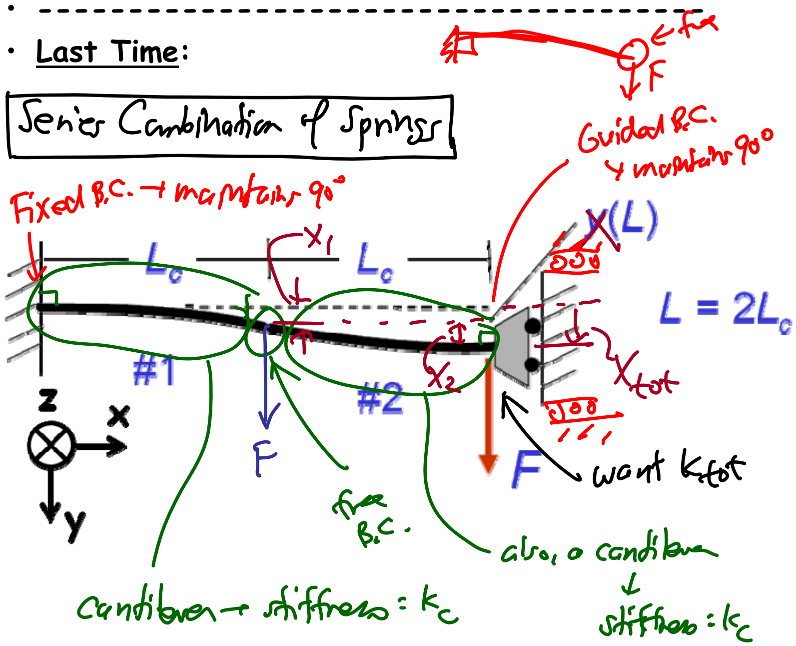
Lecture 16: Beam Combos I

- **Announcements**:
- HW#4 online and due Tuesday, March 18
- Midterm is nearing: Thursday, March 20
  - I passed out materials associated with the midterm (info sheet and old exams)
- ------------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ------------------------------------
- Last Time:

Series Combination of Springs



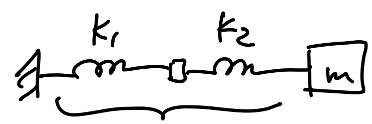
Series: $x_{tot} > x_1$, $x_{tot} > x_2$ } $x_{tot} = x_1 + x_2$

⇒ in series, F is the same along the whole beam structure

$$x_{tot} = x(L) = \frac{F}{k_{tot}} = x_1 + x_2 = 2\left(\frac{F}{k_c}\right) = F\left(\frac{1}{k_c} + \frac{1}{k_c}\right)$$

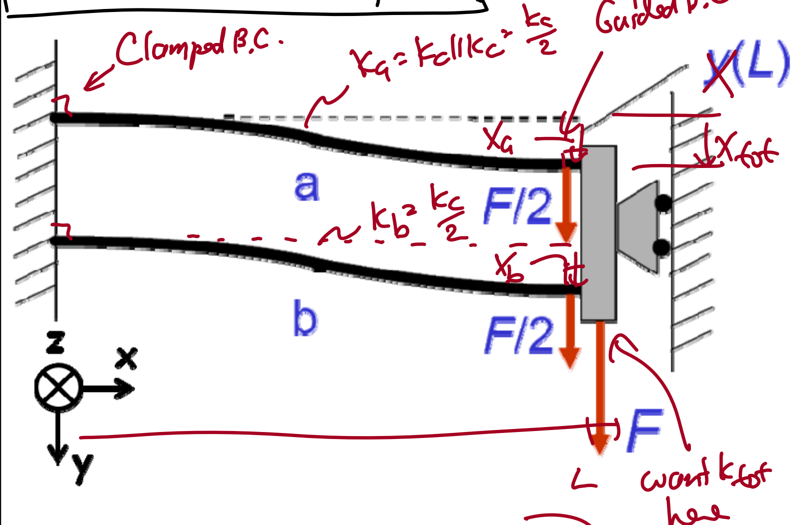
$$\boxed{\frac{1}{k_{tot}} = \frac{1}{k_c} + \frac{1}{k_c}} \rightarrow \boxed{k = k_c \| k_c}$$

