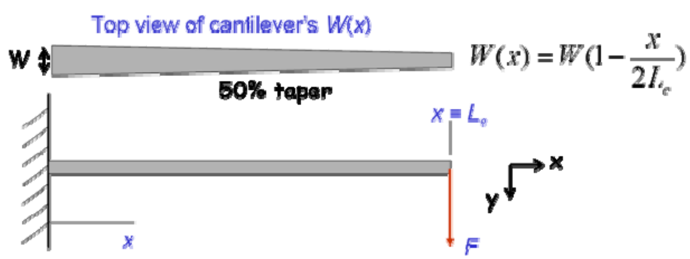
Lecture 18: Energy Methods

- Announcements:
- Pass back graded midterm today
  - Also, scroll Z scores at end of lecture
- TA, Lingqi Wu, stuck in China
  - If for long, we'll have a substitute
- HW#5 will be online soon
- Module 9 on Energy Methods is online
- ------------------------------------
- Reading: Senturia, Chpt. 10
- Lecture Topics:
  - Energy Methods
  - Virtual Work
  - Energy Formulations
  - Tapered Beam Example
  - Estimating Resonance Frequency
- ----------------------------------
- Last Time: Finished beam combos

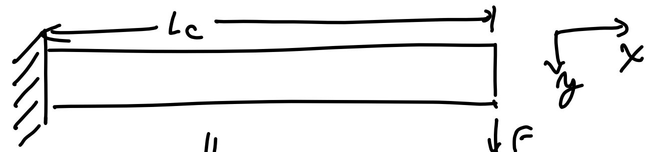
$\boxed{\text{More General Geometries}}$

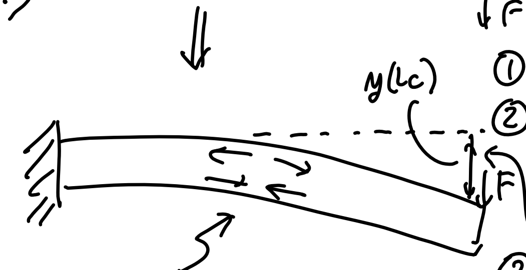
- **Euler-Bernoulli beam theory works well for simple geometries**
- **But how can we handle more complicated ones?**
- **Example: tapered cantilever beam**
- **Objective: Find an expression for displacement as a function of location x under a point load F applied at the tip of the free end of a cantilever with tapered width W(x)**



$$W(x) = W\left(1 - \frac{x}{2L_c}\right)$$

Same problem as before: Take a beam, apply a force.





① Apply force, F.

② Beam responds by bending.

③ This force has done work: $W = F \cdot y(L_c)$

④ Strain generated

→ so the beam has received an influx of stored energy

→ magnitude of " " determined by the shape the beam takes

⑤ U = Stored Energy − Work Done → 0

when we choose the right shape

→ transfer fcn $y(x) = f(x)$

This is how we get the beam's response to F!

Fundamentals of Energy Density

General Definition of Work:

$$\mathcal{W}(q_1) = \int_0^{q_1} e(q)\,dq \qquad q = \text{displacement}, \quad e = \text{effort}$$

$$\hookrightarrow \text{for EE: } \mathcal{W}(Q) = \int_0^{Q} \frac{Q}{C}\,dQ$$

Strain Energy Density

$$w = \int_0^{\epsilon_x} \sigma_x\, d\epsilon_x$$

value of strain @ position (x,y,z)

$\sigma_x(\epsilon_x) \rightarrow$ relates stress to strain @ position (x,y,z)

$$[\sigma_x = E\epsilon_x]$$

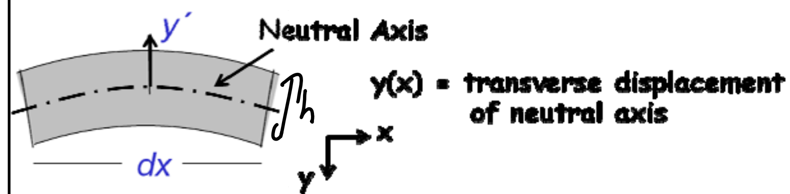
$$w = \int_0^{\epsilon_x} E\epsilon_x\, d\epsilon_x = \frac{1}{2}E\epsilon_x^2$$

Total Strain Energy, [J]

$$\mathcal{W} = \iiint \left(\frac{1}{2}E\left(\epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2\right) + \frac{1}{2}G\left(\gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2\right)\right) dV$$

volume

Bending Energy Density



First, find the bending energy $d\mathcal{W}_{bend}$ in an infinitesimal length dx:

$$d\mathcal{W}_{bend} = W dx \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{1}{2}E\epsilon_x^2(y')\,dy'$$

$$\left[\frac{1}{R} = \frac{d^2y}{dx^2}, \quad \epsilon_x = \frac{y'}{R}\right] \Rightarrow \epsilon_x(y') = y'\frac{d^2y}{dx^2}$$

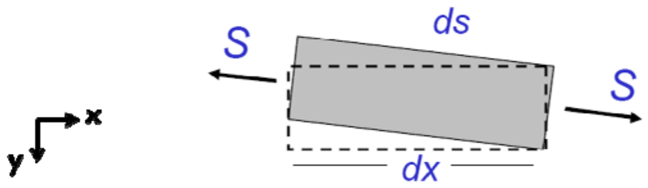
$$d\mathcal{W}_{bend} = W dx \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{1}{2}E\left[y'\frac{d^2y}{dx^2}\right]^2 dy'$$

$$= \frac{1}{2}E\underbrace{\left(\frac{Wh^3}{12}\right)}_{I_z}\left(\frac{d^2y}{dx^2}\right)^2 dx$$

fcn

$$\therefore \boxed{\mathcal{W}_{bend} = \frac{1}{2}EI_z\int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx}$$

Energy Due to Axial Load



⇒ energy related to lengthening:

$$ds = [(dx)^2 + (dy)^2]^{1/2} = dx\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}$$

binomial theorem $\approx dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right]$

$$\therefore\ \epsilon_x = \frac{ds - dx}{dx} = \frac{1}{2}\left(\frac{dy}{dx}\right)^2$$

$$dW_{axial} = S\epsilon_x dx = \frac{1}{2}S\left(\frac{dy}{dx}\right)^2 dx$$

$$\boxed{W_{axial} = \frac{1}{2}S\int_0^L \left(\frac{dy}{dx}\right)^2 dx}$$

⇒ Look @ shear strain energy in your module. (Module 9)

- Go through Module 9 pages 10-18.

- Pass back graded midterm

Midterm Statistics	
Top Score	105
Average	66
Median	68
Std. Dev.	17