PROBLEM SET #4

Issued: Wednesday, Mar. 5, 2014

Due (at 9 a.m.): Tuesday Mar. 18, 2014, in the EE C247B HW box near 125 Cory.

1. Suppose you would like to fabricate the suspended cross beam structure below using the process flow outlined in Problem Set #3. The structure is constructed entirely of doped polysilicon, i.e., the yellow and green layers are both doped polysilicon. Dimensions for most of the features are indicated in Figs. PS4.1-1 to PS4.1-3, as are points of interest to be explored in subsequent parts of this problem. The structure itself (in green) is meant to be 2 μm thick, and the interconnect layers beneath (in yellow) are meant to be in a thin doped polysilicon layer.

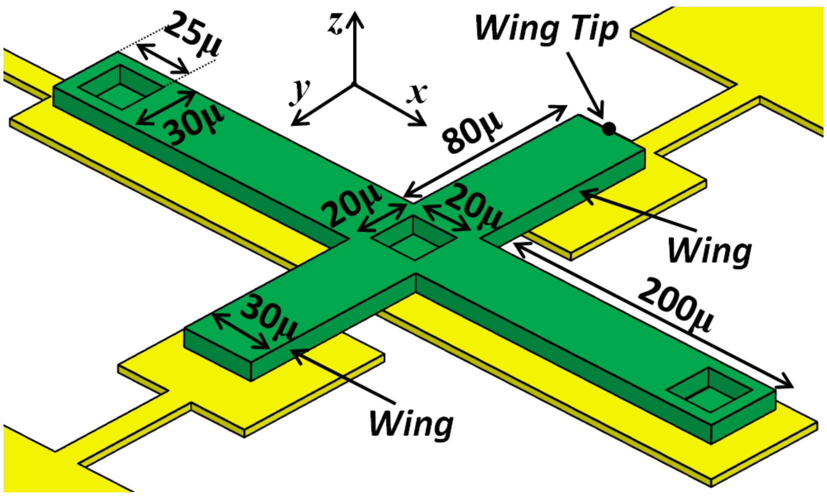


Fig. PS4.1-1 Zoom in of the polysilicon beam structure

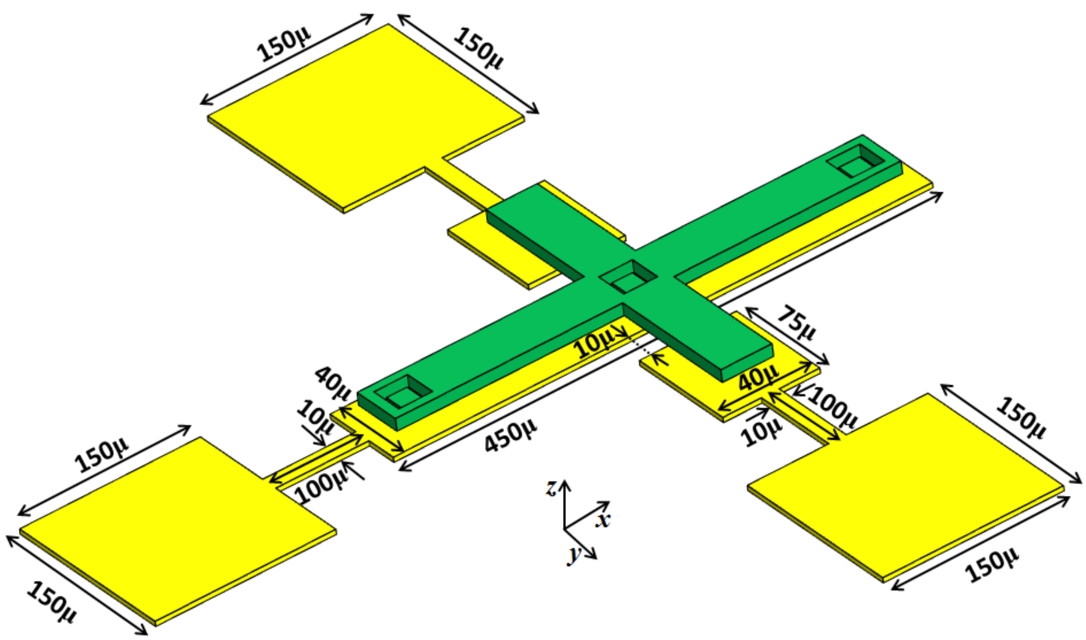


Fig. PS4.1-2 Wide-view schematic with electrode contact dimensions

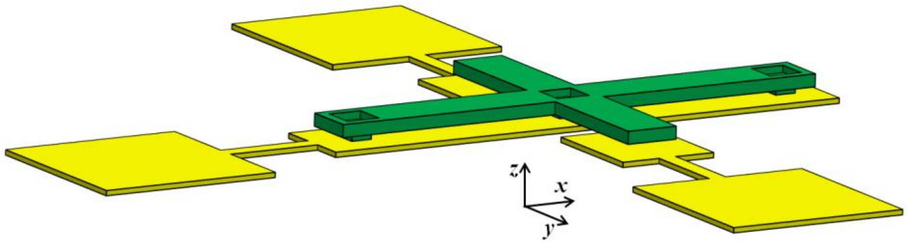


Fig. PS4.1-3 Side view showing anchor points

(a) Using the material properties given in the table below, determine if this structure will buckle after fabrication by the given process flow? Show your work and clearly state the reasons behind your conclusion.

Material Properties	Si Substrate	PolySilicon
