


EE C247B - ME C218
Introduction to MEMS Design
Spring 2014

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 12: Capacitive Transducers

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 1



Lecture Outline

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
 - ↗ Energy Conserving Transducers
 - Charge Control
 - Voltage Control
 - ↗ Parallel-Plate Capacitive Transducers
 - Linearizing Capacitive Actuators
 - Electrical Stiffness
 - ↗ Electrostatic Comb-Drive
 - 1st Order Analysis
 - 2nd Order Analysis

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 2

Basic Physics of Electrostatic Actuation

UC Berkeley

- **Goal:** Determine gap spacing g as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
 - ↪ Change the charge q
 - ↪ Change the separation g

$$\Delta W(q, g) = V\Delta q + F_e\Delta g$$

$$dW = Vdq + F_e dg$$

• **Note:** We assume that the plates are supported elastically, so they don't collapse

EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
3

Stored Energy

UC Berkeley

• Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap

No change in charge: $dq=0$

$$W = 0 + \int_0^g F_e dg'$$

$F_e = \left(\frac{q}{2}\right)\epsilon = \frac{1}{2} \frac{q^2}{\epsilon A}$
(independent of g)

$$\therefore W = \int_0^g F_e dg' = F_e g \Big|_0^g = F_e g$$

$$\therefore W(g) = \frac{1}{2} \frac{q^2}{\epsilon A} g$$

zero gap → zero stored energy

For a capacitor C :
 $q = CV \rightarrow V = \frac{q}{C}$

$\therefore W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq$
 $= \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{q^2 g}{\epsilon A} = W(q)$

Work done to charge C to q at fixed gap

Same answer when

EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
4

Charge-Control Case

UC Berkeley

- Having found stored energy, we can now find the force acting on the plates and the voltage across them:

From $dW = Vdq + F_e dg$:

⇒ Force is given by:

$$F_e = \left. \frac{\partial W(q,g)}{\partial g} \right|_{q=\text{const.}} = \frac{\partial}{\partial g} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right)$$

$$\therefore \boxed{F_e = \frac{1}{2} \frac{q^2}{\epsilon A}} \Rightarrow \text{independent of gap spacing!}$$

⇒ Voltage is given by:

$$V = \left. \frac{\partial W(q,g)}{\partial q} \right|_{g=\text{const.}} = \frac{\partial}{\partial q} \left(\frac{1}{2} \frac{q^2}{\epsilon A} g \right) = \frac{qg}{\epsilon A} \therefore \boxed{V = \frac{q}{C}} \Rightarrow \text{consistent w/ what we already know } \checkmark$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 5

Voltage-Control Case

UC Berkeley

- Practical situation: We control V
 - Charge control on the typical sub-pF MEMS actuation capacitor is difficult
 - Need to find F_e as a partial derivative of the stored energy $W = W(V,g)$ with respect to g with V held constant? But can't do this with present $W(q,g)$ formula
 - Solution:** Apply Legendre transformation and define the co-energy $W'(V,g)$

Effort (e.g., force, voltage, ...)

e

e_1

$e: \Phi(q)$

Displacement (e.g., displacement, charge)

q

q_1

Co-Energy $\rightarrow W'(e) = \int_0^{e_1} q de = \int_0^{q_1} \Phi^{-1}(e) de$

Energy $\rightarrow W(q) = \int_0^{q_1} e dq = \int_0^{q_1} \Phi(q) dq$

⇒ Can define co-energy as: $W'(e) = eq - W(q)$ (from this plot)

For a linear system, these will be equal.

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 6

Co-Energy Formulation

UC Berkeley

- For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

$$W'(V, g) = qV - W(q, g)$$

Differentially, this becomes:

$$dW'(V, g) = (q dV + V dq) - dW(q, g)$$

But $[dW(q, g) = F_e dg + V dq]$

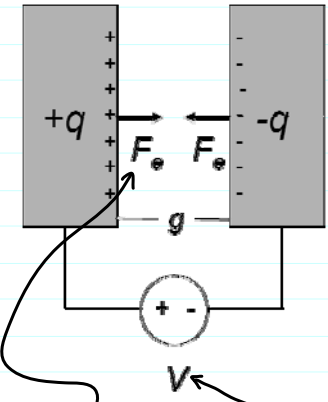
$dW'(V, g) = q dV - F_e dg$

Working Co-Energy Expression

From which:

Charge, $Q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_{g = \text{const.}}$

Force, $F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_{V = \text{const.}}$ ⇒ this gives force as a function of applied voltage



EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 7

Electrostatic Force (Voltage Control)

UC Berkeley

- Find co-energy in terms of voltage (w/ gap held constant)

$$W' = \int_0^V q(g, V') dV' = \int_0^V \left(\epsilon \frac{A}{g} \right) V' dV' = \frac{1}{2} \left(\frac{\epsilon A}{g} \right) V^2 = \frac{1}{2} C V^2$$

(as expected)

- Variation of co-energy with respect to gap yields electrostatic force:

$$F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = - \frac{1}{2} \left(- \frac{\epsilon A}{g^2} \right) V^2 = \frac{1}{2} \frac{C}{g} V^2$$

strong function of gap!

