014, in the EE C247B HW box near 125 Cory.

This homework assignment is intended to give you some early practice playing with dimensions and exploring how scaling can greatly improve or degrade certain performance characteristics of mechanical systems. Don't worry at this point if you do not understand fully some of the physical expressions used. They will be revisited later in the semester.

1. Ants are well known for their strength. They can lift as much as 50 times their own weight. It turns out that scaling has something to do with this. To explore this, suppose some atomic tests in New Mexico caused some common ants to mutate into giant ones about the same size as humans, which for his problem we'll assume means 500× larger in all dimensions. If the lifting strength of the ant goes as the cross sectional area of its muscle, can such a giant ant still lift something as much as 50 times its weight? If not, how much can it lift in terms of its own weight?

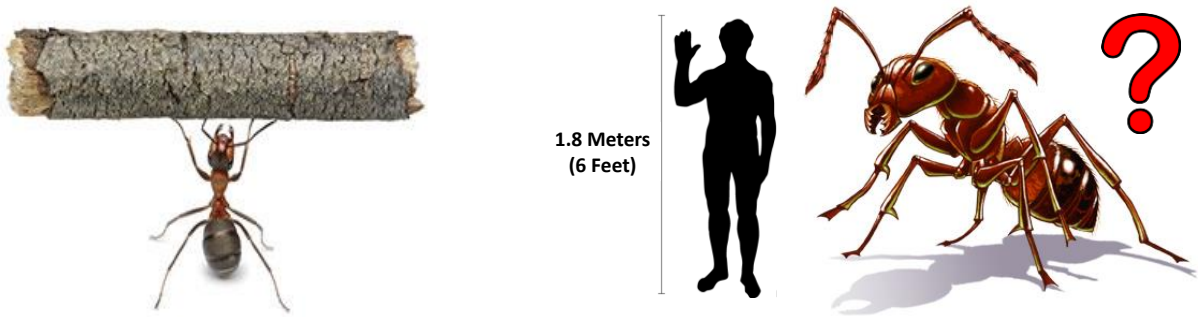


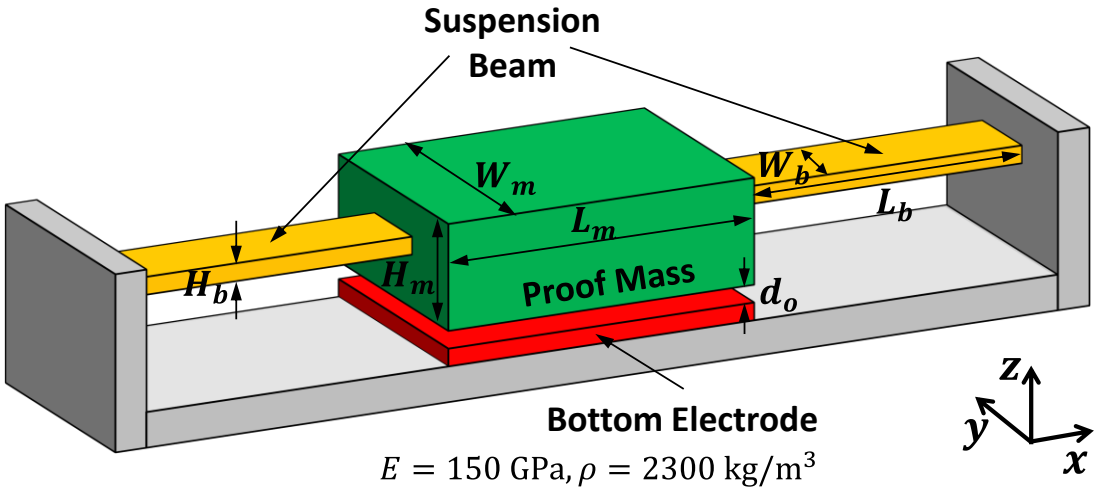
Figure PS1-1

2. Scaling to microscopic dimensions often provides benefits in some performance characteristics, but can also degrade others. To investigate, this problem explores the behavior of a simple accelerometer.