Young's Modulus [GPa]	160	150
Thermal Expansion Coeff. [10^{-6}/°C]	2.6	2.2
Poisson Ratio	0.17	0.23

(b) Write an expression for the static spring constant in z-direction at the tip of the "wing" structure and calculate its numerical value.

(c) Suppose the deposited polysilicon film has a vertical stress gradient as shown in the curve of Fig PS4.1-4, where σ_y is proportional to $z^{1/3}$ with $\sigma_1 = 10^7$ Pa at location $z = H/2$. How high above the substrate are the bottoms of the tips of the "wing" structures after fabrication by the given process flow?

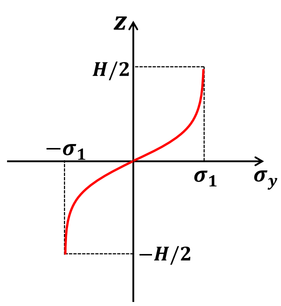


Fig. PS4.1-4 Stress gradient in PolySi

(d) One effective method to measure the stress gradient in a thin film entails measuring the radius of curvature. In some popular measurement systems, this is often done by directing a laser beam onto the surface of the film and measuring the angle θ between the wafer surface and the laser beam directed straight down. Suppose you can also use such system to measure the stress gradient in the "wing" structure, as illustrated in Fig. PS4.1-5. Write an expression of the angle θ as a function of location y and Find the angle θ_{max} at the tip of the "wing" structure as a function of the maximum stress σ_1 in the film.