Definition for "||" $\left\{ A\|B = \frac{1}{\frac{1}{A} + \frac{1}{B}} = \frac{AB}{A+B} \right.$



k_1 & k_2 in series → $k_{tot} = k_1 \| k_2$

Parallel Combination of Springs



Parallel: $x_{tot} = x_a = x_b$

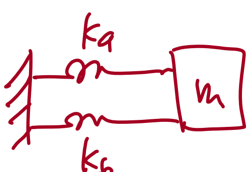
$$x_{tot} = x(L) = \frac{F}{k_{tot}} = \frac{F_a}{k_a} = \frac{F_b}{k_b}$$

$F_a = \frac{F}{2}$, $F_b = \frac{F}{2}$ — since beams are identical

$$= \left(\frac{F}{2}\right)\left(\frac{1}{k_a}\right)$$

$$\boxed{k_{tot} = 2k_a = 2k_b}$$

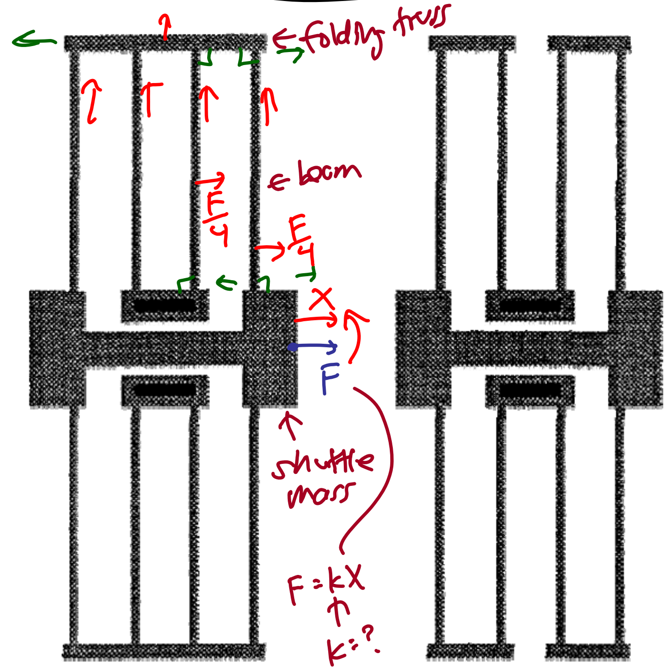
In general:



$k_{tot} = k_a + k_b$

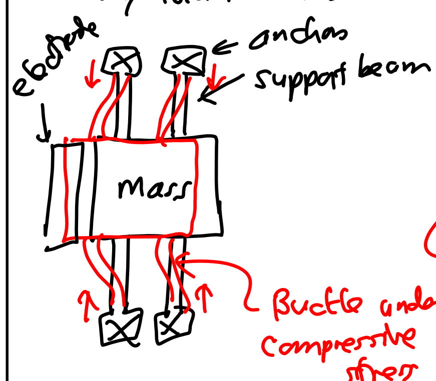
Like Capacitors in EE.

Folded-Beam Suspension Stiffness



(a) Inner fold, continuous truss (b) Inner fold, discontinuous truss

⇒ Why fold the beams?



