Lecture 12: Mechanics of Materials I

- **Announcements:**
- Module 7 on Mechanics of Materials online
- HW#3 due next Wednesday morning
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- Reading: Senturia Chpt. 3, Jaeger Chpt. 11, Handouts: "Bulk Micromachining of Silicon"
- Lecture Topics:
  - Bulk Micromachining
  - Anisotropic Etching of Silicon
  - Boron-Doped Etch Stop
  - Electrochemical Etch Stop
  - Isotropic Etching of Silicon
  - Deep Reactive Ion Etching (DRIE)
  - Wafer Bonding
- --------------------------------
- Finish up bulk micromachining Module 6
- Start through material of Module 7: Mechanics of Materials, but lectures themselves will be mostly handwritten
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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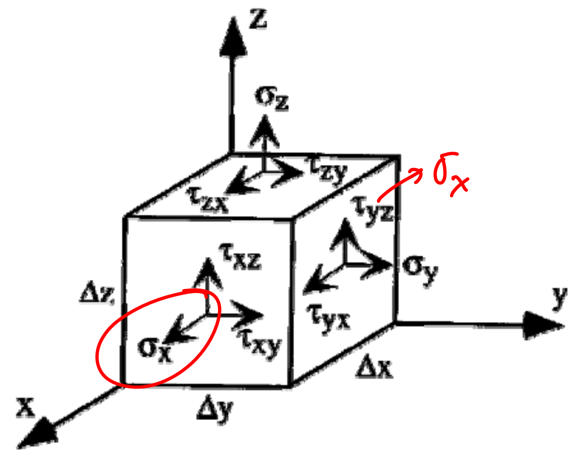
- **Last Time:** Going thru Module 6 … finish this
- Move on to Module 7

Example | Exercise the "terms"

⇒ Determine the volume change for a uniaxial stress along the x-direction



Upon application of σ_x, what is ΔV?

$$\Delta x \rightarrow \Delta x(1+\epsilon_x)$$
$$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$$
$$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$$

assuming isotropic material
↓
same ν along y & z

The resulting change in volume: ΔV

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z(1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{new volume}} - \underbrace{\Delta x \Delta y \Delta z}_{\text{original volume}}$$

$$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] $\Rightarrow$ $(1+mx)^n \simeq 1 + nmx$

$$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$$

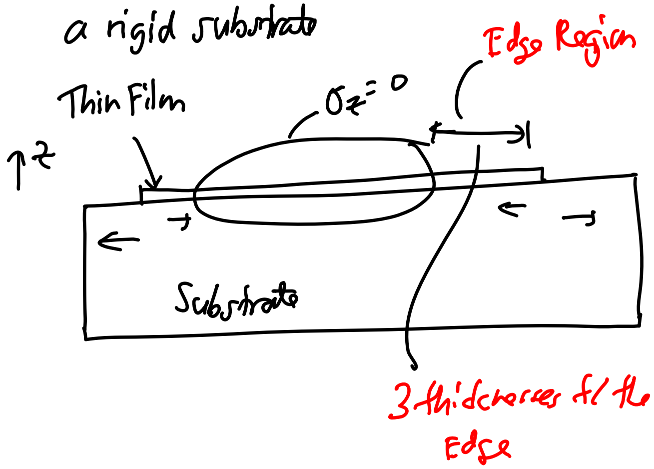
$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber): $\rightarrow$ no ΔV!

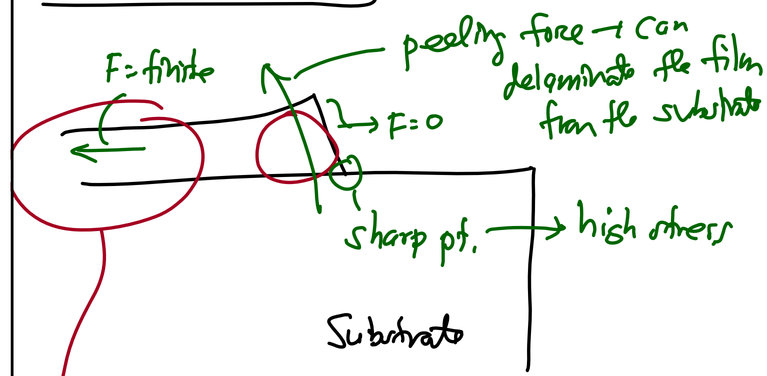
$\nu < 0.5 \rightarrow$ finite ΔV

Important Case: Plane Stress

$\Rightarrow$ common case for a thin-film coating on a rigid substrate



Zoom-in on Edge Region



Take a closer look at this region: $\sigma_z = 0$

Get two components of stress (strain)

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

Assume plane stress: $\rightarrow$ isotropic $\rightarrow$ $\sigma_x = \sigma_y = \sigma$

(symmetry in the xy-plane) $\downarrow$

$\epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right] = \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \Rightarrow \epsilon_x = \frac{\sigma}{E'}$$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$