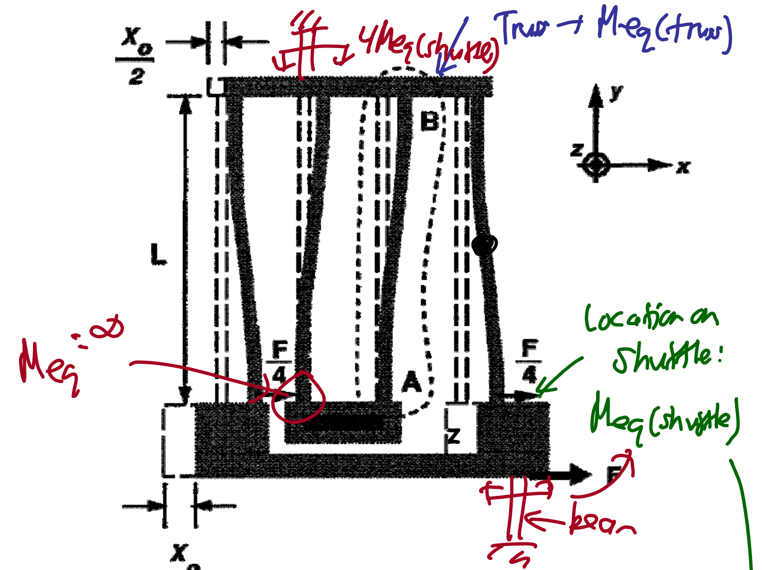
Lecture 21: Equivalent Ckts & Capacitive Transducers

- Announcements:
- First project slide due 4/11/14 (email it)
  - Subject & 3 key references
- Module 11 online
- HW#5 due next Wednesday
- ------------------------------------
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- ------------------------------------
- Last Time:
- Looking at electromechanical analogies
- Problem this lecture: Latest Microsoft updates seem to have removed the digitizing pen driver
  - Cannot use pen
  - So used combination of PowerPoint and Samsung Note to do lecture

Equivalent Dynamic Mass



Equivalent Mass:

$$\text{Equiv. Mass: } M_{eq,x} = \frac{KE_{max}}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A\int_0^l V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

for beam

velocity @ location x

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\cancel{\omega_o^2 x_o^2 \left(\frac{1}{2}\right)}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}\omega_o^2 x_o^2}}$$

$$M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

static masses: ρ(volume)

* → $$M_{eq(truss)} = \frac{\cancel{\omega_0^2 x_0^2 \left(\frac{1}{2}\right)}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}}\left(\frac{1}{4}\right)\cancel{\omega_0^2 x_0^2}}$$

$$M_{eq(truss)} = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

equiv. dynamic mass @ the truss location

Equiv. Dynamic Stiffness

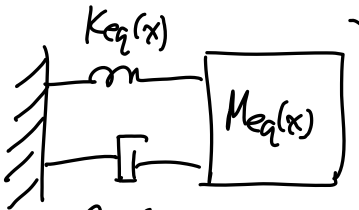
$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow \boxed{K_{eq}(x) = \omega_0^2 M_{eq}(x)}$$

