Lecture 4: Benefits of Scaling III

- Announcements:
- This is our make-up lecture for last Tuesday
- HW#1 online
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
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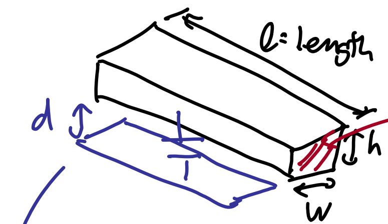
- Last Time: Covering thermal circuit modeling … which we now continue …

Review Electrical Resistance First

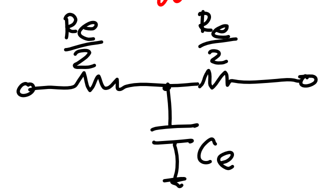
(then attack the thermal R analogy)



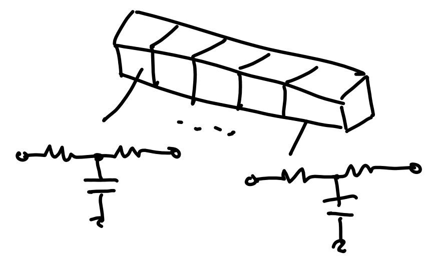
$R_e \triangleq$ electrical resistance $= \frac{l}{\sigma A}$ ← electrical conductivity

C_e = capacitance $= \frac{\epsilon_o W L}{d}$ ← permittivity

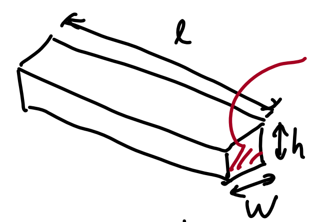
↳ Stores Energy (charge energy): $\frac{1}{2}CV^2 = E$ (V = voltage across the capacitor)



⇒ if want to be more exact:



Thermal Ckt.



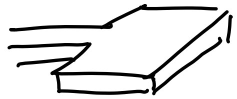
⇒ thermal capacitance: $C_{th} = \rho V C_p$ (ρ = density, V = volume, C_p = specific heat) ↘ stores thermal energy

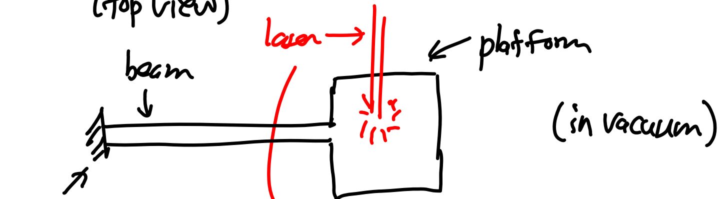
⇒ thermal resistance:

$R_{th} = \frac{l}{kA}$ (l ← length, A ← cross-sectional area, k ← thermal conductivity)

Examples

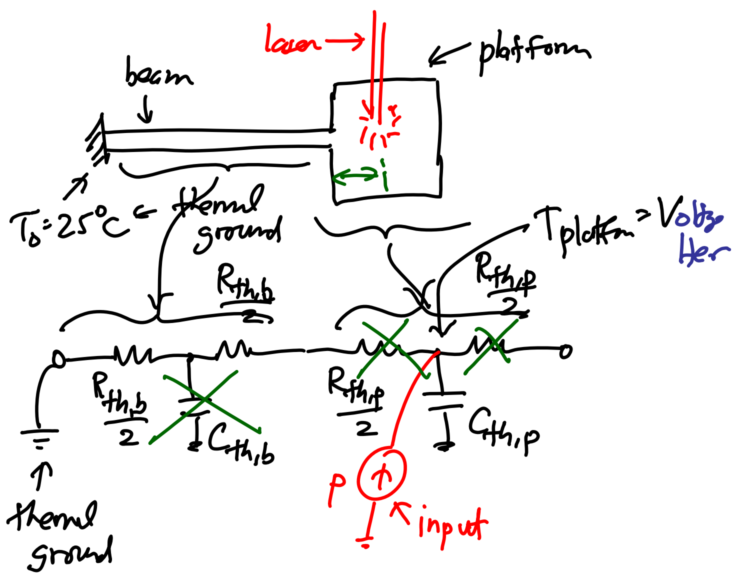
Ex. MEMS type structure (top view)





Source of power: assume all energy of laser is absorbed

What is the temperature on the platform?