- Variation of co-energy with respect to voltage yields charge:

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = \left(\frac{\epsilon A}{g} \right) V = C V$$

as expected

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 8

UC Berkeley

Spring-Suspended Capacitive Plate

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 9

UC Berkeley

Charge Control of a Spring-Suspended C

Force generated by charge q supplied by current I :

$$F_e = \left. \frac{\partial W(q, g)}{\partial g} \right|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring: $F_{\text{spring}} = kz = F_e$ (@ equilibrium)

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k}} = g \Rightarrow \text{can increase } q \text{ and drive } g \rightarrow 0$$

initial gap

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \boxed{\frac{q}{\epsilon A} \left(g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k} \right)} = V \Rightarrow V \downarrow \text{ as } g \downarrow$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 10

Voltage Control of a Spring-Suspended C

UC Berkeley

Again, $F_{\text{spring}} = kz = F_e$
 But now:

$$F_e = \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = \frac{1}{2} \frac{\epsilon A}{g^2} V^2$$

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{\epsilon A V^2}{g^2 k} = g} \Rightarrow g \text{ shows up on both sides!}$$

Charge: (for a stable gap)

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = CV = \boxed{\frac{\epsilon A}{g} V = q}$$

cubic nonlinearity in g!
 Feedback!
 If $V \uparrow \rightarrow g \downarrow \rightarrow F_e \uparrow$
 If loop gain > 1 , then this will go unstable!

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 11

Stability Analysis

UC Berkeley

- Net attractive force on the plate:

$$F_{\text{net}} = \underbrace{\frac{\epsilon A V^2}{2g^2}}_{F_e} - \underbrace{k(g_0 - g)}_{F_{\text{spring}}}$$

- An increment in gap dg leads to an increment in net attractive force dF_{net}

$$dF_{\text{net}} = \frac{\partial F_{\text{net}}}{\partial g} dg = \left[-\frac{\epsilon A V^2}{g^3} + k \right] dg$$

$F_{\text{net}} \downarrow \rightarrow dF_{\text{net}} = (-)$ If $g \downarrow \rightarrow dg = (-)$, then for stability need \rightarrow otherwise, plate collapses

Thus, need this = (+) $\Rightarrow \boxed{k > \frac{\epsilon A V^2}{g^3}}$ (for a stable uncollapsed state)

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 12

Pull-In Voltage V_{PI}

UC Berkeley

- V_{PI} = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\epsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - k(g_0 - g_{PI}) \quad (2)$$

\leftarrow pull-in gap

- Substituting (1) into (2):

$$0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - \frac{\epsilon A V_{PI}^2}{g_{PI}^3} (g_0 - g_{PI})$$

$$\frac{g_0 - g_{PI}}{g_{PI}} = \frac{1}{2} \rightarrow g_0 = \frac{3}{2} g_{PI}$$

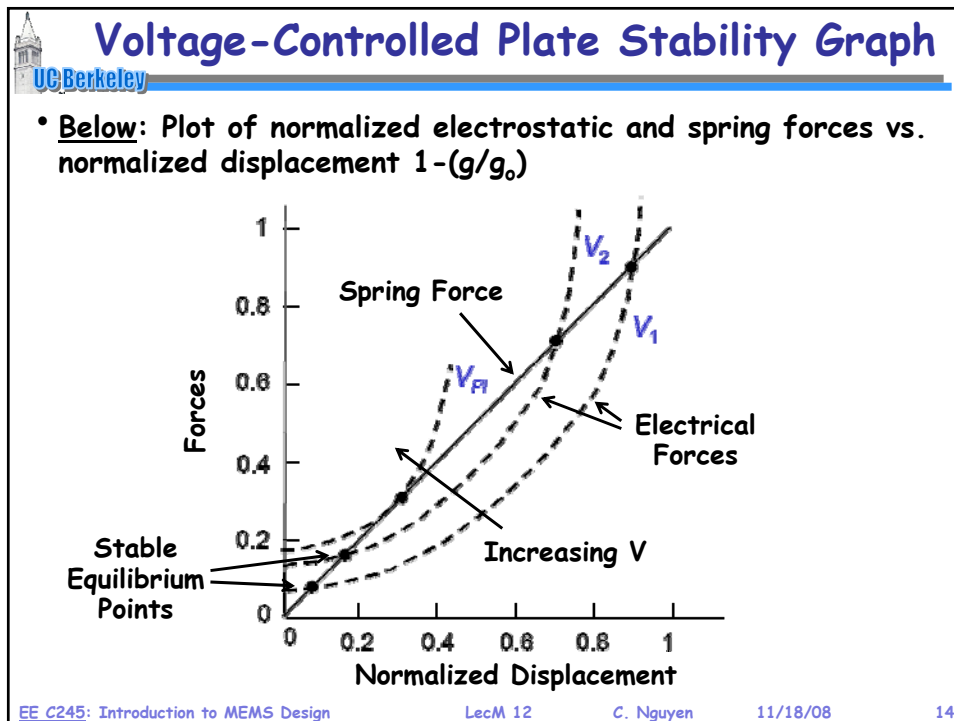
$$\therefore g_{PI} = \frac{2}{3} g_0$$


$$V_{PI} = \sqrt{\frac{k g_{PI}^3}{\epsilon A}}$$

$$\therefore V_{PI} = \sqrt{\frac{8}{27} \frac{k g_0^3}{\epsilon A}}$$

When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 13






Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through I^2R losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures


EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 15



Problems With Electrostatic Actuators


- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 16



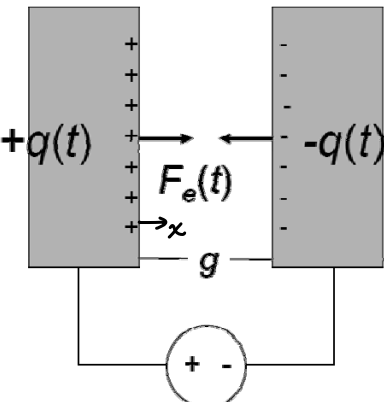
Linearizing the Voltage-to-Force Transfer Function