Problem: (fabrication)

① High T deposition
↳ stress free @ high T

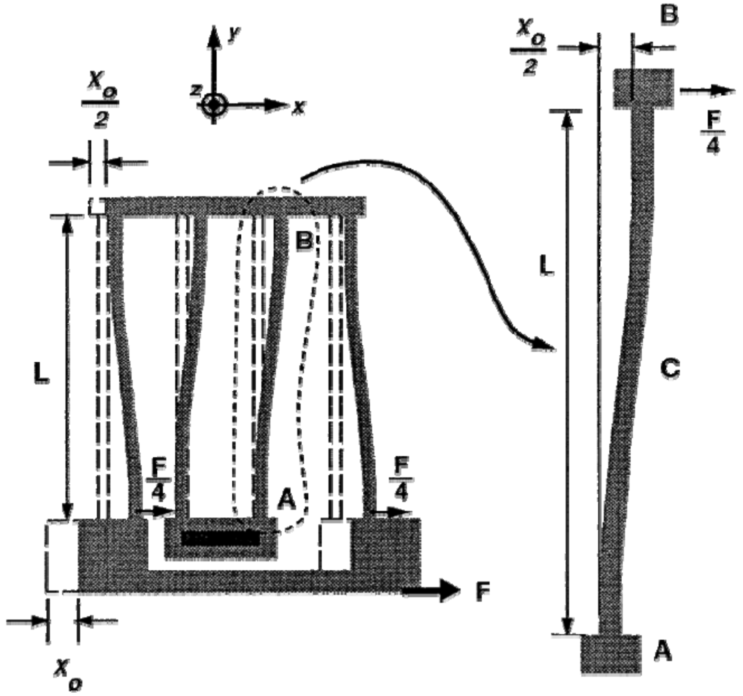
② Cool to RT → stress
Bad news if compressive!

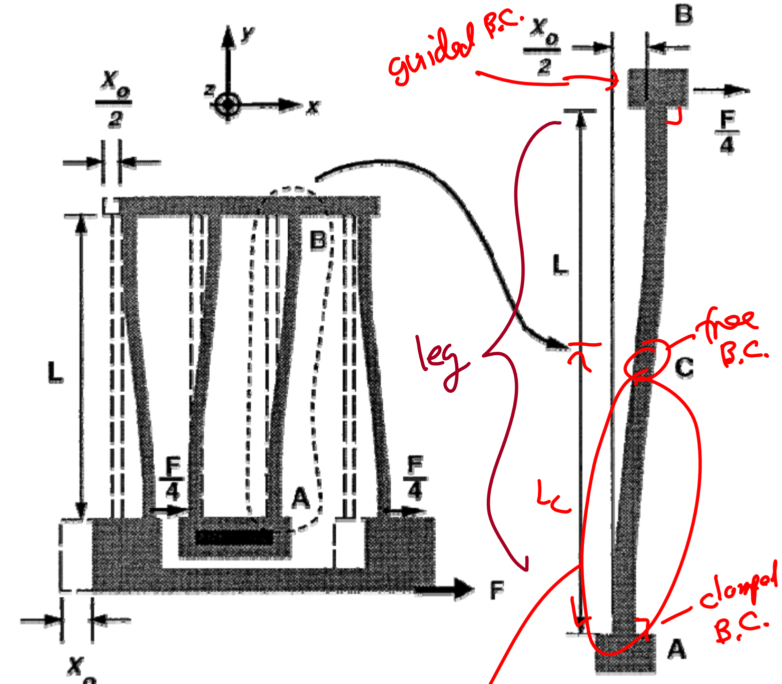
How to defend against this?

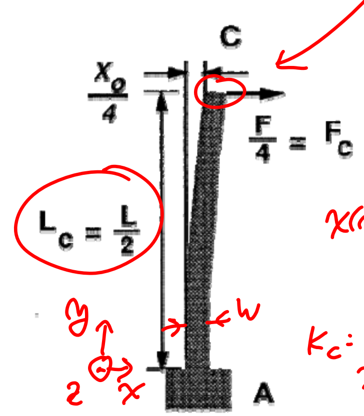
① Δ process parameters → stress free deposition
↳ can't always do this

↓

② Design → folded-beam!







$$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c}\right)$$

$$x(y) = \frac{F_c y^2}{6EI_z}(3L_c - y)$$

thickness

$$K_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3} = \frac{1}{4} E h \frac{W^3}{L_c^3}$$