Analogies:

(temperature) $T \rightarrow V$ (voltage)

(power) $P \rightarrow I$ (current)

thermal ← → electrical

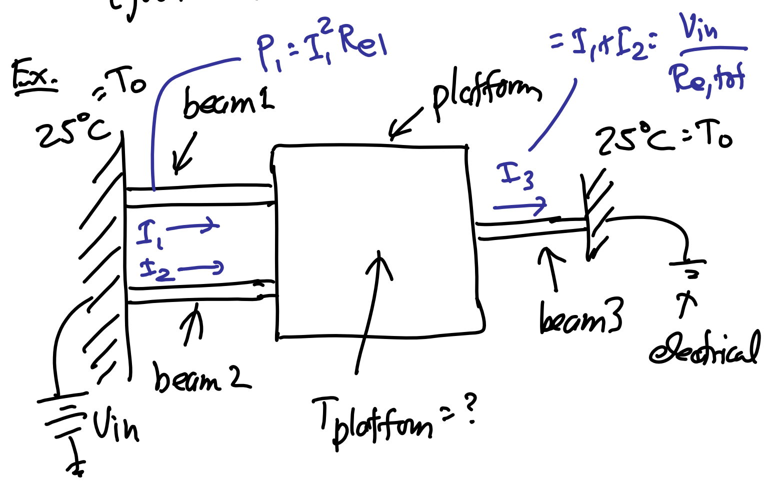
To Analyze:

① Remove small elements to simplify the ckt.

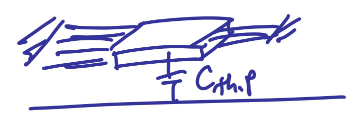
② $T_{platform} = PR_{th,b}$ (in steady-state)

(just like $V = IR$ in an electrical system)

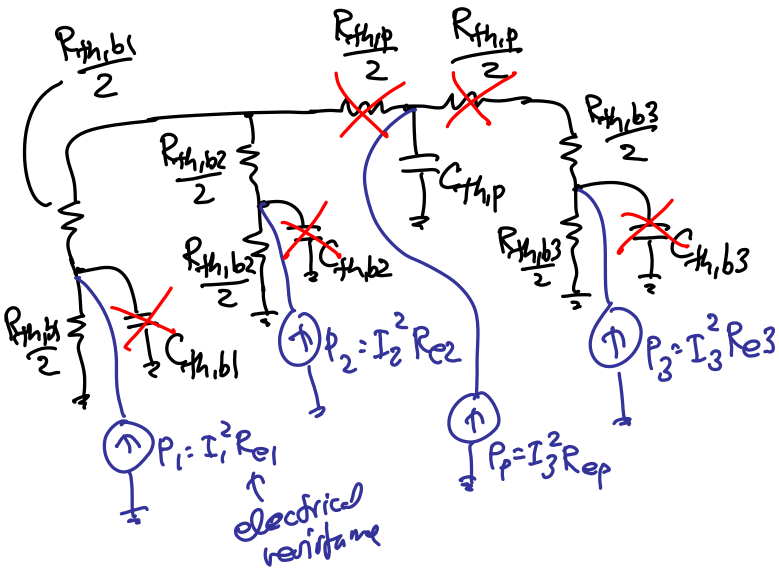
Ex.



→ coupled thermal, electrical system



⇒ Draw the thermal ckt:



To Analyze:

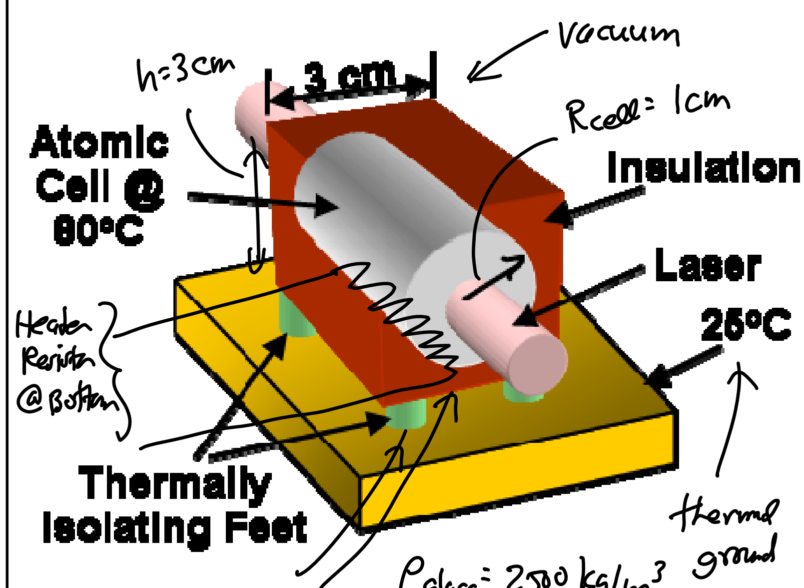
(1) $I_3 = I_1 + I_2 = \frac{V_{in}}{R_{e,tot}}$ (electrical analysis)