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 17



Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage V_p together with the intended input (or drive) voltage $v_i(t)$, where $V_p \gg v_i(t)$



$v(t) = V_p + v_i(t)$

$$F_e(t) = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x} \left(\frac{1}{2} C [v(t)]^2 \right)$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} [v(t)]^2 = \frac{1}{2} (V_p + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} [V_p^2 + 2V_p v_i(t) + [v_i(t)]^2] \frac{\partial C}{\partial x}$$

$$[V_p \gg v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2} V_p^2 \frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_p \frac{\partial C}{\partial x} v_i(t)}_{\text{AC drive signal}}$$

$$C(x) = \frac{\epsilon A}{g_0 \cdot x} = C_0 \left(1 - \frac{x}{g_0}\right)^{-1} \approx C_0 \left(1 + \frac{x}{g_0}\right)$$

$[x \ll g_0]$ $\sim \text{const.} \therefore \text{linear}$

$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_0}{g_0} \Rightarrow F_e(t) = \frac{1}{2} \frac{C_0}{g_0} V_p^2 + V_p \frac{C_0}{g_0} v_i(t)$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 18

Differential Capacitive Transducer

UC Berkeley

- The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$

Do the math.

Assume matched gaps.

$$F_{net}(t) = \frac{1}{2} \frac{\partial C}{\partial x} \{ [V_R(t)]^2 - [V_L(t)]^2 \}$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} \{ (V_p^2 + 2V_p V(t) + [V(t)]^2) - (V_p^2 - 2V_p V(t) + [V(t)]^2) \}$$

$$\therefore F_{net}(t) = 2V_p \frac{\partial C}{\partial x} V(t) = 2V_p \frac{C_0}{g_0} N(t) \Rightarrow \text{Linear w/ } V(t)! \text{ (gap match limited)}$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 19

**Remaining Nonlinearity
(Electrical Stiffness)**

UC Berkeley

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 20

Parallel-Plate Capacitive Nonlinearity

UC Berkeley

- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:**

V_a or v_A

$v_a = |v_a| \cos \omega t$

V_A

t

V_a or $v_A = V_A + v_a$

Total Value

DC Component (upper case variable; upper case subscript)

AC or Signal Component (lower case variable; lower case subscript)

Conductive Structure

Electrode

k_m

d_1

m

x

F_{dI}

v_1

V_1

V_P

EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
21

Parallel-Plate Capacitive Nonlinearity

UC Berkeley

- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Expression for $\partial C/\partial x$:**

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_0 \left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + \frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

$$\frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \dots\right)$$

where

$$A_1 = -\frac{2}{d_1}$$

$$A_2 = \frac{3}{d_1^2}$$

$$A_3 = -\frac{4}{d_1^3}$$

⋮

Conductive Structure

Electrode

k_m

d_1

m

x

F_{dI}

v_1

V_1

V_P

EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
22

Parallel-Plate Capacitive Nonlinearity

UC Berkeley

- Thus, the expression for force from the left side becomes:

$$F_{d1} = \frac{1}{2} \frac{\partial C}{\partial x} (V_P - V_i - N_i)^2 = \frac{1}{2} \frac{\partial C}{\partial x} (V_{P1} - N_i)^2$$

{small displacements: $x \ll d_1$ }

$$F_{d1} = \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) (1 + A_1 x) (V_{P1}^2 - 2V_{P1}N_i + N_i^2)$$

$$= \frac{1}{2} \left(-\frac{C_{o1}}{d_1} \right) \left\{ V_{P1}^2 - 2V_{P1}N_i + N_i^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x N_i + A_1 x N_i^2 \right\}$$

@ resonance: $x = \frac{Q F_{d1}}{j k} \approx \frac{Q}{j k} \frac{\partial C}{\partial x} V_{P1} V_i$

Thus:

$$N_i = |N_i| \cos \omega_o t \rightarrow x = |x| \sin \omega_o t$$

\uparrow
 x 90° phase-shifted from N_i

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 23

Parallel-Plate Capacitive Nonlinearity

UC Berkeley

- Retaining only terms at the drive frequency:

$$F_{d1}|_{\omega_o} = \underbrace{V_{P1} \frac{C_{o1}}{d_1}}_{\text{Drive force arising from the input excitation voltage at the frequency of this voltage}} |v_1| \cos \omega_o t + V_{P1}^2 \frac{C_{o1}}{d_1^2} |x| \sin \omega_o t$$

Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
- ↳ A negative spring constant
- ↳ Since it derives from V_p , we call it the electrical stiffness, given by:

$$k_e = V_{P1}^2 \frac{C_{o1}}{d_1^2} = V_{P1}^2 \frac{\epsilon A}{d_1^3}$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 24

Electrical Stiffness, k_e

UC Berkeley

- The electrical stiffness k_e behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

$$= \sqrt{\frac{k_m}{m} \left(1 - \frac{k_e}{k_m}\right)^{1/2}}$$

$$\omega_o' = \omega_o \left(1 - \frac{V_{P1}^2 \epsilon A}{k_m d_1^3}\right)^{1/2}$$

