Lecture 13: Mechanics of Materials II

- Announcements:
- Module 7 on Mechanics of Materials online
- HW#3 due Wednesday morning
- HW#4 online soon
- No office hours this coming Wednesday
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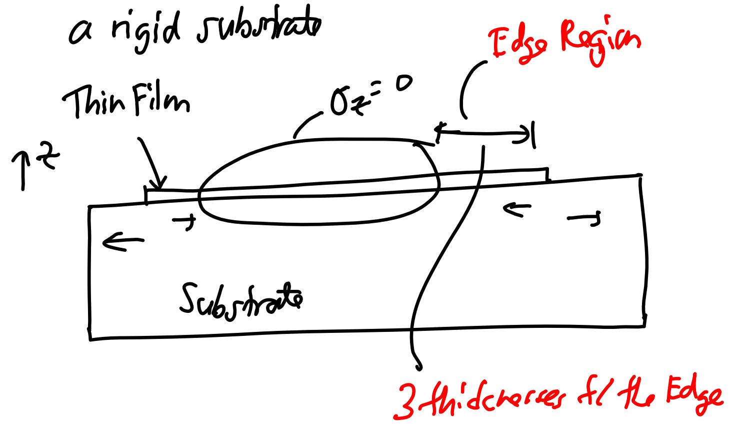
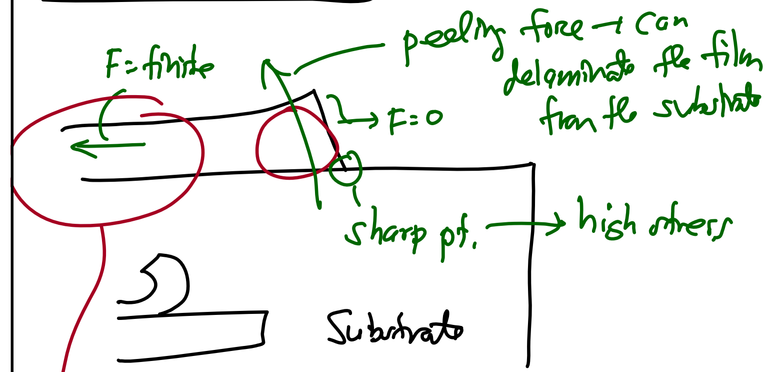
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- ---------------------------------
- Last Time: Going thru Module 7 … continue this

$\boxed{\text{Important Case: Plane Stress}}$

⇒ common case for a thin-film coating on a rigid substrate



Zoom-in on Edge Region



Take a closer look at this region: $\sigma_z = 0$

Get two components of stress (strain)

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

Assume plane stress: → isotropic → $\sigma_x = \sigma_y = \sigma$

(symmetry in the xy-plane) $\quad \epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right] = \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \Rightarrow \epsilon_x = \frac{\sigma}{E'}$$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. linear thermal expansion coefficient

$$\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Exp. Coefficient}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{kelvin}^{-1}]$$

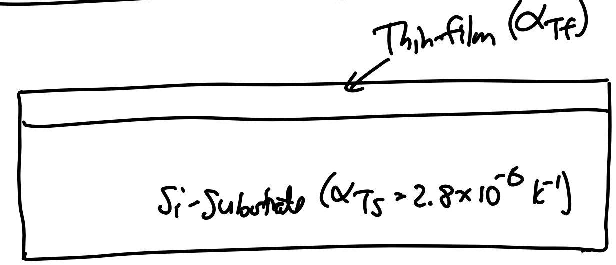
Remarks.

1. α_T values tend to be in the 10^{-6} to 10^{-7} range
2. $10^{-6} K^{-1}$ = 1 μstrain/K
3. In 3D, get a volume thermal exp. coefficient.

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

4. For moderate ΔT's → $\alpha_T \approx$ constant
   → for larger ΔT, then $\alpha_T = f(T)$

Ex. Thin-Film Thermal Stress



Assume.

1. Substrate is much thicker than the film.
2. Film is deposited stress free @ T_d ← deposition temperature
3. Then the whole thing is cooled to room temperature, T_r.

The thermal strain of the substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate.

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But the film is attached to the substrate

⇒ thickness sub >> thickness film

∴ substrate wins!

* → thus, the actual strain experienced by the film is that of the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

→ Note that this is a biaxial strain. (Assumes the film is deposited isotropically onto the substrate.)

$$\sigma_{f,mismatch} = \underbrace{\left(\frac{E}{1-\nu}\right)}_{E'} \epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\ K^{-1}$

$E' = 4GPa$

$$\underbrace{\text{deposited @ } 250°C\text{, then cool to RT} = 25°C}_{\Delta T = 225K}$$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

$[\mu = 10^{-6},\ m = 10^{-3},\ k = 10^{3},\ G = 10^{9}]$

$$\sigma_{f,mismatch} = (\underset{10^9}{4G})(1.5\times10^{-2}) = 60.5MPa$$

Stress is (+) → tensile

SiO_2 → [(-) would be compressive]