Lecture 24: Equivalent Circuits II

- **Announcements:**
- Module 13 now online
- HW#6 due this Friday at 9 a.m.
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- Equivalent Circuits II Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- ------------------------------------
- **Last Time:**
- Finished comb-drive
- Project Notes:

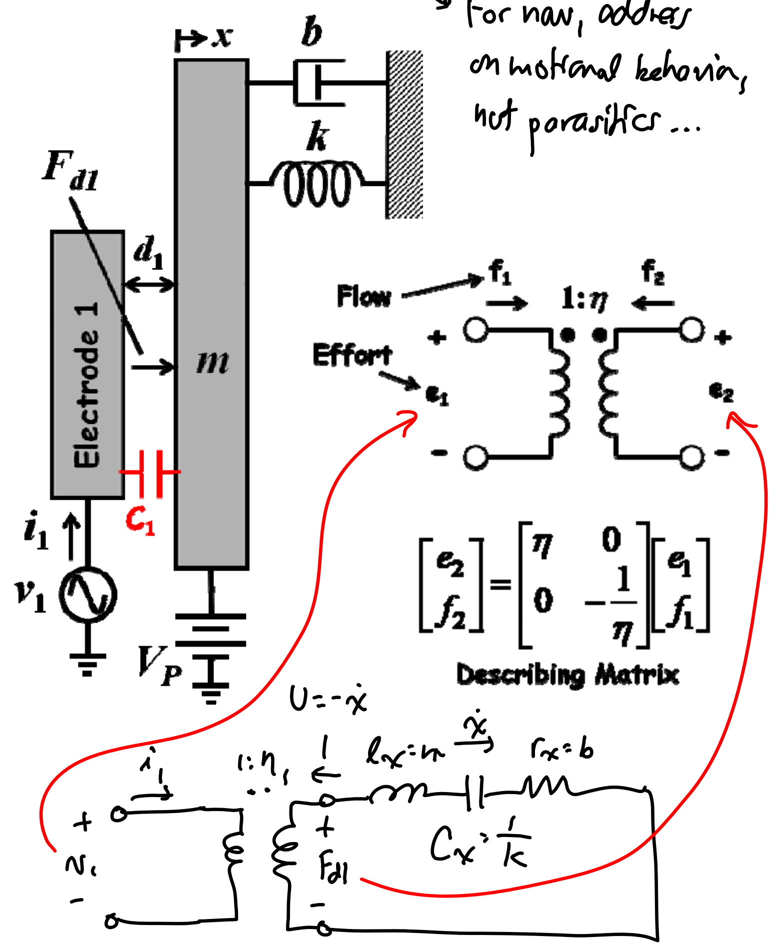
I want this table:

Parameter	Before Miniaturization	After Miniaturization
Power Cons.	100W	1mW

- Start Module 13 and go through slides 1-9

$\boxed{\text{Input Electrical Equiv. Ckt.}}$

For now, address only motional behavior, not parasitics...



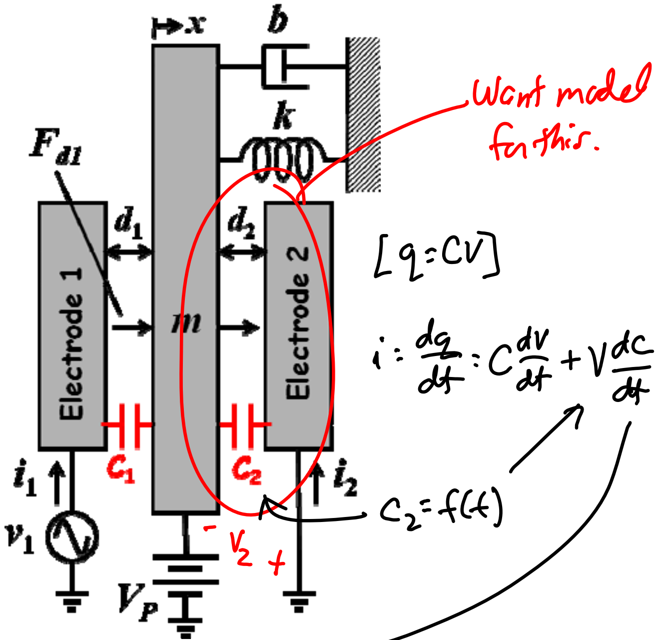
$$\underbrace{F_{d1}}_{e_2} = \underbrace{-V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1}$$

$$\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right| \qquad \frac{\epsilon_0 A}{d^2}$$