Frequency is now a function of dc-bias V_{P1}

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 25

Voltage-Controllable Center Frequency

UC Berkeley

- Quadrature force \Rightarrow voltage-controllable electrical stiffness:

$$k_e = \frac{\epsilon_o A_o}{d^3} V_P^2$$

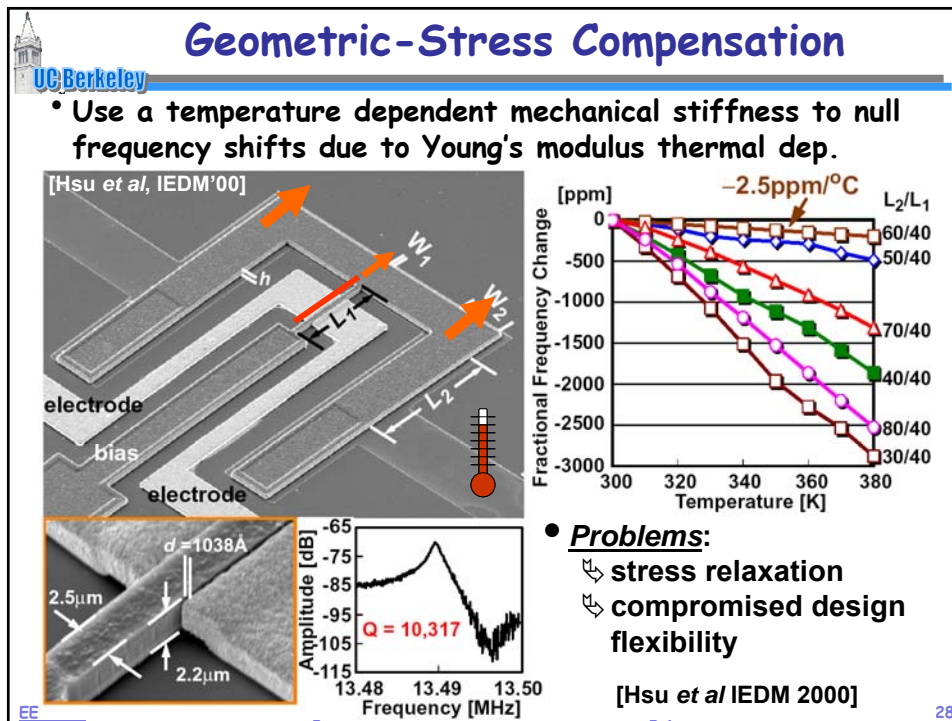
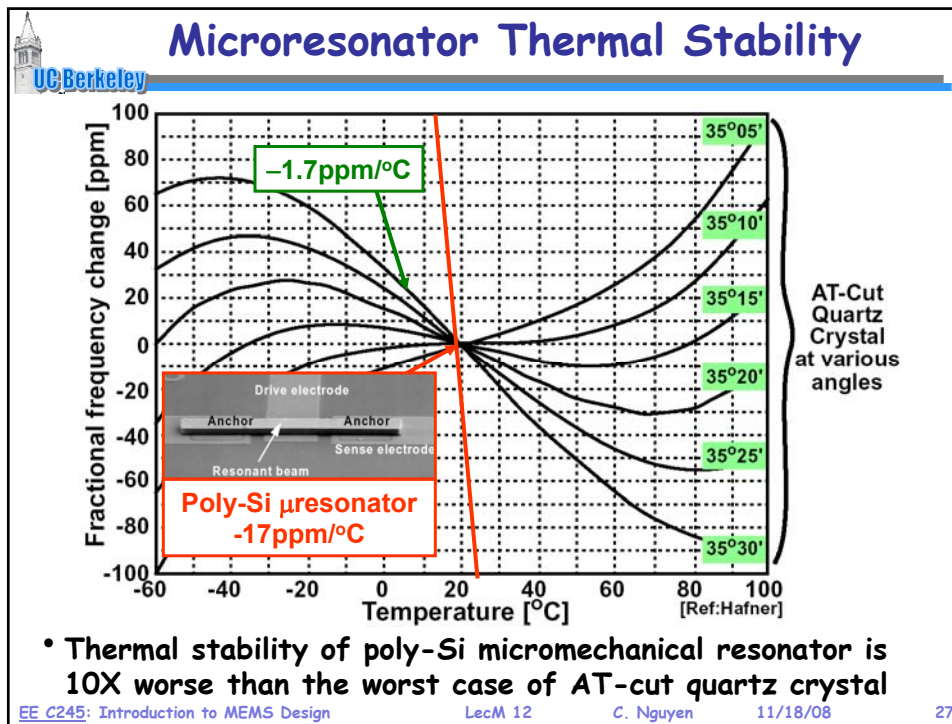
Electrode Overlap Area A_o
Gap d