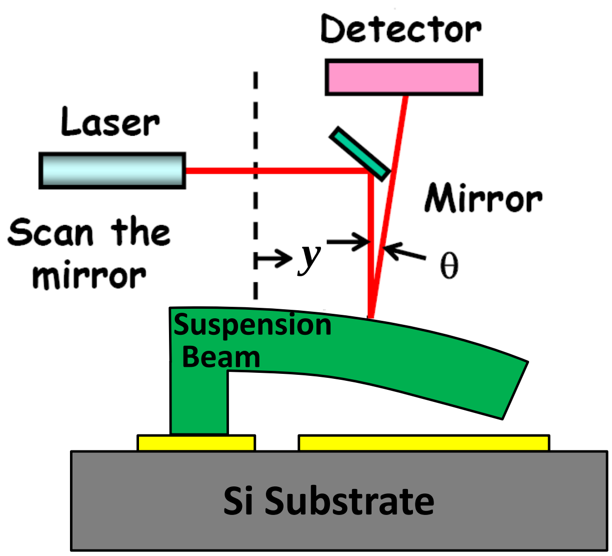


Fig. PS4.1-5

2. Diagnostic structures are essential to any MEMS layout. This fact is perhaps most evident when a process fails and requires debugging. If inplane stress is the problem, one useful diagnostic structure is the vernier stress gauge as shown in Fig. PS4.2-1, which mechanically amplifies the strain caused by residual stress in a film and is capable of measuring both tensile and compressive stress. Here, the residual stress generates a small displacement δ_t in the "test beam". This displacement in turn generates an angular deflection along the "slope beam", which then generates a much larger displacement δ_i at the vernier that can be visually read via optical microscope. By using this structure, the strain in the "test beam" is effectively amplified to a larger displacement at the vernier site.

Suppose the vernier gauge is constructed from a 2 μm thick polysilicon layer.

(a) If the inplane residual stress in the film is isotropic with a magnitude σ_f, what is the change in the test beam length (δ_t)? Derive an expression for the bending profile of the slope beam as a function of δ_t and then as a function of residual stress σ_f. (Assume the *y*-direction loading on the slope beam generated from the residual stress in the test beam will not change the slope beam *x*-direction stiffness)

(b) Write an expression for displacement at the vernier (δ_i) as a function of residual stress σ_f in the film. What value of L_c maximizes the vernier displacement (δ_i) for a given residual stress σ_f?

(c) Suppose you find the 2nd finger from the top on the indicator beam aligns exactly with the 2nd finger from the top on the fixed vernier. Indicate whether the residual stress in the film is tensile or compressive and find its numeric value. (The vernier gauge dimensions are given in Table PS4.2.)

(d) What is the maximum compressive residual stress σ_{max} that the vernier stress gauge can measure?

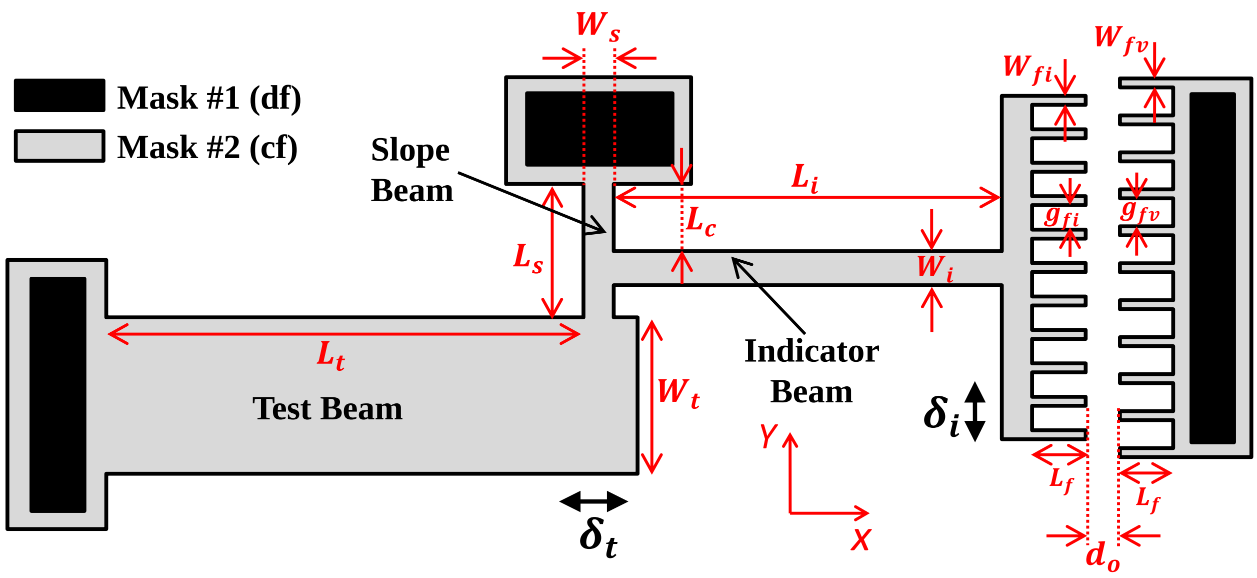


Fig. PS4.2-1

Table PS4.2

ρ	E	ν	L_t	W_t	L_s	W_s	L_i	W_i	L_c
2300 kg/m^3	150 GPa	0.2	400 μm	200 μm	20 μm	1 μm	400 μm	2 μm	10 μm

