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## Electrostatic Comb Drive

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## Electrostatic Comb Drive

- Use of comb-capacitive transducers brings many benefits
  - ↳ Linearizes voltage-generated input forces
  - ↳ (Ideally) eliminates dependence of frequency on dc-bias
  - ↳ Allows a large range of motion

Comb-Driven Folded Beam Actuator

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## Typical Drive & Sense Configuration

2-port Lateral Micromechanism     $N_f = \# \text{ shuttle fingers}$

Simple Analysis:

$$F_{d1} = \frac{1}{2} \frac{\partial C_1}{\partial x} (V_1 - V_{p1})^2 = \frac{1}{2} \left( -\frac{\epsilon_0 h}{d_1} \right) (V_1^2 - 2V_{p1}V_1 + V_{p1}^2) (2N_f)$$

$$F_{d2} = \frac{1}{2} \frac{\partial C_2}{\partial x} (V_2 - V_{p2})^2 = \frac{1}{2} \left( \frac{\epsilon_0 h}{d_2} \right) (V_2^2 - 2V_{p2}V_2 + V_{p2}^2) (2N_f)$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left( \frac{\epsilon_0 h}{d} \right) (V_2^2 - V_1^2 - 2(V_{p2}V_2 - V_{p1}V_1) + V_{p2}^2 - V_{p1}^2) (2N_f)$$

For  $V_1 = V_2, V_1 = -V_2$   
 $F_{net} = 2(2N_f) \left( \frac{\epsilon_0 h}{d} \right) V_{p1} V_1$

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## Comb-Drive Force Equation (2<sup>nd</sup> Pass)

- In our 1<sup>st</sup> pass, we accounted for
  - ↳ Parallel-plate capacitance between stator and rotor
- ... but neglected:
  - ↳ Fringing fields
  - ↳ Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

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**UC Berkeley** **Comb-Drive Force With Ground Plane Correction**

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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**UC Berkeley** **Capacitance Expressions**

- Case:  $V_r = V_p = 0V$
- $C_{sp}$  depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,s}x + C'_{sp,r}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

Region 2 Region 3

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,s} - C'_{sp,r}) V_s^2$$

(for  $V_r = V_p = 0$ )

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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**UC Berkeley** **Simulate to Get Capacitors → Force**

- Below: 2D finite element simulation

$w = l = g = z_0 = 2 \mu m$

Capacitance [pF/m] Lateral force [pN/V<sup>2</sup>/finger]

Vertical displacement of rotor,  $\Delta z$  [μm]

Curves shown:  $C_{rs} + C_{sp,s}$ ,  $C_{sp}$ ,  $C_{rs}$ ,  $C_{sp,r}$ ,  $C_{sp,s}$ , Force

$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,s} - C'_{sp,r}) V_s^2$$

20-40% reduction of  $F_{e,x}$

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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### Vertical Force (Levitation)

$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2} \frac{dC_{sp}}{dz} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dz} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dz} (V_s - V_r)^2$$

• For  $V_r = 0V$  (as shown):  $F_{e,z} = \frac{1}{2} N_X \left[ \frac{d(C'_{sp,e} + C'_{rs})}{dz} \right] V_s^2$

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### Simulated Levitation Force

• Below: simulated vertical force  $F_z$  vs.  $z$  at different  $V_P$ 's [f/ Bill Tang Ph.D., UCB, 1990]  
 ↳ See that  $F_z$  is roughly proportional to  $-z$  for  $z$  less than  $z_0$  → it's like an electrical stiffness that adds to the mechanical stiffness

$$F_z \approx \gamma_z V_P^2 \frac{(z_0 - z)}{z_0} = k_e (z_0 - z)$$

↑  
Electrical Stiffness

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### Vertical Resonance Frequency

Vertical resonance frequency  $\omega_z = \sqrt{\frac{k_z + k_e}{k_z}}$  where  $k_e = \left(\frac{\gamma_z}{z_0}\right) V^2$

Vertical resonance frequency at  $V_P = 0V$  = Lateral resonance frequency

• Signs of electrical stiffnesses in MEMS:  
 Comb (x-axis) →  $k_e = 0$   
 Comb (z-axis) →  $k_e > 0$   
 Parallel Plate →  $k_e < 0$

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### Suppressing Levitation

• Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb  
 • **Penalty:** x-axis force is reduced

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### Force of Comb-Drive vs. Parallel-Plate

gap =  $g = 1 \mu\text{m}$ ,  
 thickness =  $t = 2 \mu\text{m}$   
 finger length =  $L = 100 \mu\text{m}$   
 overlap length  $x = 75 \mu\text{m}$

- Comb drive (x-direction)  
 $\hookrightarrow V_1 = V_2 = V_S = 1\text{V}$   

$$F_{e,x} = \frac{\epsilon_0 t}{g} V_S^2$$
- Differential Parallel-Plate (y-direction)  
 $\hookrightarrow V_1 = 0\text{V}, V_2 = 1\text{V}$   

$$F_{e,y} = \frac{1}{2} \frac{\epsilon_0 t x}{g^2} V_2^2$$

$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\epsilon_0 t x}{g^2} V_2^2}{\frac{2 \epsilon_0 t}{g} V_S^2} = \frac{1}{2} \frac{x}{g}$

Parallel-plate generates a much larger force; but at the cost of linearity

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