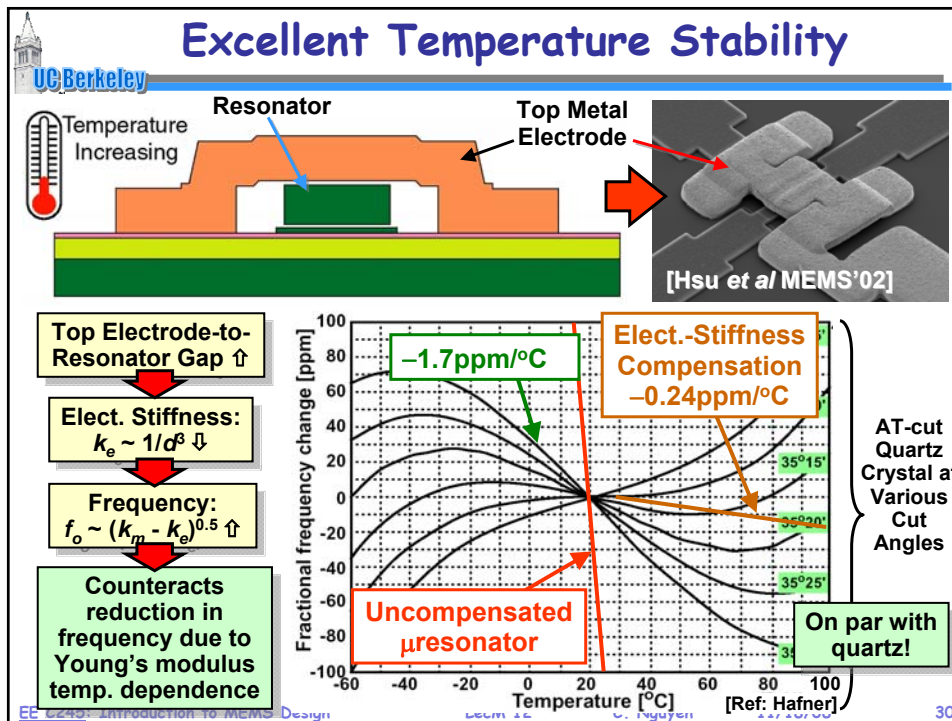
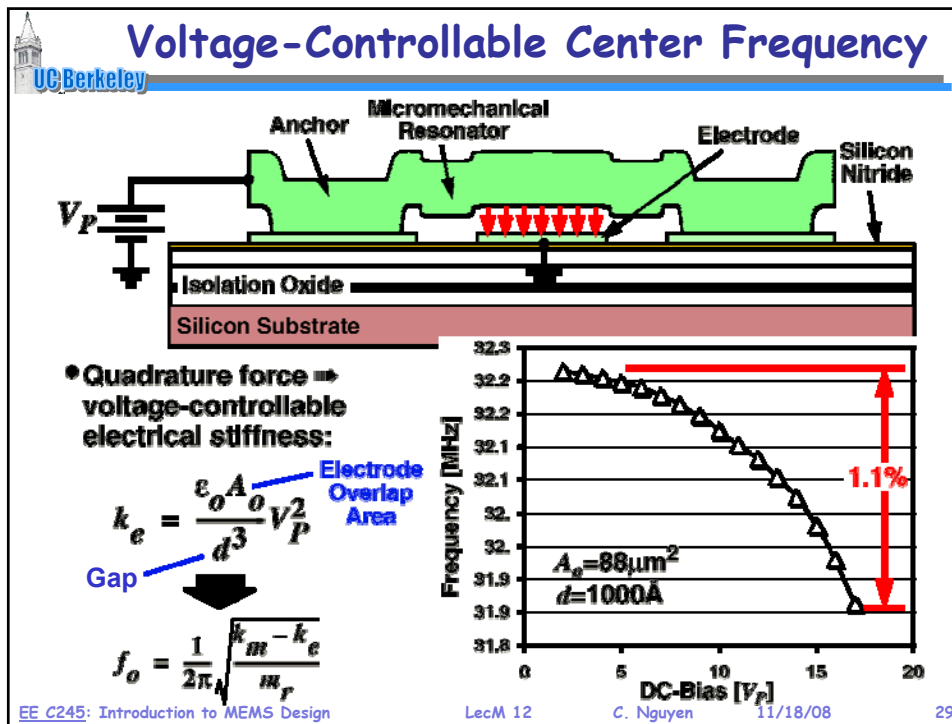
$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_m - k_e}{m_r}}$$

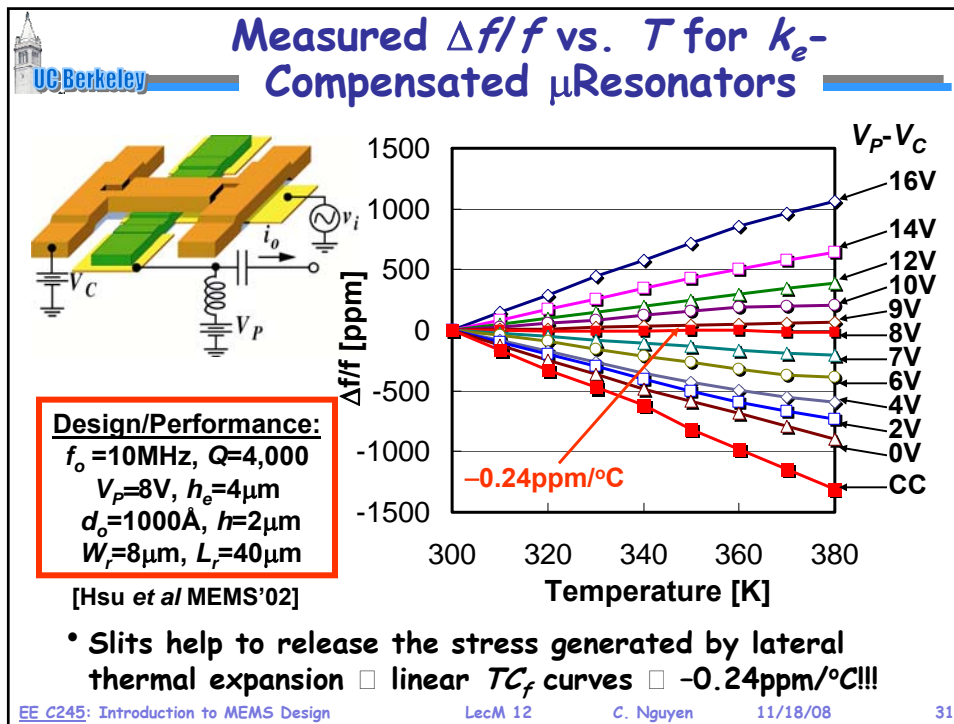
DC-Bias [VP]	Frequency [MHz]
0	32.25
5	32.22
10	32.18
15	32.12
20	31.85

$A_o = 88 \mu\text{m}^2$
 $d = 1000 \text{ \AA}$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 26







Can One Cancel k_e w/ Two Electrodes?

