Lecture 2: Benefits of Scaling I

- Announcements:
- The notes from last time are online
- Modules 1 & 2 are online
- HW#1 will soon be online
  - Due in two weeks
- Discussion Section will be M 5-6 p.m. in 521 Cory
  - But we won't have the room on 2/24, so either a new room will be found for just that day, or a new room for discussions will be found
- TA Office Hours will be held in 367 Cory (for now)
- I will be gone this coming Tuesday, next week
  - No lecture
  - Make-up lecture will probably be next Friday, 1/31, in the afternoon, with specific time TBD and broadcast via email
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - — GHz micromechanical resonators
    - — Chip-scale atomic clock
    - — Micro gas chromatograph
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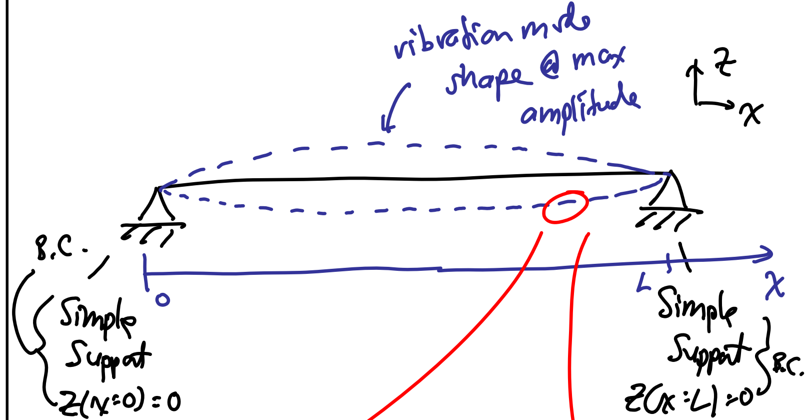
- Start going through module 2

Scaling of Guitar Strings

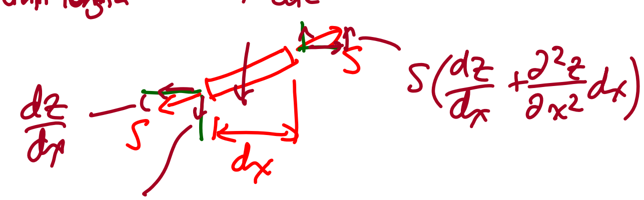
guitar string ≡ transversely vibrating stretched wire



Get equation for resonance freq: (fundamental mode)

Free Body Diagram

mass per unit length $\rightarrow \overbrace{m'dx}^{mass}\dfrac{d^2z}{dt^2}$ } — inertial force = ma



$S\dfrac{dz}{dx}$ ← z directed component of force from tension

⇒ condition for dynamic equilibrium:

$$S\left(\frac{dz}{dx} + \frac{\partial^2 z}{\partial x^2}dx\right) - S\frac{dz}{dx} - m'dx\frac{\partial^2 z}{\partial t^2} = 0$$