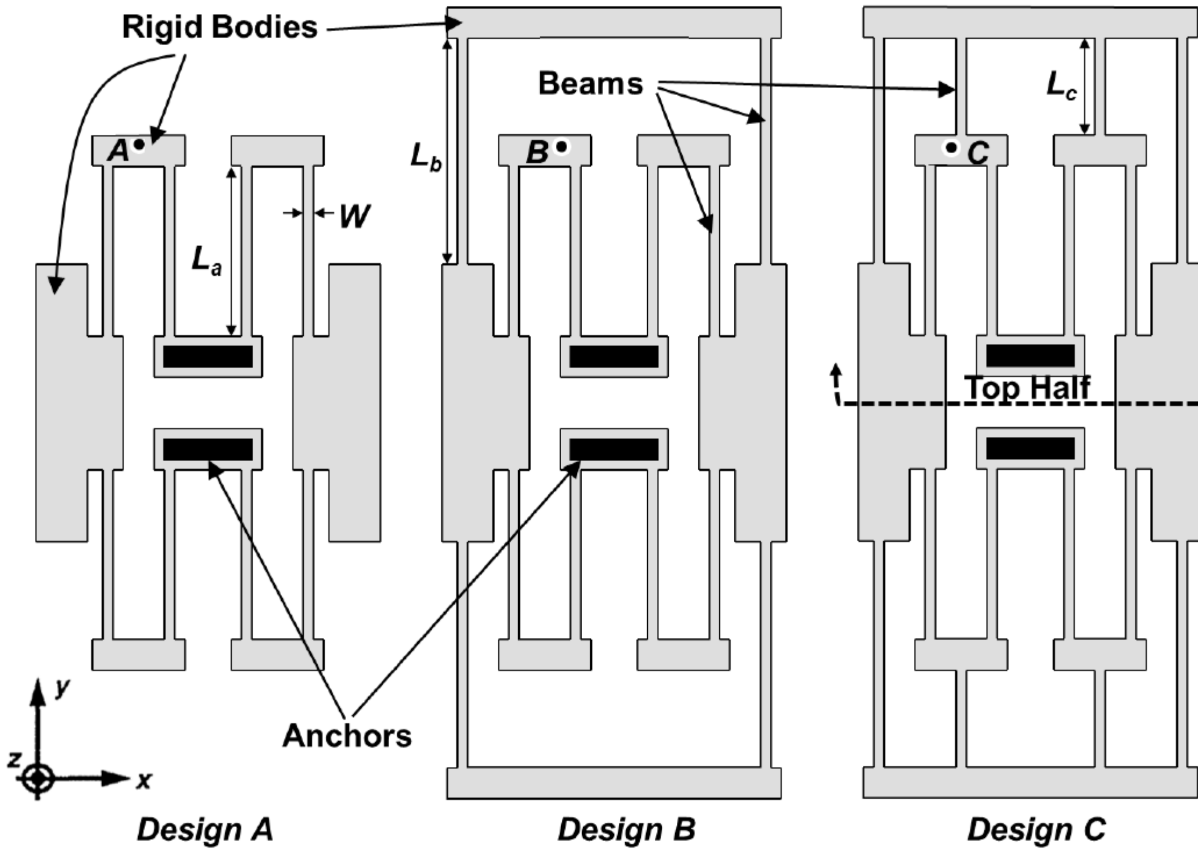
L_f	d_o	W_{fi}	W_{fv}	g_{fi}	g_{fv}
5 μm	2 μm	1 μm	1 μm	2 μm	2.1 μm

3. Fig. PS4.3 below presents top views of a 2μm-thick micromechanical structure with increasing suspension complexity from *Design A* to *Design C*. For each structure, everything is suspended 2 μm above the substrate except for the anchoring locations indicated as the darkly shaded regions. Key dimensions for the beams and data on the structural material used in this problem are given in the box below the figure. Also, assume that all folding trusses and shuttles are rigid in all directions, including the vertical (i.e., *z*) direction. As indicated in the box, assume that all suspension and coupling beam widths are 2 μm.