- What if we don't like the dependence of frequency on V_p ?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:


Subtracts from the F_{d1} term, as expected

$$F_{d2}|_{\omega_o} = -V_{p2} \frac{C_{o2}}{d_2} |v_2| \cos \omega_o t$$

$$+ V_{p2}^2 \frac{C_{o2}}{d_2^2} |x| \sin \omega_o t$$

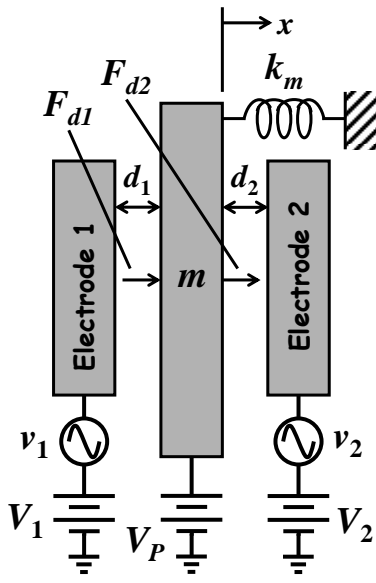
Adds to the quadrature term $\rightarrow k_e$'s add, no matter the electrode configuration!

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 32




Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
 - ↪ Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
 - ↪ Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_0)
 - ↪ Noise can alias due to nonlinearity
- Range of motion is small
 - ↪ For larger motion, need larger gap ... but larger gap weakens the electrostatic force
 - ↪ Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
33



Electrostatic Comb Drive

Electrostatic Comb Drive

EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
34

Electrostatic Comb Drive

UC Berkeley

- Use of comb-capacitive transducers brings many benefits
 - ↳ Linearizes voltage-generated input forces
 - ↳ (Ideally) eliminates dependence of frequency on dc-bias
 - ↳ Allows a large range of motion

Comb-Driven Folded Beam Actuator

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 35

Comb-Drive Force Equation (1st Pass)

UC Berkeley

Top View

Side View

$$C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \left[\frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$

$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} (V_p - V_i)^2 = \frac{2}{2} \frac{\epsilon_0 h}{d} (V_p^2 - 2V_p V_i + V_i^2) \approx -2V_p \frac{\epsilon_0 h}{d} V_i = F_d$$

When $V_i = (+) \rightarrow F_d = (-)$ ✓

↳ But wait! This ignores other practical effects! (No dependence on λ ! LINEAR!)

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 36

Lateral Comb-Drive Electrical Stiffness

Top View

Side View

- Again: $C(x) = \frac{2Nshx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2Nsh}{d}$
- No $(\partial C/\partial x)$ x-dependence \rightarrow no electrical stiffness: $k_e = 0!$
- Frequency immune to changes in V_p or gap spacing!

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 37

Typical Drive & Sense Configuration

2-port Lateral Microresonator

N_f : # shuttle fingers

Simple Analysis:

$$F_{d1} = \frac{1}{2} \frac{\partial C_1}{\partial x} (V_1 - V_{P1})^2 = \frac{1}{2} \left(-\frac{\epsilon_0 h}{d} \right) (N_1^2 - 2V_{P1}V_1 + V_{P1}^2) (2N_f)$$

$$F_{d2} = \frac{1}{2} \frac{\partial C_2}{\partial x} (V_2 - V_{P2})^2 = \frac{1}{2} \left(\frac{\epsilon_0 h}{d} \right) (N_2^2 - 2V_{P2}V_2 + V_{P2}^2) (2N_f)$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left(\frac{\epsilon_0 h}{d} \right) (N_2^2 - N_1^2 - 2(V_{P2}V_2 - V_{P1}V_1) + V_{P2}^2 - V_{P1}^2) (2N_f)$$

For $V_1 = V_2, V_1 = -V_2$

$$F_{net} = 2(2N_f) \left(\frac{\epsilon_0 h}{d} \right) V_{P1} V_1$$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 38

Comb-Drive Force Equation (2nd Pass)

UC Berkeley

- In our 1st pass, we accounted for
 - ↪ Parallel-plate capacitance between stator and rotor
- ... but neglected:
 - ↪ Fringing fields
 - ↪ Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

The diagram shows a cross-section of a comb-drive actuator. On the left, a 3D view shows a rotor (green) moving vertically between stator fingers (hatched). Labels include 'Stator', 'Rotor', 'Ground Plane', 'L' (finger length), and 'x' (displacement). On the right, a 2D schematic shows the electrical circuit with a voltage source V_s connected to the stator and a voltage source V_r connected to the rotor. Parameters t (finger thickness), g (gap), and z_0 (substrate thickness) are also indicated.