(2) Get P_i's. (powers)

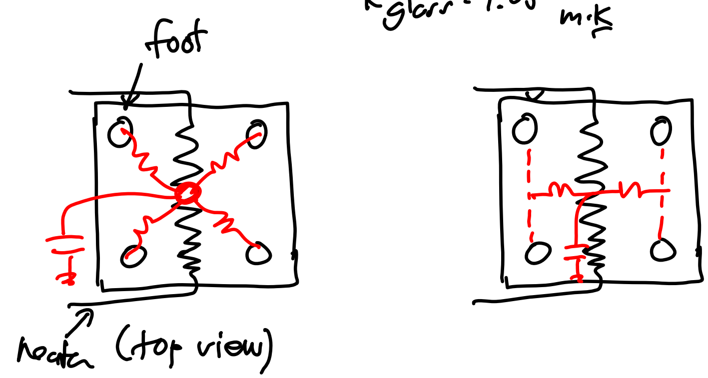
(3) Use superposition to solve the thermal ckt.
handle one power source at a time & sum the temperatures (i.e., thermal voltages) to get the total temperature at any node

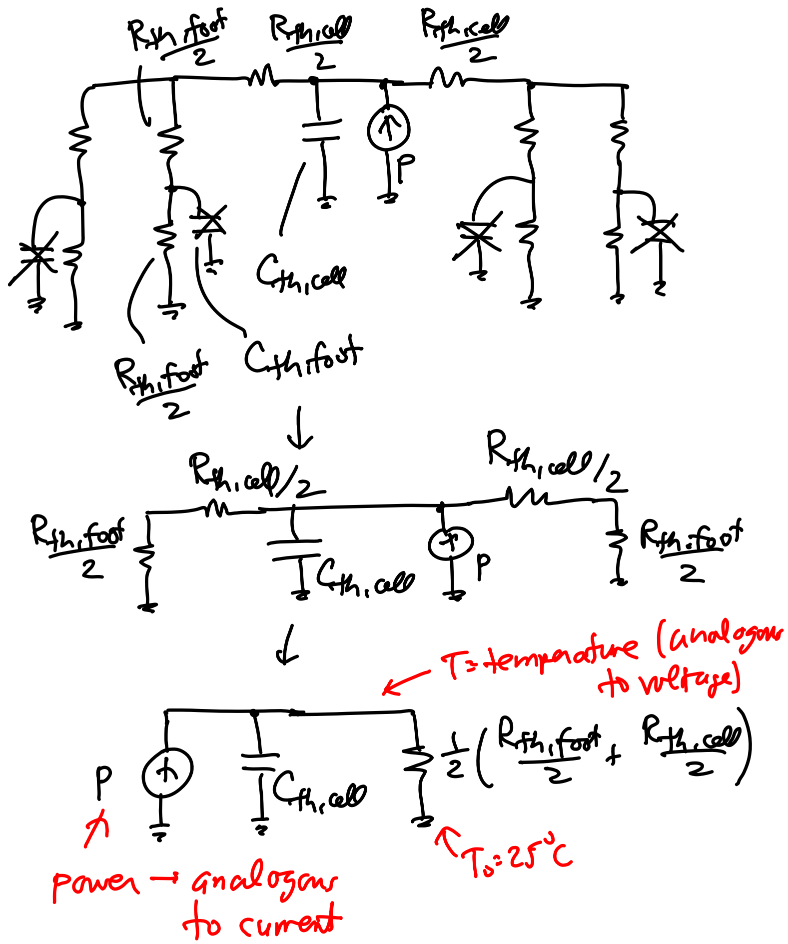
Example: Thermal Ckt.