(a) Write an expression for the static spring constant in the *x*-direction at location *A* in Design *A* and calculate its numerical value (with units).

(b) Write an expression for the static spring constant in the *x*-direction at location *B* in Design *B* and calculate its numerical value (with units).

(c) Write an expression for the static spring constant in the *x*-direction at location *C* in the top half of Design *C* (i.e., assume the bottom half has been cut away, anchors and all) and calculate its numerical value (with units).



Beam Dimensions: L_a = 100 μm, L_b = 150 μm, L_c = 50 μm, W = 2 μm Structural Material Properties: Young's Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3; Poisson ratio, ν = 0.226

Fig. PS4.3

4. Thermoelastic damping (TED) can occur in any material subject to periodic stress. It is pronounced in flexural mode resonators when heat moves from compressed parts to tensioned parts during vibration. For example, as shown in Fig. PS4.4-1, when a clamped-clamped beam (CC-Beam) resonator undergoes bending, the tensile and compressive parts in the structure will generate temperature gradient. Thermal conductivity in the material will allow these hot and cold regions to equilibrate, which causes heat flux and energy loss, thereby limits Q of the resonator. In most cases, for low frequency CC-Beam resonators, TED is the main loss mechanism and dominates the quality factor. The magnitude of the TED in a clamped-clamped beam is given by the equations in Module 7 of Lecture 12.

Suppose you want to design a 100 MHz clamped-clamped beam resonator with very high quality factor. You have the freedom to choose either quartz or single crystalline silicon as the structure material, the properties of which are shown in Table PS4.4-2. Below are the design guidelines and limitations you should follow:

- The CC-beam resonator length (L) and width (W) should be no smaller than 1 μm due to lithography resolution limit.
- The beam thickness h should be less than 3 μm.
- In order to have minimum mode shape distortion, the length/thickness ratio L/h should be no smaller than 5, and the beam width W should be exactly 5× smaller compared with the beam length L.

Come up with a design of 100 MHz clamped-clamped beam resonator that attains the highest Q if TED is the dominant loss. Clearly state the material you pick, the beam thickness h you choose, the key dimensions of the beam (L, W) and the theoretical quality factor Q it can reach. (Assume the resonator is operating in vacuum at room temperature 300 K.)

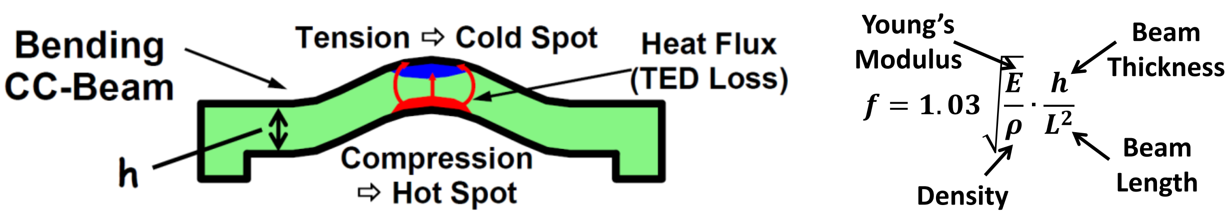


Fig. PS4.4-1

Table PS4.4-2

Property	Silicon	Quartz	Units
Thermal expansion coefficient	2.60	13.7	ppm/K
Young's modulus	170	78	GPa
Material density	2330	2600	kg/m^3
Heat capacity	0.7	0.75	J/(g·K)
Thermal conductivity	150	10	W/(m·K)