(a) Figure PS1-2 shows the schematic of a macro scale accelerometer and lists its material properties and critical dimensions. Assuming $L_b >> H_b$, and assuming each suspension beam is rigidly attached to the chassis, the *z*-direction stiffness at the "proof mass end" of suspension beams can be approximated by the expression

$$k_z = 2EW_b\left(\frac{H_b}{L_b}\right)^3$$

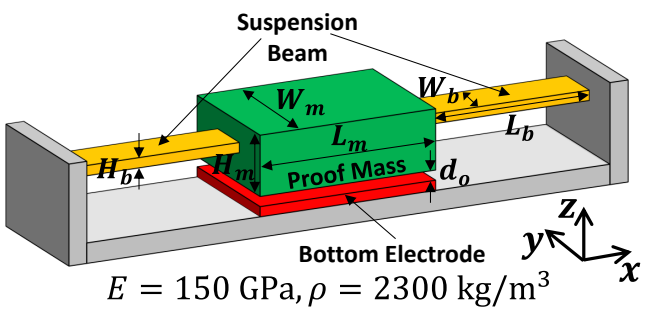
where E and ρ are the Young's modulus and density of the structural material, respectively, and dimensions are given in Figure PS1-2. What is the total *z*-direction stiffness at the proof mass location? What is the z-direction displacement of the proof mass due to gravity? Will the proof mass contact the bottom electrode due to gravity? (The mass of the suspension beams can be neglected in this question.)



$E = 150$ GPa, $\rho = 2300$ kg/m^3
$L_m = 300$ mm, $H_m = 50$ mm, $W_m = 200$ mm
$L_b = 260$ mm, $H_b = 4$ mm, $W_b = 50$ mm, $d_o = 1$ mm

Figure PS1-2

(b) Assume now we use MEMS technology to scale down the size of the accelerometer by 1000× to achieve dimensions as shown in Figure PS1-3. What is the z-direction displacement of the proof mass due to gravity now? Will the proof mass contact the bottom electrode?



$E = 150$ GPa, $\rho = 2300$ kg/m^3
$L_m = 300$ µm, $H_m = 50$ µm, $W_m = 200$ µm
$L_b = 260$ µm, $H_b = 4$ µm, $W_b = 50$ µm, $d_o = 1$ µm

Figure PS1-3

(c) The resonance frequency for the structure is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

where k is the stiffness in the direction of resonance, and m is the mass of the system, which for simplicity, should be assumed for this problem to be the mass of the proof mass, i.e., neglect the mass of the suspension beams. What is the resonance frequency of the structure before scaling? What is it after scaling by (1/1000)×?

(d) Provide an expression and a numerical value for the overlap capacitance C_o between the proof mass and the bottom electrode. By what factor does it scale if all dimensions are scaled by (1/1000)×?

In the next part of this problem, we mainly explore the behavior of the micro-scale accelerometer shown in Figure PS1-3.

(e) Write an expression for the z-directed displacement Δz that ensues after the chassis experiences a constant z-directed acceleration of magnitude a. What is Δz for a 10g acceleration? for the micro scale accelerometer in Figure PS1-3? By what factor does it scale if all dimensions in Figure PS1-3 are scaled by (1/2)×?

(f) Write an expression for the change in bottom electrode-to-proof mass capacitance ΔC that ensues after the chassis experiences a constant z-directed acceleration of magnitude a. What is ΔC for a 10g acceleration? By what factor does ΔC change if all dimensions (including the bottom electrode-to-proof mass gap spacing) are scaled by (1/2)×?