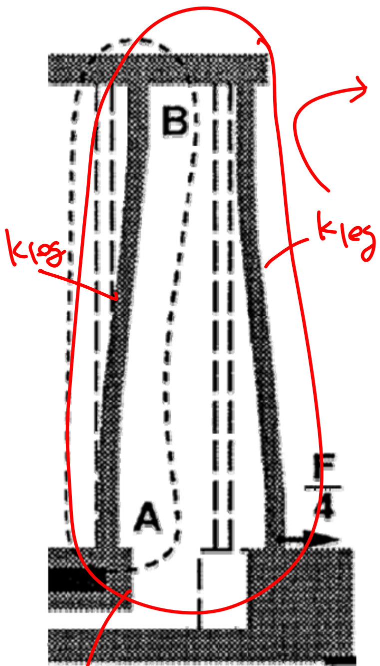
$I_z = \frac{1}{12} h W^3$

cantilever length

⇒ $k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}$ ← full beam length

Stiffness of a cantilever of length $L_c = \frac{L}{2}$

From before: $k_{leg} = k_c \| k_c = \frac{k_c}{2}$



$k_{pair} = k_{leg} \| k_{leg}$

legs in series, because the total displacement X_{tot} equals the sum of the displacements of the individual legs

$F_{pair} = \frac{F}{4}$ ← applied to the shuttle

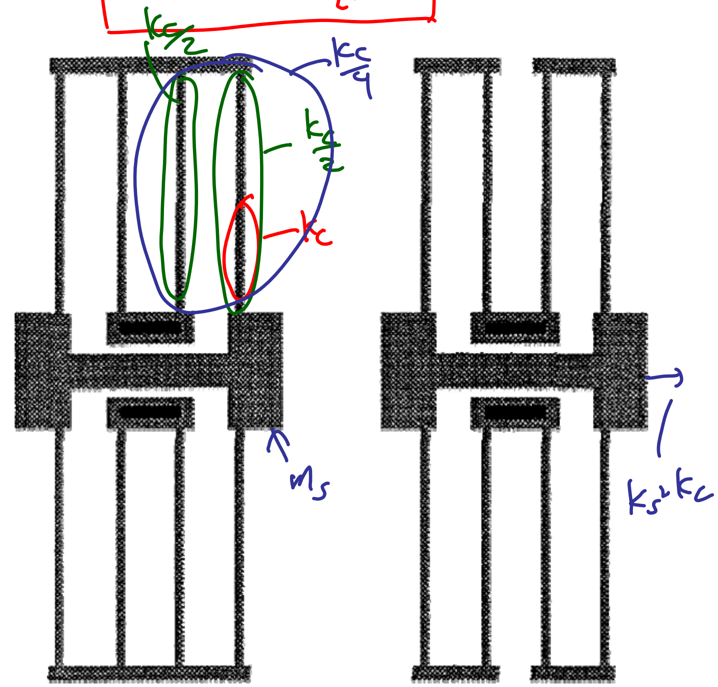
$x = \frac{F_{pair}}{k_{pair}} = \frac{F_{pair}}{k_{leg} \| k_{leg}}$

$= \frac{F}{4}\left(\frac{1}{k_{leg}} + \frac{1}{k_{leg}}\right)$

Thus,

$x = \left(\frac{F}{4}\right)\left(\frac{2}{k_c} + \frac{2}{k_c}\right) = \frac{F}{k_c} = \frac{F}{k_{tot}}$

