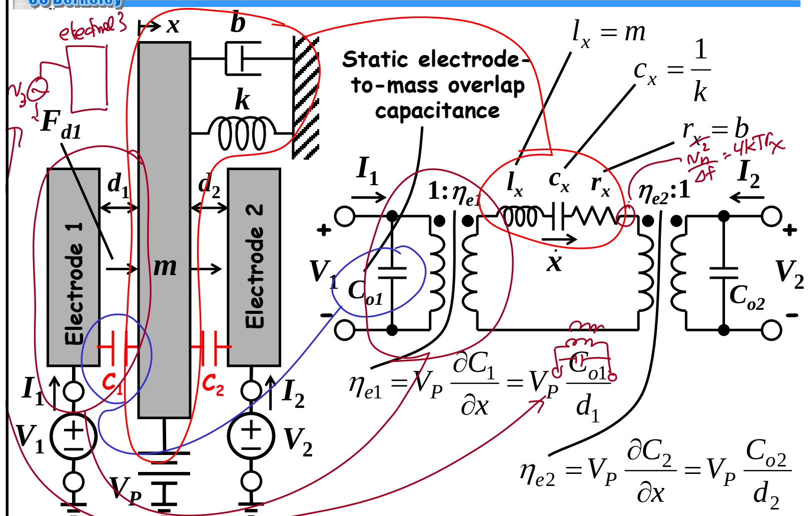
Lecture 25: Circuit Analysis & Gyros

- Announcements:
- Module 14 & 15 now online
- HW#6 due this Friday at 9 a.m.
- Project Outbrief Signups: pick two days
  - Wednesday, May 14? okay
  - Thursday, May 15? okay
  - Friday, May 9? okay
- -----------------------------------
- Equivalent Circuits II Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- -----------------------------------
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- -----------------------------------
- Last Time:

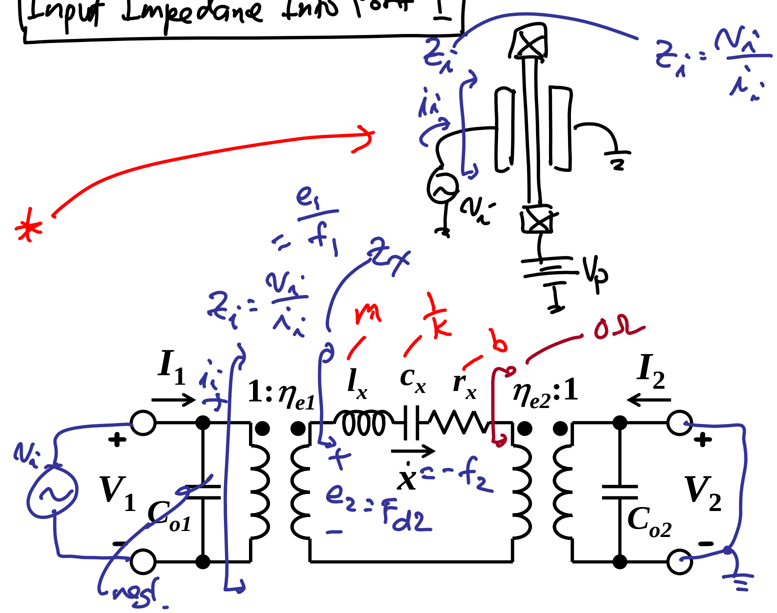
Complete Electrical-Port Equiv. Circuit



$$\eta_{e1} = V_P \frac{\partial C_1}{\partial x} = V_P \frac{C_{o1}}{d_1}$$

$$\eta_{e2} = V_P \frac{\partial C_2}{\partial x} = V_P \frac{C_{o2}}{d_2}$$

Input Impedance Into Port I



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{array}{l} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{array}$$

$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \longrightarrow \frac{v_i}{i_i} = z_i = -\frac{1}{\eta^2}\frac{F_{d2}}{(-\dot{x}_2)}$$

$$z_i = \frac{1}{\eta_{e1}^2} z_x$$

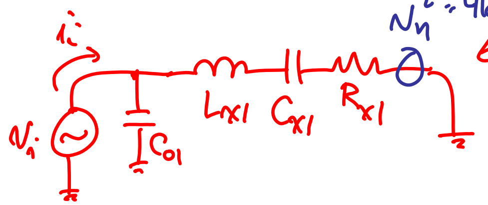
$$z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

$$= j\omega\underbrace{\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}}^{m} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}}^{k} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}^{b}$$

to model noise!