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 39

Comb-Drive Force With Ground Plane Correction

UC Berkeley

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

The diagram shows an equivalent circuit model. It consists of three nodes: stator (s), rotor (r), and ground plane (p). Capacitors are connected between these nodes: $\frac{1}{2} C_{sp}$ between stator and ground, C_{rp} between rotor and ground, $\frac{1}{2} C_{rs}$ between stator and rotor, and $\frac{1}{2} C_{rs}$ between rotor and stator. The rotor node is connected to a voltage source V_r and the stator node to a voltage source V_s . A vertical axis z_0 is shown on the left.

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 40

Capacitance Expressions

UC Berkeley

- Case: $V_r = V_p = 0V$
- C_{sp} depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

Region 2

Region 3

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 41

Comb-Drive Force With Ground Plane Correction

UC Berkeley

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

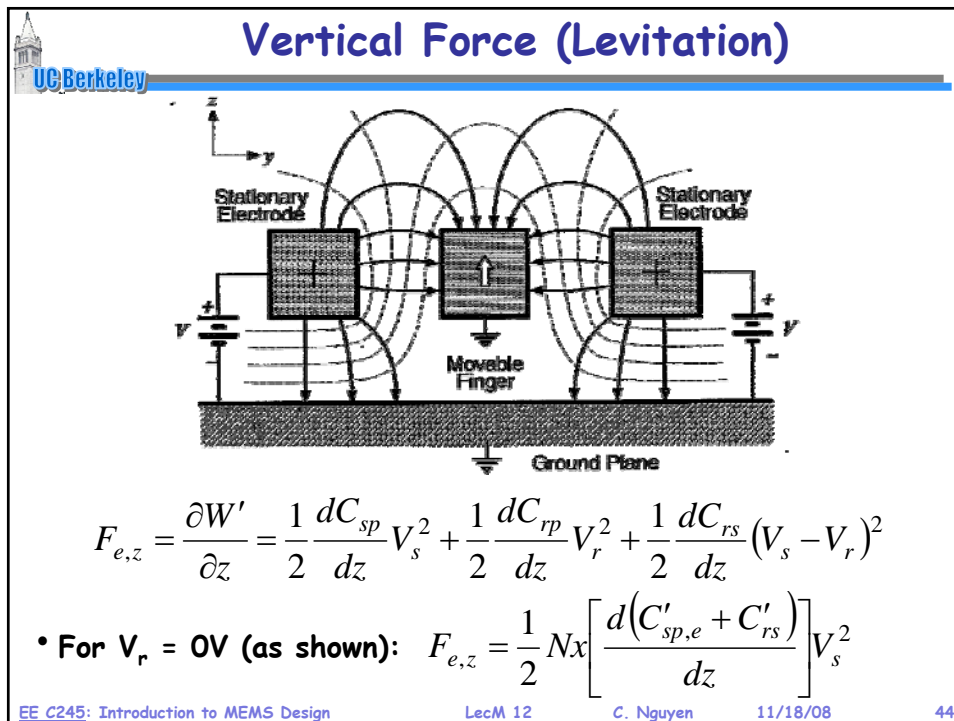
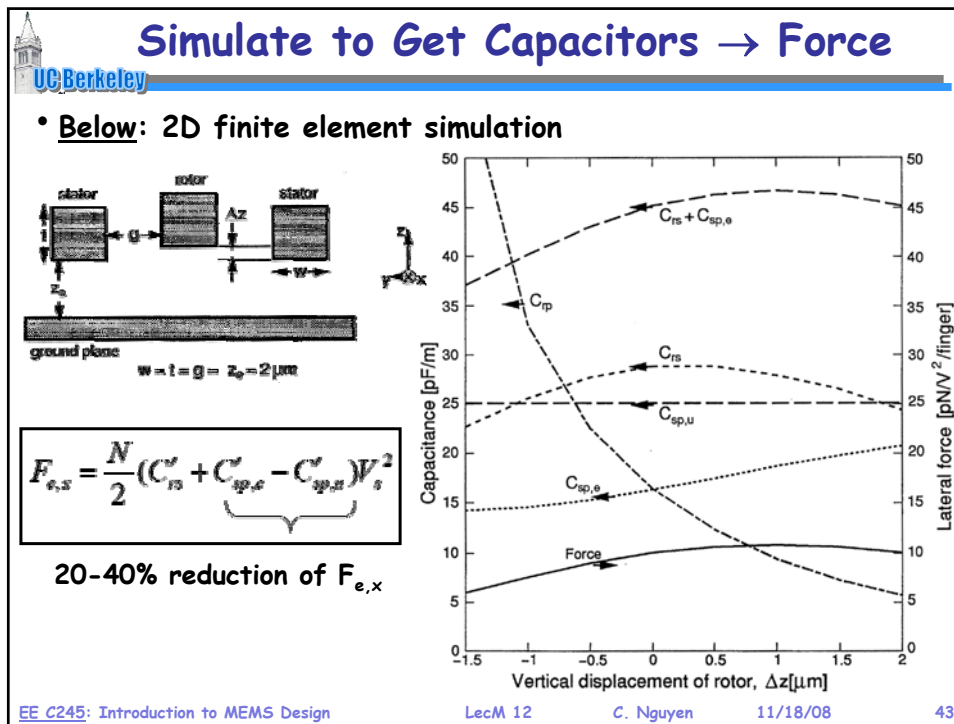
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