⇒ large equiv. mass & large stiffness go hand-in-hand

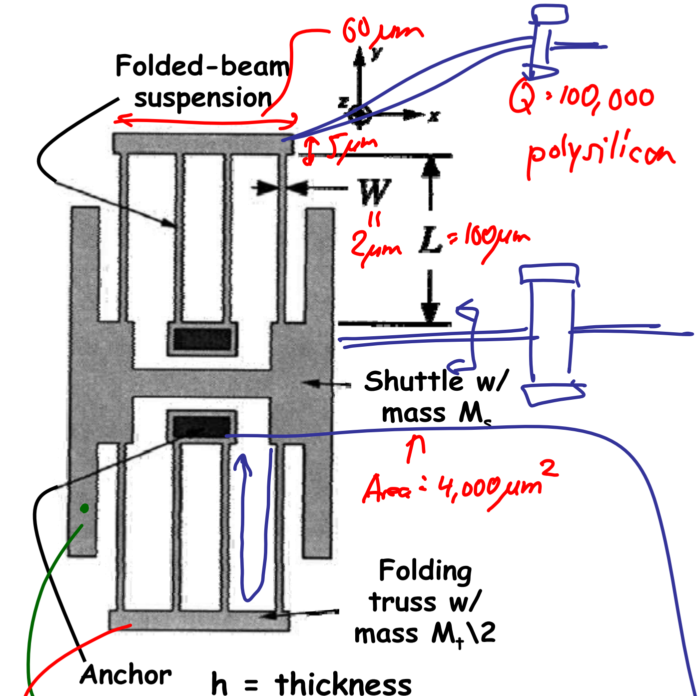
Equiv. Dynamic Damping

$$Q = \frac{\omega_0 M_{eq}(x) \sim L}{C_{eq}(x) \sim R} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

damping



specified @ single location, x



$K_{eq(shuttle)} = 4.8$ N/m

$M_{eq(shuttle)} = 2.16 \times 10^{-11}$ kg

$C_{eq(shuttle)} = 1.02 \times 10^{-10}$ kg/s

$K_{eq(truss)} = 19.2$ N/m

$M_{eq(truss)} = 8.64 \times 10^{-11}$ kg

$C_{eq(shuttle)} = 4.08 \times 10^{-10}$ kg/s

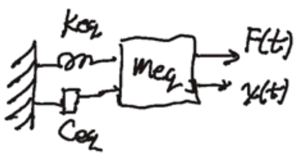
$K_{eq(anchor)} = \infty$

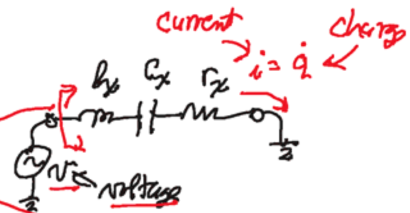
$M_{eq(anchor)} = \infty$

$C_{eq(anchor)} = \infty$

- Now, using Papyrus on Samsung Note 1
- Slow going … will need to make up some time in the next lecture → maybe go for 2 hours next lecture

Electromechanical Analogies





$F(t) = F\cos(\omega t) \to x(t) = X\cos(\omega t)$ (off resonance)

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

→ using phasor concepts

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

Impedance looking in

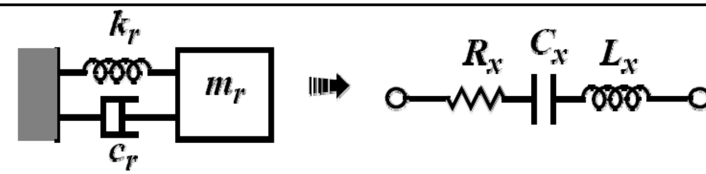
$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega C_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/C_x)}{j\omega} i + r_x i$$

⇒ by analogy

$$\boxed{F \to v \quad m_{eq} \to l_x \quad c_{eq} \to r_x \quad \dot{x} \to i \quad k_{eq} \to \frac{1}{C_x}}$$

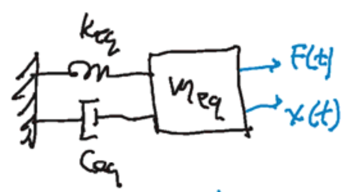
[Parameter Relationships in the Current Analogy]



- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Lowpass Biquad Transfer Function



$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

→ convert to full phasor form

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega} X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\omega\frac{c_{eq}}{k_{eq}}\right]^{-1}$$

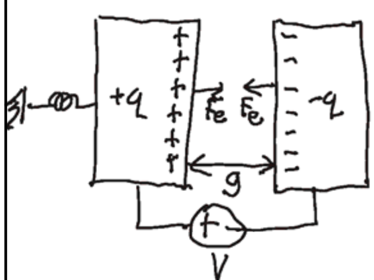
$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2,\quad Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \to \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]^{-1}$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}}$$

- Go through pages 11-22 of Module 11
- Then, start into Module 12

Energy Conserving Transducer



Goal: Determine gap spacing g as a function of input variables V, I → q

Assume the plates are supported elastically

How do we proceed?

① Determine the energy of the system.

② What can I Δ to Δ the energy in the system?

$$\Delta W(q,g) = V\Delta q + F_e \Delta g$$

$$dW = Vdq + F_e dg$$

Stored Energy