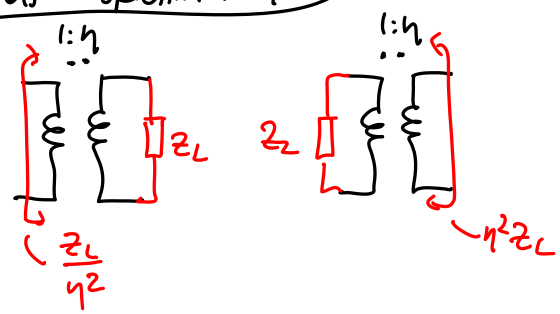
$$\overline{v_n^2} = 4kTR_x\Delta f$$

*

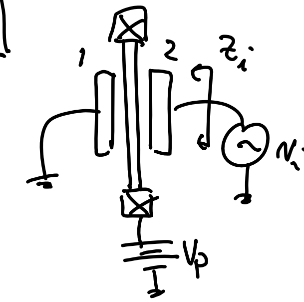


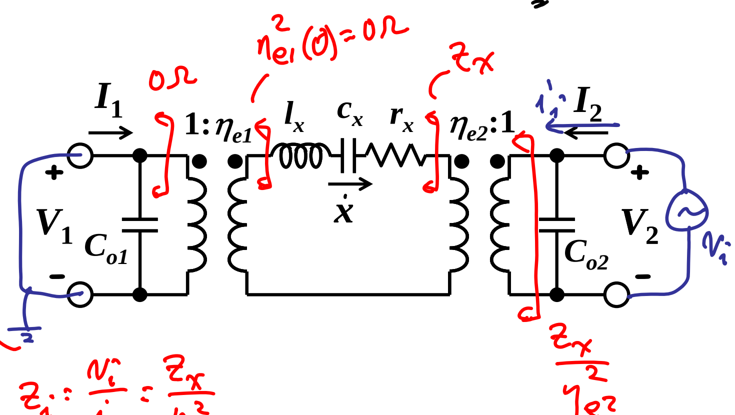
Purely Electrical Equiv. Ckt.

Xformer Inspection Analysis



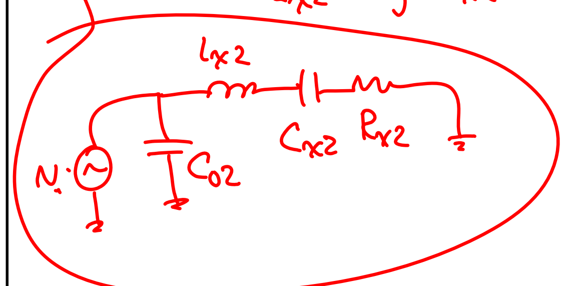
Input Impedance Into Port 2



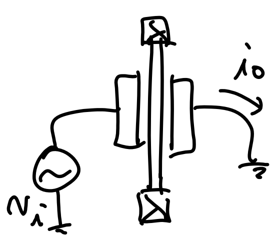


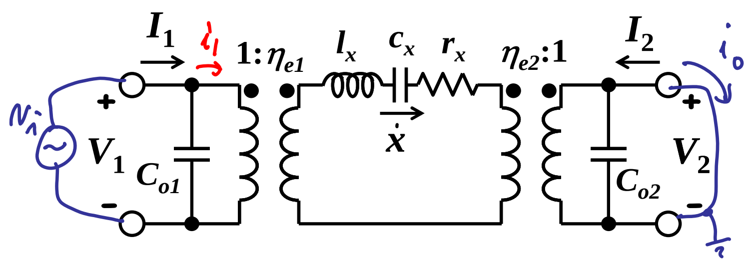
$$z_i = \frac{v_i}{i_i} = \frac{z_x}{\eta_{e2}^2}$$

$$= j\omega\underbrace{\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \underbrace{\frac{1}{j\omega(\eta_{e2}^2 c_x)}}_{\frac{1}{j\omega C_{x2}}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1 to Port 2 Transconductance → $G = \frac{i_o}{v_i}$





$$\dot{x} = \frac{1}{\eta_{e1}} i_1, \quad i_o = \eta_{e2}\dot{x} \rightarrow i_o = \frac{\eta_{e2}}{\eta_{e1}} i_1 = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{Z_i}\right)$$

$$= \frac{\eta_{e2}}{\eta_{e1}} v_i \left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$$

$$\frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}$$

$$\boxed{\frac{i_o}{v_i}(j\omega) = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1}, \quad L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}},\ C_{x12} = \eta_{e1}\eta_{e2}c_x,\ R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}}$$

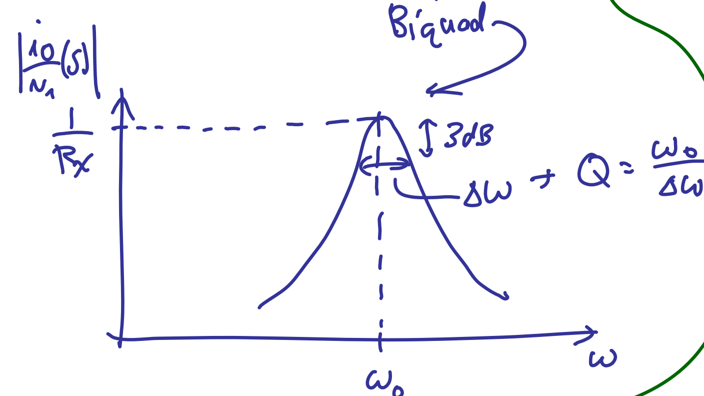
Separate freq. response & magnitude:

$$\frac{i_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_xC_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left[\frac{1}{L_xC_x} = \omega_o^2,\quad Q = \frac{\omega_o L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o}{Q}\right]$$

$$\frac{i_o}{v_i}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain Term}}\ \underbrace{\frac{s\left(\frac{\omega_o}{Q}\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}}_{\text{Freq. Shaping Term}} = \underbrace{\frac{1}{R_x}}_{\text{resonance magnitude}} H(s)$$

Freq. Shaping Term → Bandpass Biquad



significance: Can just solve the ckt. @ resonance, then multiply by a proper $H(s)$!