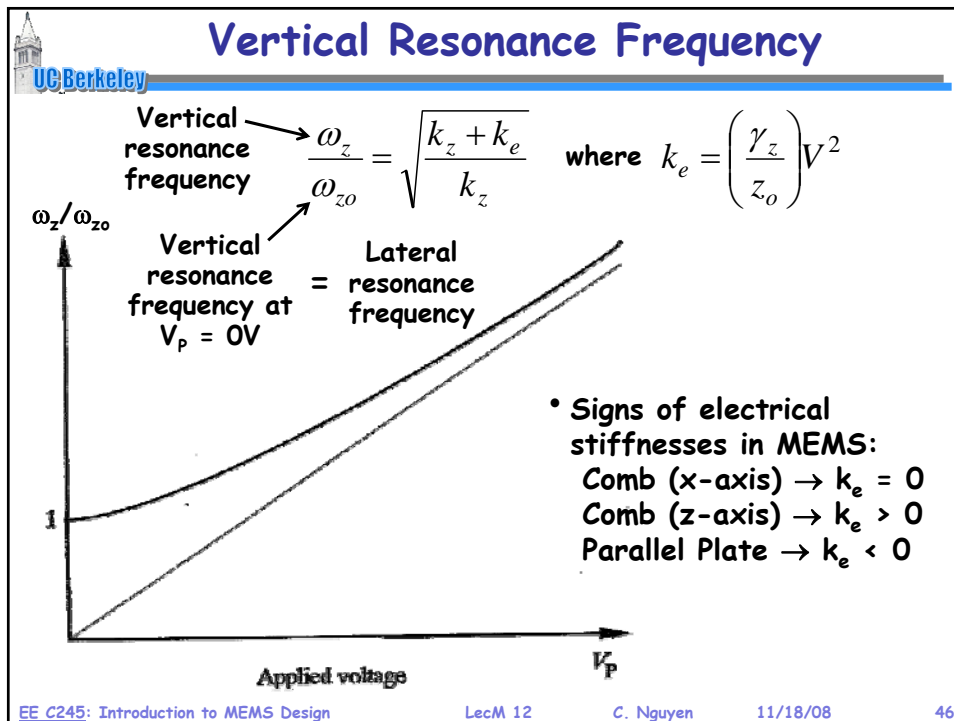
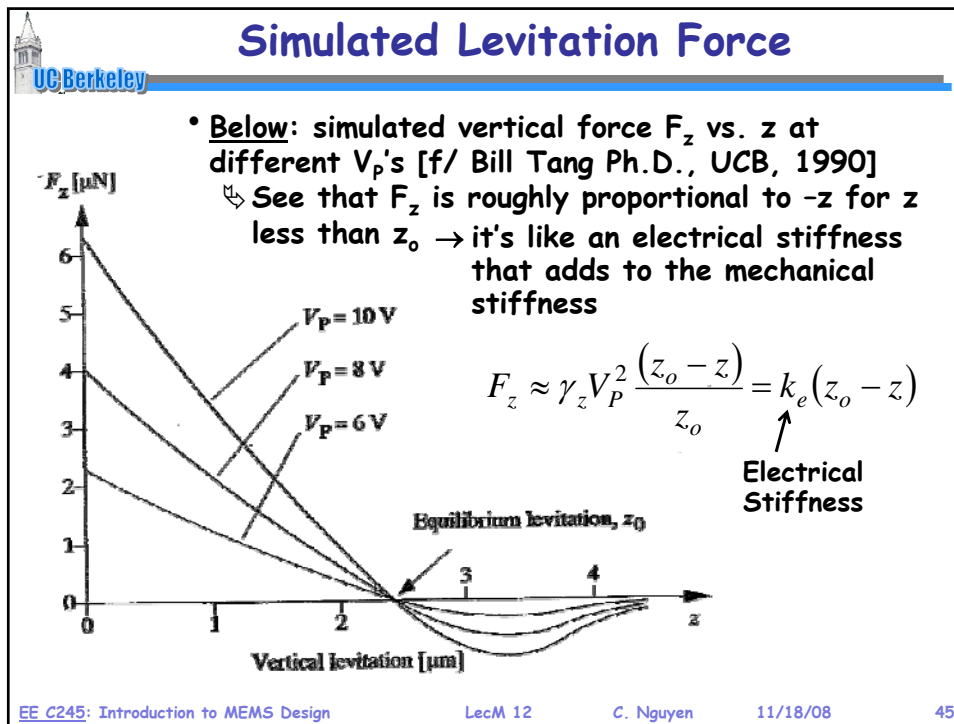
$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,e} - C'_{sp,u}) V_s^2$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 42





Suppressing Levitation

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 47

Force of Comb-Drive vs. Parallel-Plate

$\text{Gap} = d_o = 1 \mu\text{m}$
 $\text{Thickness} = h = 2 \mu\text{m}$
 $\text{Finger Length} = L_f = 100 \mu\text{m}$
 $\text{Finger Overlap} = L_d = 75 \mu\text{m}$

- **Comb drive (x-direction)**
 $\Rightarrow V_1 = V_2 = V_s = 1V$

$$F_{e,x} = \frac{1}{2} \frac{\epsilon_o h}{d_o} V_s^2$$
- **Differential Parallel-Plate (y-direction)**
 $\Rightarrow V_1 = 0V, V_2 = 1V$

$$F_{e,y} = \frac{1}{2} \frac{\epsilon_o h L_d}{d_o^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{1}{2} \frac{\epsilon_o h L_d}{d_o^2} V_2^2}{\frac{1}{2} \frac{\epsilon_o h}{d_o} V_s^2} = \frac{L_d}{d_o}$$

Parallel-plate generates a much larger force; but at the cost of linearity

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 48