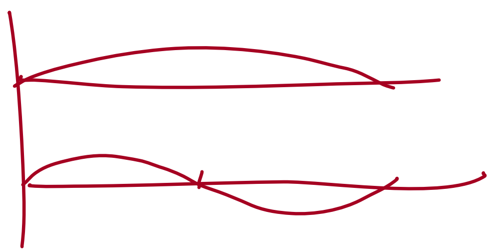
↓ solve

$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$

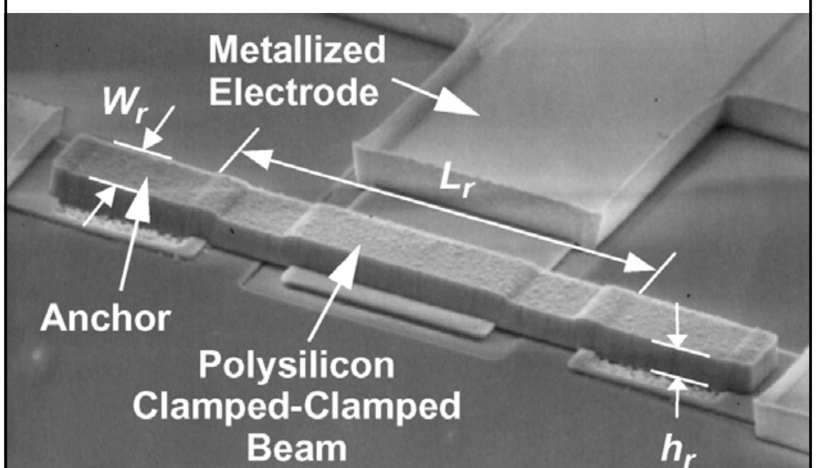
→ if $L\downarrow$ → $f_i\uparrow$

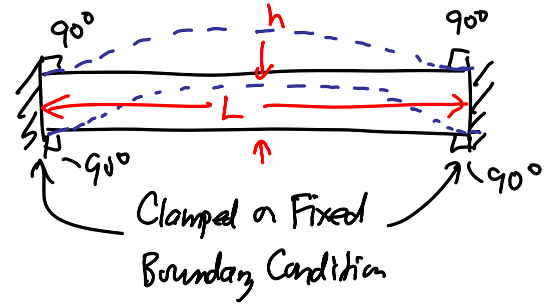
← frequency

i: mode $= 1, 2, 3, \ldots$



$\boxed{\text{Clamped-Clamped Beam}}$





⇒ Eq. for Resonance Freq.:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \qquad (1)$$

acoustic velocity

where $E \triangleq$ Young's Modulus of Elasticity [GPa]

$\rho \triangleq$ density [kg/m³]

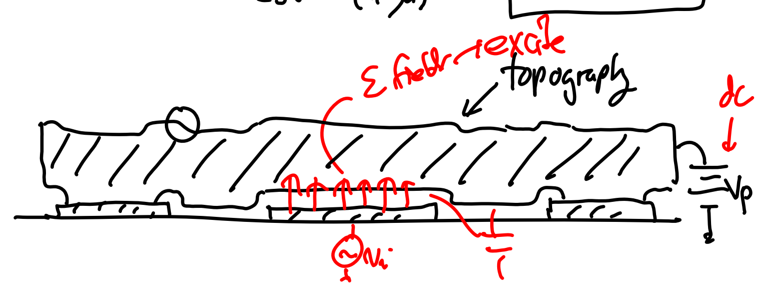
$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example: $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150GPa$, $\rho = 2300\ kg/m^3$

$$\therefore f_o = (1.03)\sqrt{\frac{150G}{2300}}\,\frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_o = 10.4MHz}$$



Scaling:

2x, ½x

① Scale all dimensions equally by a factor S

$$f_o \sim \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_o = \frac{1}{S^2}$ → even faster rise in freq.!

(...but problems...)

Example:

$L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = \boxed{1.04\text{ GHz}}$

ignore width effect → really need ~3μm

questionable thing to do

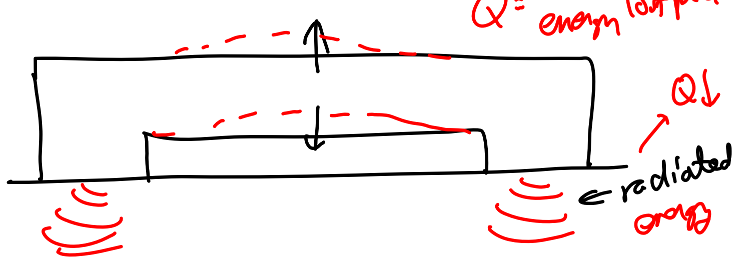
Remarks:

① Eq.(1) not accurate when $L \simeq W \simeq h$

② When $L \simeq h$ (or when it isn't more than 10xh)

→ get anchor loss problems

$$Q = \frac{\text{energy per cycle}}{\text{energy lost per cycle}}$$



Solns:

③ Solution: use nanodimensions! ✓

→ ex.: h=300nm, L~1μm

Problem: power handling ↓ when size ↓

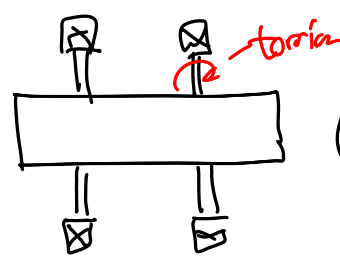
→ Soln: use massive numbers in arrays!

④ Better Soln: use other geometries

Free-Free Beam: nodal points

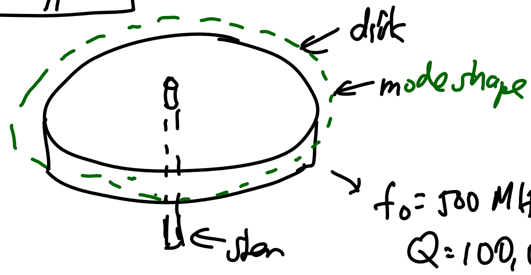
(side view)

no vertical motion → less loss from pumping into the substrate



(top view)

Even Better Approach



$f_0 = 500$ MHz
$Q = 100,000$