(g) Many MEMS-based accelerometers sense ΔC as a measure of the magnitude of acceleration experienced. Figure PS1-4 presents one possible circuit that converts a change in capacitance to a readable output voltage. Here, the fixed capacitor C_o has the same value as the bottom electrode-to-proof mass capacitance of the accelerometer at rest. (The accelerometers electrode-to-proof mass capacitance is represented by C(z) in the figure.) For this circuit, write an expression for the output voltage V_{out} as a function of acceleration a, assuming the op amp is ideal. How does the output voltage change as all dimensions are scaled by (1/2)×? (The circuit is biased with V_P = 10V)

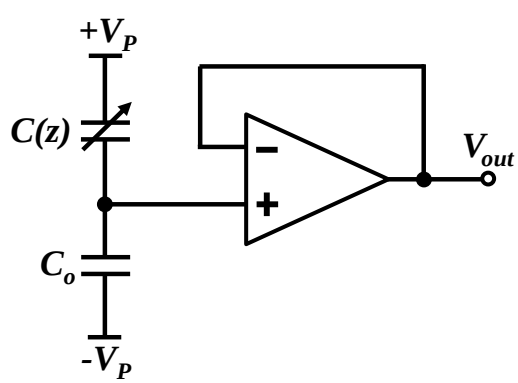


Figure PS1-4

(h) The resolution of a sensor is the smallest input it can detect. Assuming the smallest change in capacitance that the electronic circuit in Fig. PS1-4 can detect is 1fF, what is the resolution of the overall accelerometer (in volts)? Can the micro-scale accelerometer measure the force of gravity, i.e., 1g?

(i) Assume that a 150 mm (6″) diameter wafer has a useful area of 100 mm × 100 mm over which accelerometers can be fabricated. (Here, the edges of the wafer are for handling, so do not yield working devices.) A dicing saw is used to cut the wafer into individual dies and the width of each cut is 50 μm. The cost per sensor is given by $C(n,d) = (\$3000 + \$1 \times n + \$2 \times d)/d$, where n is the number of cuts through the wafer and d is the num-

ber of dies. Here, the fixed \$2 cost per sensor is due to post processing, packaging, and testing costs. Assuming that the minimum die size that can be reliably handled is 1 mm × 1 mm, what is the lowest achievable fabrication cost per sensor (to the nearest cent)? [Hint: it would be helpful to define *d(n)* and to find *n*.]

3. The general equation for transverse free vibrations of a prismatic beam, such as shown in Figure PS1-5, can be expressed as

$$EI\frac{\partial^4 v}{\partial x^4}dx = -\rho A dx\frac{\partial^2 v}{\partial t^2} \tag{2}$$

where E and ρ are the Young's modulus and density of the structural material, respectively; I is the moment of inertia; A is the cross-sectional area of the beam; v is the y-directed displacement variable; x is indicated in the figure, and t is time. You might not understand all of these variables right now, but you will deeper into the course. For now, just treat this problem as a math problem, designed to jog your memory on how to solve differential equations.

Find a general form of the solution to (2) that gives the mode shape of the beam, as indicated in Figure PS1-6. (The mode shape is the shape of the beam at maximum amplitude during resonance.) This should be in terms of x, the resonance radian frequency ω, and some constants governed by boundary conditions, i.e., it should be expressed as $V = f(x,\omega)$, where V is a function describing the mode shape. You need not determine the values of the constants, but you should show them as variables (the same way you've done before in math courses).

[Hint: Assume a solution $v = V\,(C\cos\omega t + D\sin\omega t)$, then find V.]