$\boxed{k_{tot} = k_c = \frac{24EI_z}{L^3}}$



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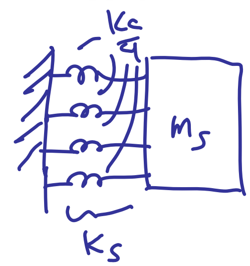
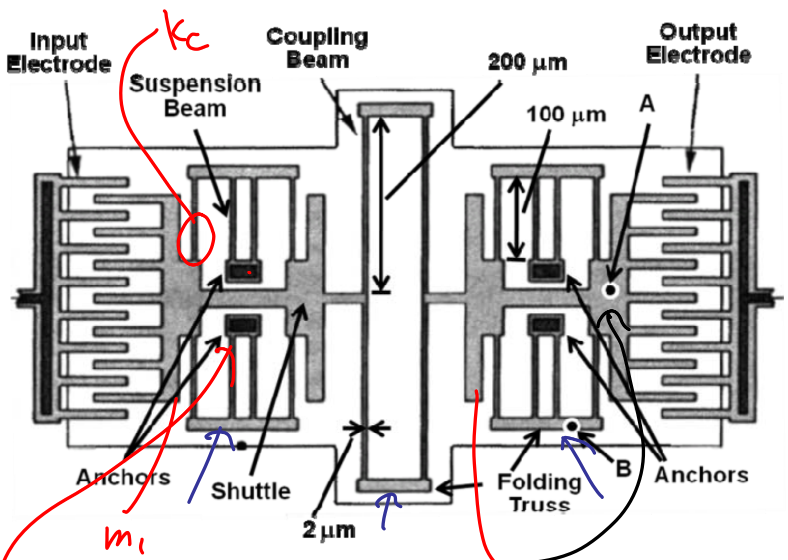
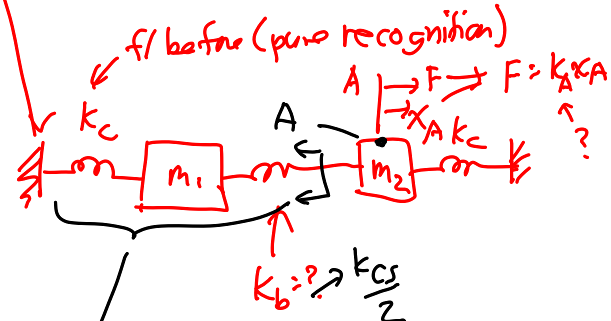


$\boxed{k_s = 4\left(\frac{k_c}{4}\right) = k_c}$

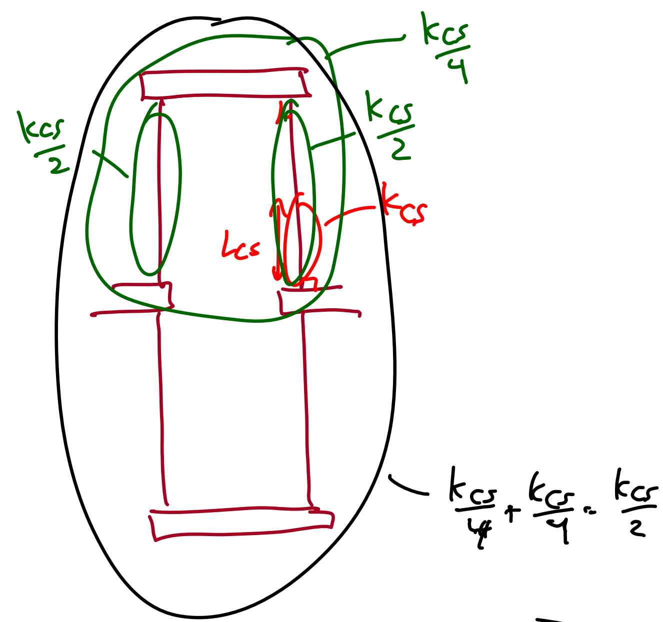
Micromechanical Filter Example



⇒ Find the stiffness @ pt. A
(shuttles & trusses are rigid)



$k_c \| \frac{k_{cs}}{2}$



$\therefore \ k_A = k_c + k_c \| \frac{k_{cs}}{2}$ where $k_c = \frac{24EI_z}{L_b^3}$

$k_{cs} = \frac{24EI_z}{L_{cs}^3}$