$V_P \uparrow \rightarrow \eta_1 \uparrow$

gap $\downarrow$ $\eta_1 \uparrow\uparrow$

Output Current Into Ground



$[q = CV]$

$$i = \frac{dq}{dt} = C\frac{dv}{dt} + V\frac{dC}{dt}$$

$$i_2 = C_2(x,t)\cancelto{0}{\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

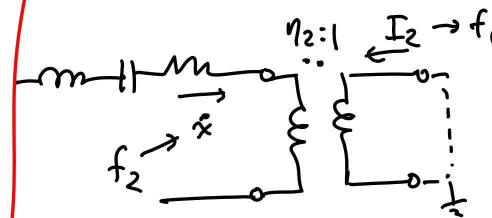
In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

motional current

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$$

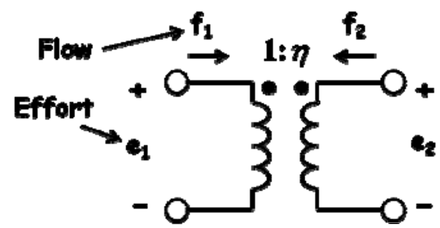
90° phase lag (+) (+) → $I_2 =$ 90° lag ~~(x)~~ when $x = (+)$



$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

$$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow I_2 = -\eta_2\dot{x}$$

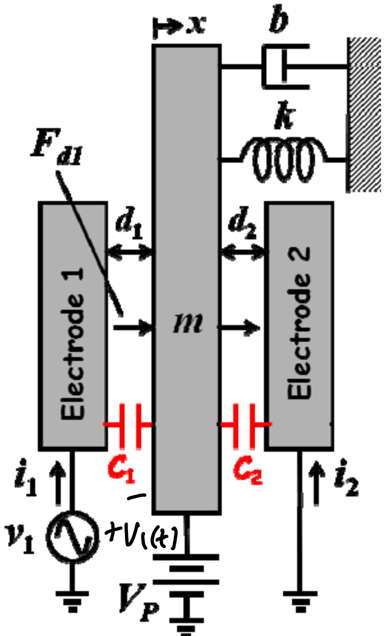
$$\therefore \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

due to mass motion

$$[V_1(t) = v_1 - V_P] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + [v_1 - V_P]\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

lower case — $f(t)$

negl. ($v_1 x$ term is small)

$$\therefore I(j\omega) = \underbrace{j\omega C_1 v_1}_{\text{Feedthrough Current}} + \cancel{j\omega v_1 \frac{\partial C_1}{\partial x} X} \underbrace{- j\omega V_P \frac{\partial C_1}{\partial x} X}_{\text{Motional Current}} \quad *$$

@ DC (or @ low freq.): $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\left(\frac{\partial C_1}{\partial x}\right)v_1$

* @ resonance: $x = \frac{QF_{d1}}{jk} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = X$

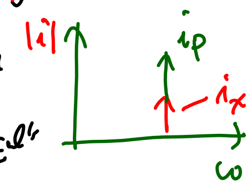
↑ 90° phase lag f/ $v_1 \to x$

Thus (@ resonance) $\to \omega_o$

$$I_1(j\omega) = j\omega_o C_1 v_1 + \cancel{j}\omega_o\left(V_P\frac{\partial C_1}{\partial x}\right)^2\frac{Q}{\cancel{j}k}v_1$$

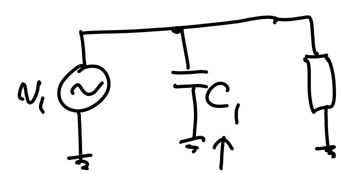
$$= \underbrace{j\omega_o C_1 v_1}_{i_p} + \underbrace{\omega_o \frac{Q}{k}\eta_{e1}^2 v_1}_{i_x}$$

"electrical"



90° phase-shifted from v_1 ↓ This is a capacitor in shunt w/ the input port:

In phase w/ v_1. ↓ ∴ this is an effective resistance seen "looking into electrode 1" → Motional Resistance *



* Motional Resistance:

$$R_{x1} = \frac{v_1}{I_1} = \frac{k}{\omega_0 Q \eta_{e1}^2} = \frac{m\omega_0}{Q\eta_{e1}^2} = \boxed{\frac{b}{\eta_{e1}^2} = R_{x1}}$$

"motion"

The equivalent ckt better get this right!

$$\boxed{\frac{\overline{v_n^2}}{\Delta f} = 4kTR_{x1}}$$ ← noise!

• Look at Module 13, slide 16