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# EE C247B - ME C218

## Introduction to MEMS Design


### Spring 2014

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Lecture Module 14: Sensing Circuits

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## Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - ↳ Detection Circuits
    - ↳ Velocity Sensing
    - ↳ Position Sensing

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### Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

*Consider the mechanical device by itself first, w/ output shorted*

*Now, taking RL into account*

$$\frac{V_o}{V_i}(s) = \frac{\omega_0 Q}{R_{x12}} \cdot \frac{1}{\omega} \cdot \frac{R_L}{R_{x12} + R_L} \cdot \frac{1}{\omega} \cdot \frac{1}{R_{x12}}$$

$$\frac{V_o}{V_i}(s) = \frac{1}{R_{x12}} \cdot \frac{1}{\omega} \cdot \frac{R_L}{R_{x12} + R_L} \cdot \frac{1}{\omega} \cdot \frac{1}{R_{x12}}$$

$$\frac{V_o}{V_i}(s) = \frac{R_L}{R_{x12} + R_L} \cdot \frac{1}{\omega} \cdot \frac{1}{R_{x12}}$$

*Solve the problem @ resonance first, then multiply by H(s)*

*ω<sub>0</sub> + C<sub>x</sub> cancel @ resonance*

*ω altered*

*Q' = Q \* (R<sub>x12</sub> / (R<sub>x12</sub> + R<sub>L</sub>))*

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### Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

*Since this structure has completely symmetrical I/O ports:*

*Work @ resonance: (to simplify the analysis)*

$$\frac{V_o}{V_i} = \frac{R_0}{R_x + R_0} \text{ (@ resonance)}$$

*Then, generalize to off resonance:*

$$\frac{V_o}{V_i} = \frac{R_0}{R_x + R_0} \cdot \frac{1}{\omega} \cdot \frac{1}{R_x} \cdot \frac{1}{\omega} \cdot \frac{1}{R_x}$$

*where Q' = Q \* (R<sub>x</sub> / (R<sub>x</sub> + R<sub>0</sub>))*

*C<sub>x</sub> & L<sub>x</sub> cancel*

*Voltage Representing Velocity*

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### Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

cantilever

Since this structure has completely symmetrical I/O ports:

Brute force approach:

$$\frac{N_O}{V_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{s^2 R_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{\frac{R_D}{L_x C_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$

$$\frac{N_O}{V_i}(s) = \frac{R_D}{R_x + R_D} \frac{s(\omega_0 Q')}{s^2 + s(\omega_0/Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} (H)(s, Q')$$

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### Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

cantilever

Again, have port-to-port I/O symmetry:


Brute force approach:

$$\frac{N_O}{V_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}} = \frac{C_x/C_D}{s^2 R_x C_x + 1 + s^2 L_x C_x + \frac{C_x}{C_D}}$$

$$= \frac{C_x/C_D}{1 + C_x/C_D} \frac{1}{s^2 + s \frac{R_x}{L_x} + \frac{(1 + C_x/C_D)}{L_x C_x}}$$

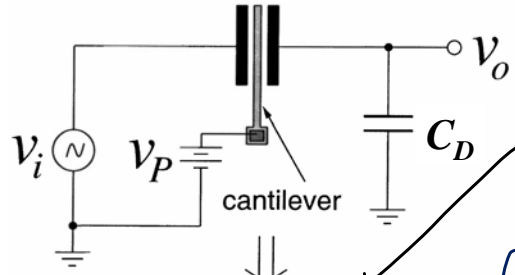
$\omega_0^2 = \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_x/C_D)$   
 $Q' = \frac{\omega_0' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_x/C_D}$

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## Position-to-Voltage Conversion

• To sense position (i.e., displacement), use a capacitive load



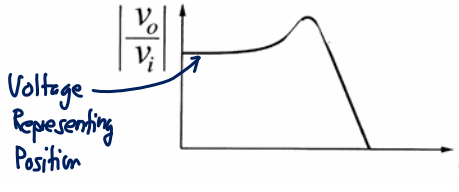
$$\frac{V_o}{V_i}(s) = \underbrace{\frac{C_x/C_D}{1 + C_x/C_D}}_{\text{DC Gain Term}} \underbrace{\frac{(\omega_0')^2}{s^2 + (\frac{\omega_0'}{Q'})s + (\omega_0')^2}}_{\text{Low-Pass Biquad}}$$

→ To maximize gain → 1, need  $C_D \ll C_x$ .  
 (must minimize  $C_D$ )


Note: Can use similar short-cut to the R case.

- ① Get DC response → C's dominate.
- ② Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \text{LP}(s, \omega_0', Q') \cdot \omega_0' Q'$$



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## Velocity Sensing Circuits

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### Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

Work @ resonance: (to simplify the analysis)

$$\frac{N_0}{N_i} = \frac{R_0}{R_x + R_0} \text{ (@ resonance)}$$

Then, generalize to off resonance:

$$\frac{N_0}{N_i} = \frac{R_0}{R_x + R_0} \cdot H(s, Q'), \text{ where } Q' = Q \left( \frac{R_x}{R_x + R_0} \right)$$

*Handwritten notes: Cx & Lx cancel*

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### Problems With Purely Resistive Sensing

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Now, we get: (approximately)

$$\frac{N_0}{N_i}(s) \sim \frac{R_0}{R_x + R_0} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_0) C_p}$     Depend on both  $R_0$  &  $C_p$ .

