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## Brute Force Methods for Resonance Frequency Determination

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## Basic Concept: Scaling Guitar Strings

**Guitar String**

Vib. Amplitude vs Freq. (110 Hz)

Low Q vs High Q

Vibrating "A" String (110 Hz)

**Freq. Equation:**

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_r}{m_r}}$$

Stiffness (k<sub>r</sub>) and Mass (m<sub>r</sub>)

**μMechanical Resonator**

Metallized Electrode, Anchor, Polysilicon Clamped-Clamped Beam, W<sub>r</sub>, L<sub>r</sub>, h<sub>r</sub>

[Bannon 1996]

**Performance:**  
L<sub>r</sub>=40.8μm  
m<sub>r</sub> ~ 10<sup>-13</sup> kg  
W<sub>r</sub>=8μm, h<sub>r</sub>=2μm  
d=1000Å, V<sub>p</sub>=5V  
Press.=70mTorr

f<sub>o</sub>=8.5MHz  
Q<sub>vac</sub>=8,000  
Q<sub>air</sub>~50

Transmission [dB] vs Frequency [MHz]

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## Anchor Losses

**Fixed-Fixed Beam Resonator**

Elastic Wave Radiation

Anchor, Electrode, Gap, Anchor

**Q = 300 at 70MHz**

**Problem:** direct anchoring to the substrate ⇒ anchor radiation into the substrate ⇒ lower Q

**Solution:** support at motionless nodal points ⇒ isolate resonator from anchors ⇒ less energy loss ⇒ higher Q

**Free-Free Beam Resonator**

Supporting Beams, Anchor, Free-Free Beam

**Q = 15,000 at 92MHz**

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## 92 MHz Free-Free Beam μResonator

- Free-free beam μmechanical resonator with non-intrusive supports ⇒ reduce anchor dissipation ⇒ higher Q

Drive Electrode, Support Beams, Flexural-Mode Beam, Anchor, Ground Plane and Sense Electrode

**Design/Performance:**  
L<sub>r</sub>=13.1μm, W<sub>r</sub>=6μm  
h=2μm, d=1000Å  
V<sub>p</sub>=28-76V, W<sub>e</sub>=2.8μm  
f<sub>o</sub>=92.25MHz  
Q=7,450 @ 10mTorr

[Wang, Yu, Nguyen 1998]

Transmission [dB] vs Frequency [MHz]

92.25 MHz  
Q = 7,450

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### Higher Order Modes for Higher Freq.

**2nd Mode Free-Free Beam**

**3rd Mode Free Free Beam**

Distinct Mode Shapes

Transmission [dB] vs Frequency [MHz]  
Phase [degree] vs Frequency [MHz]  
 $Q = 11,500$

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### Flexural-Mode Beam Wave Equation

Derive the wave equation for transverse vibration:  
Dynamic Equilibrium Condition for Forces in the y-direction:  
 $F - (F + \frac{\partial F}{\partial x} dx) - \rho A dx \frac{\partial^2 u}{\partial t^2} = 0$  (1) *neglect the ∂F/∂x dx term*  
and the moment equilibrium condition:  $-F dx + \frac{\partial M}{\partial x} dx \approx 0$  (2)  
Combining (1) & (2):  
 $\frac{\partial^2 M}{\partial x^2} dx = -\rho A dx \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^2}{\partial x^2} (-EI \frac{\partial^2 u}{\partial x^2}) = -\rho A \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^4 u}{\partial x^4} = \left(\frac{EI}{\rho A}\right) \frac{\partial^4 u}{\partial t^4}$   
 $\left[\frac{\partial^2 u}{\partial x^2} = -\frac{M}{EI} \rightarrow M = -EI \frac{\partial^2 u}{\partial x^2}\right]$   
 $I_y = \frac{Wh^3}{12}$

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### Example: Free-Free Beam

- Determine the resonance frequency of the beam
- Specify the lumped parameter mechanical equivalent circuit
- Transform to a lumped parameter electrical equivalent circuit
- Start with the flexural-mode beam equation:

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A}\right) \frac{\partial^4 u}{\partial x^4}$$

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### Free-Free Beam Frequency

- Substitute  $u = u_1 e^{i\omega t}$  into the wave equation:  
 $\frac{\partial^4 u}{\partial x^4} = \left(\omega^2 \frac{\rho A}{EI}\right) u$  (1)
- This is a 4th order differential equation with solution:  
 $u(x) = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx$  (2)  
*Give the mode shape during resonance vibration.*
- Boundary Conditions:

At $x = 0$	At $x = l$	
$\frac{\partial^2 u}{\partial x^2} = 0$	$\frac{\partial^2 u}{\partial x^2} = 0$	$M = 0$ (Bending moment)
$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial M}{\partial x} = 0$ (Shearing force)

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### Free-Free Beam Frequency (cont)

- Applying B.C.'s, get  $A=C$  and  $B=D$ , and
 
$$\begin{bmatrix} (\cosh kl - \cos kl) & (\sinh kl - \sin kl) \\ (\sinh kl + \sin kl) & (\cosh kl - \cos kl) \end{bmatrix} \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} = 0 \quad (3)$$
- Setting the determinant = 0 yields
 
$$\cos kl = \frac{1}{\cosh kl}$$
- Which has roots at
 
$$k_1 l = 4.730 \quad k_2 l = 7.853 \quad k_3 l = 10.996$$
- Substituting (2) into (1) finally yields:
 
$$k^n = \frac{\rho A}{EI} \omega^2 \rightarrow f_n = \frac{(k_n l)^2}{2\pi l^2} \sqrt{\frac{EI}{\rho A}} \quad \left[ \text{Free-Free Beam Frequency Equation} \right]$$

*These values of  $k_n l$  correspond to the different modes of vibration!*

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### Higher Order Free-Free Beam Modes

Mode	$n$	Nodal Points	$k_n l$	$f_n/f_1$
Fundamental ( $f_1$ )	1	2	4.730	1.000
1st Harmonic	2	3	7.853	2.757
2nd Harmonic	3	4	10.996	5.404
3rd Harmonic	4	5	14.137	8.932
4th Harmonic	5	6	17.279	13.344

← More than 10x increase

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### Mode Shape Expression

- The mode shape expression can be obtained by using the fact that  $A=C$  and  $B=D$  into (2), yielding
 
$$u_x = \mathcal{A} \left[ \left( \frac{\mathcal{A}}{\mathcal{B}} \right) (\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]$$
- Get the amplitude ratio by expanding (3) [the matrix] and solving, which yields
 
$$\frac{\mathcal{A}}{\mathcal{B}} = \frac{\sin kl - \sinh kl}{\cosh kl - \cos kl}$$
- Then just substitute the roots for each mode to get the expression for mode shape
 

Fundamental Mode ( $n=1$ )  
[Substitute  $k_1 l = 4.730$ ]

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