⇒ determine the power needed to get this atomic cell to 80°C (from RT) & how fast

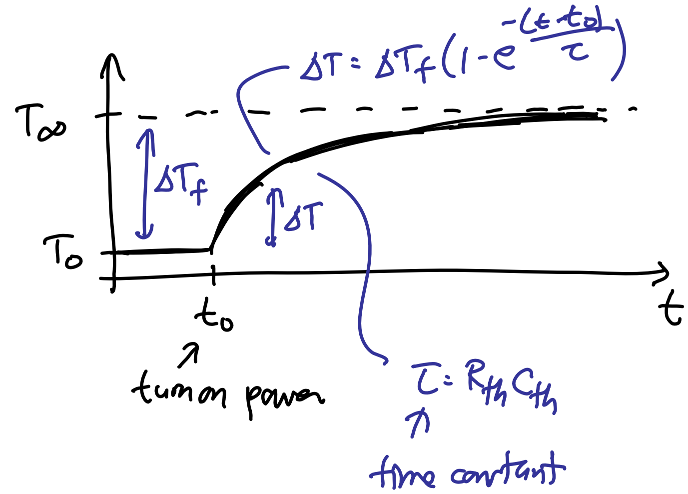


all glass: $\rho_{glass} = 2500\ kg/m^3$
$C_{p,glass} = 0.5\ J/(g\cdot K)$
$k_{glass} = 1.05\ \frac{W}{m\cdot K}$



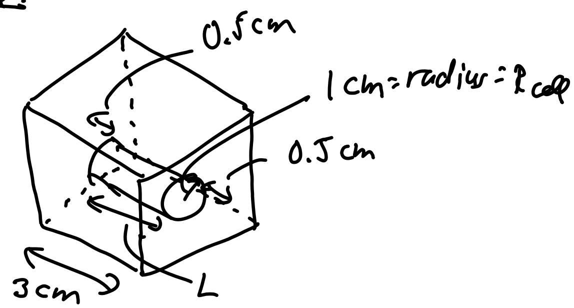


$\Delta T = T - T_0 = P R_{th}$ (in steady-state)



Find $C_{th,cell}$:

First, find the cell volume:



$$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$$

$$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7\ cm^3$$

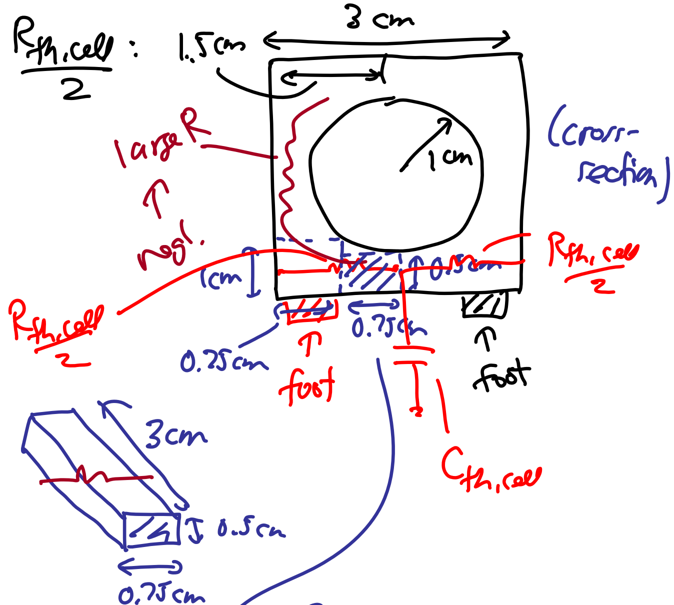
$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$

$$= \left(2500\ \frac{kg}{m^3}\right)\left(1000\ \frac{g}{kg}\right)\left(\frac{1}{100^3}\ \frac{m^3}{cm^3}\right)$$

$$\times\ (20.7\ cm^3)\left(0.5\ \frac{J}{g \cdot K}\right)$$

$$\Rightarrow C_{th,cell} = 25.9\ J/K$$

Find $\frac{R_{th,cell}}{2}$:



$$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{k(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{k(3)(1)} = \frac{1}{k}\left(\frac{1}{8}+\frac{1}{4}\right) = \frac{3}{8}\frac{1}{k}$$

$$\left[R_{th} = \frac{l}{kA}\right]$$

$$\therefore \frac{R_{th,cell}}{2} = \frac{3}{8}\frac{1}{1.05} \times \left(100\ \frac{cm}{m}\right)$$

$$= \underline{35.7\ K/W}$$