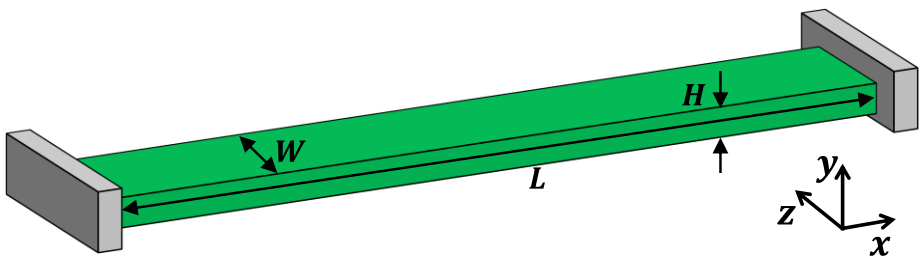


Figure PS1-5

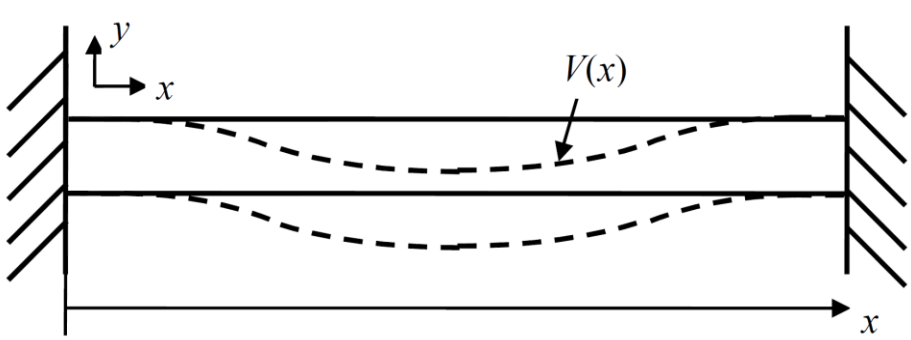
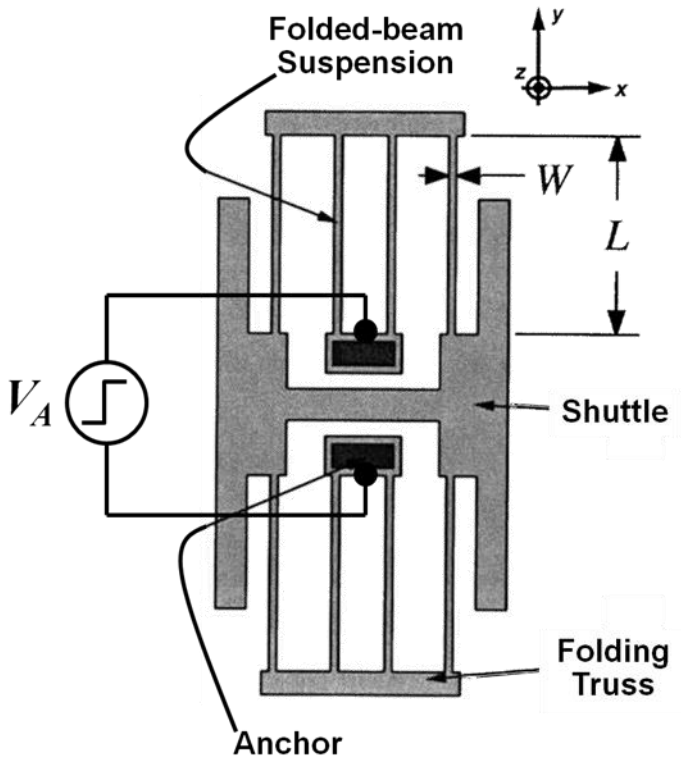


Figure PS1-6

4. Suppose a step-function voltage V_A were suddenly applied across the anchors of a 2μm-thick polysilicon folded-beam suspended structure as shown in the (top view) figure below. In this structure, the dark shaded regions denote anchors to the substrate (which is thermal ground); and the lightly shaded areas are suspended 2 μm above the substrate. Assume that the structure is in vacuum.



Structural Material Properties:
Young's Modulus, E = 150 GPa
Density, ρ = 2,300 kg/m^3
Poisson ratio, ν = 0.226
Sheet Resistance = 10 Ω/□
Specific Heat, c_p = 0.77 J/(g K)
Thermal Cond., k = 30 W/(m K)
Geometric Dimensions:
L = 50μm; W = 2μm
Thickness, h = 2μm
Folding Truss Area = 5 × 50 μm^2
Shuttle Area = 8,000 μm^2
Ignore the thermal and electrical resistance of the shuttle.

Use the data in the box above to answer the following questions.

(a) With what time constant will the shuttle reach its steady-state temperature after the voltage V_A steps from 0V to 1V? Give a formula and a numerical answer with units.

(b) If the final step function value of V_A is 1V, what is the steady-state temperature on the shuttle? Give a formula and a numerical answer with units.