Impact depends on where  $\omega_p$  is relative to  $\omega_b$

Includes  $C_o$ , line C, bond pad C, and next stage C

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### Problems With Purely Resistive Sensing

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- In general, the sensor output must be connected to the inputs of further signal conditioning circuits  $\rightarrow$  input  $R_i$  of these circuits can load  $R_D$

These change w/ hook-up  $\rightarrow$  not good.

**Problem:** need a sensing circuit that is immune to parasitics or loading.

$\rightarrow$

**Soln:** use op amps.

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### The TransR Amplifier Advantage

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- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance  $C_p$  and  $R_i$
- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground  $\Rightarrow$  No voltage across  $C_p$   
 $\hookrightarrow C_p$  effectively isn't there!

$$V_0 = -\frac{R_2}{R_x} \oplus(s) i_i$$

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## Position Sensing Circuits

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## Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{1}{s} \cdot \mathcal{H}(s, \omega_0, Q) \cdot \omega_0^2 \delta^2$$

Integration yields displacement.  
 To maximize gain, minimize  $C_D$ .  
 ⇒ Problem: parasitic capacitance  
 $C_D \rightarrow C_D + C_{p_i} + C_{p_b}$   
 ⇒ DC Gain:  $\frac{C_x / (C_x C_{p_i} + C_{p_b})}{1 + C_x / (C_D + C_{p_i} + C_{p_b})}$   
 Remedy: Suppress  $C_p$  via use of op amps.

Output will get smaller!

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**The Op Amp Integrator Advantage**

• The virtual ground provided by the ideal op amp eliminates the parasitic capacitance  $C_p$

$N_o = -i_o(R_2 \parallel \frac{1}{sC_2})$   
 $\approx -\frac{N_i}{R_x} \cdot \frac{1}{sC_2} \Rightarrow \frac{N_o}{N_i}(s) = -\frac{1}{R_x C_2} \frac{1}{s}$

*well defined → good!*

*Can drive next stage's  $R_i$  w/o interference to transfer function!*

$R_2 \gg \frac{1}{sC_2}$  (for biasing)

$R_o = 0\Omega$

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**Differential Position Sensing**

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### Differential Position Sensing

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- Example: ADXL-50

Tethers with fixed ends  
 $C_1$   
 $C_2$

Proof Mass  
 Sense Finger  
 Applied Acceleration  
 Suspension Beam in Tension

Fixed Electrodes

Capacitive divider

$$V_o = -V_p + (2V_p) \frac{C_1}{C_1 + C_2}$$

$$= \frac{-V_p C_1 - V_p C_2 + 2V_p C_1}{C_1 + C_2} = V_p \left( \frac{C_1 - C_2}{C_1 + C_2} \right) = V_o$$

Issue: Parasitic Capacitance

$$V_o = \frac{C_1 - C_2}{C_1 + C_2 + C_p} V_p$$

As before,  $C_p$  reduces gain  $\rightarrow$  Soln: Use op amp!

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### Buffer-Bootstrapped Position Sensing

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+ $V_P$   
 $-V_P$

Includes capacitance from interconnects, bond pads, and  $C_{gs}$  of the op amp

Unity Gain Buffer

$C_p$

$C_{gd}$  = gate-to-drain capacitance of the input MOS transistor

Interconnect  
 Ground Plane  
 1x

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across  $C_p$
  - It's effectively not there!

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### Effect of Finite Op Amp Gain

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Total ADXL-50 Sense C ~ 100fF

Unity Gain Buffer

$V_0$

$+V_P$

$-V_P$

$N_o = A_o(V_+ - V_-) = A_o(N_i - N_o) \rightarrow N_o(1 + A_o) = A_o N_i \rightarrow \frac{N_o}{N_i} = \frac{A_o}{1 + A_o}$

Get  $Z_i = \frac{V_i}{i_i}$ :  $i_i = (N_i - N_o) s C_p = N_i (1 - \frac{A_o}{1 + A_o}) s C_p = N_i \frac{1}{1 + A_o} s C_p$

$\therefore \frac{N_o}{N_i} = Z_i = \frac{1}{s \left[ \frac{C_p}{1 + A_o} \right]}$   $C_{eff} = \frac{C_p}{1 + A_o}$

No longer zero!

Ex:  $A_o = 100, C_p = 2pF$   
 $\Rightarrow C_{eff} = \frac{2pF}{101} = 20fF$   
 Not negligible compared w/ ADXL-50  $C_{tot} \sim 100fF!$

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### Integrator-Based Diff. Position Sensing

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$+V_P$

$-V_P$

$i_1$

$i_2$

$i_o$

$C_p$

$R_2$

$C_F$

$R_2 \gg \frac{1}{sC_2}$  (for biasing)

$R_o = 0\Omega$

$V_0$

Can drive next stage's  $R_1$  w/o interference to transfer function!

$i_o = i_1 + i_2 = N_p(sC_1) - N_p(sC_2) = N_p s(C_1 - C_2)$

$\therefore N_o = -i_o \left( \frac{1}{sC_F} \right) = -N_p \left( \frac{C_1 - C_2}{C_F} \right)$

$\frac{N_o}{V_p} = -\frac{C_1 - C_2}{C_F}$   $\Rightarrow$  A seemingly perfect differential sensor/amplifier output! ... but only when the op amp is ideal...

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