Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element

Given an x-directed uniaxial stress, σ_x:

$$\Delta x \rightarrow \Delta x(1+\epsilon_x)$$
$$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$$
$$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$$

The resulting change in volume ΔV

$$\Delta V = \Delta x \Delta y \Delta z(1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x \Delta y \Delta z = \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

{Assume small strains} $\Rightarrow \Delta V = \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$[(1+mx)^n \approx 1+nmx] \Rightarrow \approx \Delta x \Delta y \Delta z\left[1+\epsilon_x - 2\nu\epsilon_x - 2\nu\epsilon_x^2 - 1\right]$ (negl.)

$$\Delta V \approx \Delta x \Delta y \Delta z(1-2\nu)\epsilon_x$$

For $\nu = 0.5$ (rubber) $\rightarrow$ no ΔV!

$\nu < 0.5 \rightarrow$ finite ΔV

Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)

Thin film | Plane stress region | Edge region | Substrate

- At regions more than 3 thicknesses from edges, the top surface is stress-free $\rightarrow \sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$
$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane $\rightarrow \sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

Copyright © 2014 Regents of the University of California

Edge Region of a Tensile (σ>0) Film

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here

Shear stresses

$F \neq 0$

$F = 0$

Extra peel force

Discontinuity of stress at the attached corner → stress concentration

Peel forces that can peel the film off the surface

Linear Thermal Expansion

- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

Linear thermal expansion coefficient $\triangleq \alpha_T = \dfrac{d\varepsilon_x}{dT}$ [Kelvin^{-1}]

Remarks:

- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of μstrain/K, where 10^{-6} K^{-1} = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient → $\dfrac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

α_T As a Function of Temperature

x10^{-6}
Thermal expansion
T (K)
Ni-Co-Fe alloy
W
Pyrex
Si
SiO$_2$

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

Thin-Film Thermal Stress

Thin Film (α_{Tf})

Silicon Substrate (α_{Ts} = 2.8 x 10^{-6} K^{-1})

Substrate much thicker than thin film

- Assume film is deposited stress-free at a temperature T_r, then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$\varepsilon_s = -\alpha_{Ts}\Delta T$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate: $\varepsilon_{f,free} = -\alpha_{Tf}\Delta T$ ↻ over

Copyright © 2014 Regents of the University of California

Linear Thermal Expansion

But the film *is* attached to the substrate, so the actual strain in the film is the same as that in the substrate.

$$\epsilon_{f,attached} = -\alpha_{Ts} \Delta T$$

Thus:

$$\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T$$

↳ Note that this is biaxial strain
↳ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\,K^{-1}$, $E' = 4GPa$
deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$
↳ $\sigma_{f,mismatch} = (4)(1.5\times10^{-2}) = 60.5MPa$ ← stress is (+), ∴ tensile [(-) would be compressive] (e.g., SiO_2)

MEMS Material Properties

Material Properties for MEMS

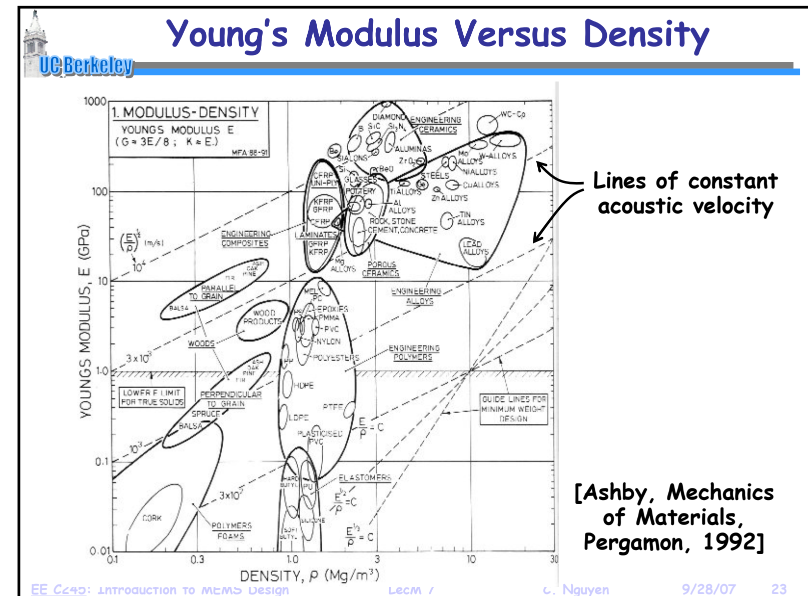
Material	Density, ρ, Kg/m³	Modulus, E, GPa	E/ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: (m/s)²

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

Young's Modulus Versus Density



[Ashby, Mechanics of Materials, Pergamon, 1992]

Copyright © 2014 Regents of the University of California

Yield Strength

- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move

Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials

Tensile Strength

Stress

Fracture

Proportional Limit

(Si @ T=30°C)

Ductile (Mild Steel)

Brittle (Si)

(or Si @ T>900°C)

Strain

[Maluf]

Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f, MPa	$\frac{\sigma_f}{E}$ (-) x 10^{-3}	$\frac{\sigma_f^2}{E}$ MJ/m³
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

Young's Modulus Versus Strength

4. MODULUS-STRENGTH

YOUNGS MODULUS, E (GPa)

STRENGTH σ_f (MPa)

Lines of constant maximum strain

[Ashby, Mechanics of Materials, Pergamon, 1992]

Copyright © 2014 Regents of the University of California

Quality Factor (or Q)

Clamped-Clamped Beam μResonator

Resonator Beam, W_r, L_r, h, Electrode, v_i, i_o, V_P, $C(t)$

$Q \sim 10{,}000$

Frequency:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

Stiffness, Young's Modulus, Density, Mass

Mass ⇨ (e.g., $m_r = 10^{-13}$ kg) ⇨ Smaller mass ⇨ higher freq. range and lower series R_x

$$i_o = V_P \frac{dC}{dt}$$

Note: If V_P = 0V ⇨ device off

Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

- Example: series LCR circuit

$$\Rightarrow \quad Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit

$$\Rightarrow \quad Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

|H(jω)|, BW_{-3dB}, 3dB, f_o, f

Selective Low-Loss Filters: Need Q

Resonator Tank — Coupler — Resonator Tank — Coupler — Resonator Tank: General BPF Implementation

Typical LC implementation:

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000

Tank Q: 30,000; 20,000; 10,000; 5,000; 4,000

Increasing Insertion Loss

Transmission [dB] vs. Frequency [MHz] (998–1002)

Copyright © 2014 Regents of the University of California

Oscillator: Need for High Q

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability

Sustaining Amplifier
Ideal Sinusoid: $v_o(t) = V_o\sin(2\pi f_o t)$
$\omega_o = 2\pi/T_o$
Frequency-Selective Tank
Higher Q
Real Sinusoid: $v_o(t) = (V_o + \varepsilon(t))\sin(2\pi f_o t + \theta(t))$
Tighter Spectrum
Zero-Crossing Point

Attaining High Q

- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode

Quartz, Electrodes, Thickness-Shear Mode, C_o, L_x, C_x, R_x, $Q > 10{,}000$

Energy Dissipation and Resonator Q

$$\frac{1}{Q} = \frac{1}{Q_{defects}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{viscous}} + \frac{1}{Q_{support}}$$

Material Defect Losses; Gas Damping; Thermoelastic Damping (TED); Anchor Losses

At high frequency, this is our big problem!

Bending CC-Beam; Tension ⇨ Cold Spot; Heat Flux (TED Loss); Compression ⇨ Hot Spot; Elastic Wave Radiation (Anchor Loss)

Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED} f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

Bending CC-Beam; Tension ⇨ Cold Spot; Heat Flux (TED Loss); Compression ⇨ Hot Spot; h

- ζ = thermoelastic damping factor
- α = thermal expansion coefficient
- T = beam temperature
- E = elastic modulus
- ρ = material density
- C_p = heat capacity at const. pressure
- K = thermal conductivity
- f = beam frequency
- h = beam thickness
- f_{TED} = characteristic TED frequency

Copyright © 2014 Regents of the University of California

TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm²
Material density	2.33	2.60	g/cm³
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

[from Roszhart, Hilton Head 1990]

Q vs. Temperature

Quartz Crystal

Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle

Aluminum Vibrating Resonator

[from Braginsky, Systems With Small Dissipation]

Even aluminum achieves exceptional Q's at cryogenic temperatures

Polysilicon Wine-Glass Disk Resonator

Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V

1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- *Below*: 20 μm diameter disk

Design/Performance:
R=10μm, t=2.2μm, d=800Å, V_P=7V
f_o=1.51 GHz (2nd mode), Q=11,555

[Wang, Butler, Nguyen MEMS'04]

Copyright © 2014 Regents of the University of California

Disk Resonator Loss Mechanisms

(Not Dominant in Vacuum)
Gas Damping
Strain Energy Flow
Nodal Axis
Electronic Carrier Drift Loss
(Dwarfed By Substrate Loss)
No motion along the nodal axis, but motion along the finite width of the stem
Disk
Stem
Stem Height
λ/4 helps reduce loss, but not perfect
Hysteretic Motion of Defect
(Dwarfed By Substrate Loss)
Substrate
Substrate Loss Thru Anchors
(Dominates)

MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:

$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness [m]
R = substrate radius of curvature [m]

θ
Slope = 1/R
x
Detector
Laser
Mirror
Scan the mirror
x
θ
t
h
Si-substrate
R

MEMS Stress Test Structure

- Simple Approach: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where

$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

E = Young's modulus [Pa]
I = (1/12)Wh³ = moment of inertia
L, W, h indicated in the figure

  - Limitation: Only compressive stress is measurable

W
L
h = thickness
Poly 2
100 μm

Copyright © 2014 Regents of the University of California

More Effective Stress Diagnostic

- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress

Expansion → Compression
Contraction → Tensile

Q Measurement Using Resonators

Resonator Data
$R = 32\ \mu m$, $h = 3\ \mu m$
$d = 80$ nm, $V_p = 3$ V

Folded-Beam Comb-Driven Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator

Comb-Drive Resonator in Action

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

Copyright © 2014 Regents of the University of California

Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator

Measurement of Young's Modulus

- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Young's modulus

Equivalent mass

h = thickness

- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Copyright © 2014 Regents of the University of California