# EE C247B – ME C218
# Introduction to MEMS Design
# Spring 2014

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Lecture Module 14: Sensing Circuits



## Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing

## Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

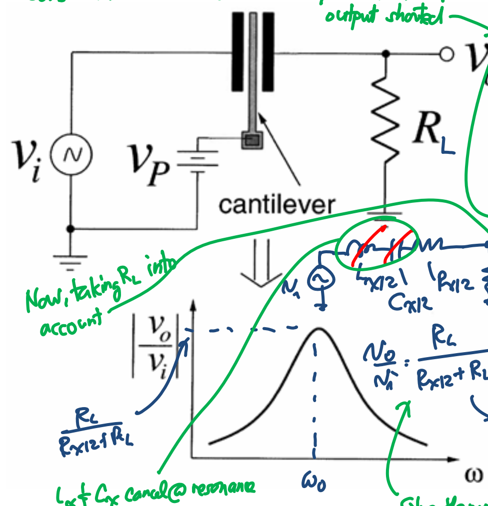
Consider the mechanical device by itself first, w/ output shorted

$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_0 Q}{k} \Theta(s)$$

$$[F_{d1} = \eta_{e1} v_i] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_0 Q}{k}\Theta(s)$$

$$[i_0 = \eta_{e2}\dot{x}] \Rightarrow \frac{i_0}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_0 Q}{k}\Theta(s) = \frac{\eta_{e1}\eta_{e2}Q}{m\omega_0}\Theta(s)$$

$$\frac{1}{R_{x12}}$$




Now, taking $R_L$ into account

$$\frac{v_o}{v_i} = \frac{R_L}{R_{x12}+R_L} \qquad \frac{i_o}{v_i}(s) = \frac{1}{R_{x12}}\Theta(s)$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_L}{R_{x12}+R_L}\Theta(s)} \qquad Q' = Q\left(\frac{R_{x12}}{R_{x12}+R_L}\right)$$

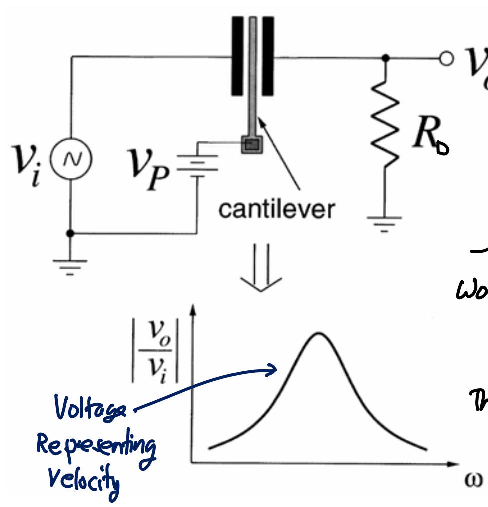
w/ altered Q

Solve the problem @ resonance first, then multiply by $\Theta(s)$



## Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load




Since this structure has completely symmetrical I/O ports:

$$R_x = \frac{b}{\eta_e^2}, \quad C_x = \frac{\eta_e^2}{k}, \quad L_x = \frac{m}{\eta_e^2}$$

$C_x$ & $L_x$ cancel

Work @ resonance: (to simplify the analysis)

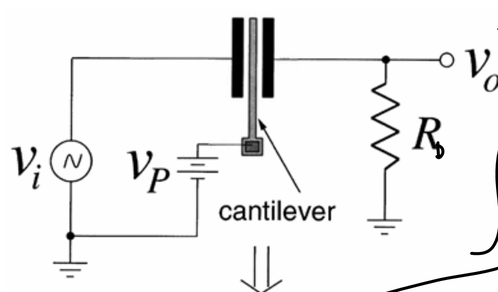
$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \quad (@\ \text{resonance})$$

Then, generalize to off resonance:

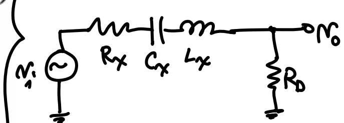
$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}\Theta(s,Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

## Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load




Since this structure has completely symmetrical I/O ports:




$$\left[Q = \frac{\omega_0 L_x}{R_x} \to Q' = \frac{\omega_0 L_x}{R_x+R_D} \to \frac{R_x+R_D}{L_x} = \frac{\omega_0}{Q'}\right]$$

$$\left[\frac{1}{L_x C_x} = \omega_0^2\right]$$

Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_DC_x}{sR_xC_x + 1 + s^2L_xC_x + sR_DC_x} = \frac{s\frac{R_D}{L_x}}{s^2 + s\frac{R_x+R_D}{L_x} + \frac{1}{L_xC_x}}$$

$$= \frac{R_D}{R_x+R_D}\,\frac{s\left(\frac{R_x+R_D}{L_x}\right)}{s^2+s\left(\frac{R_x+R_D}{L_x}\right)+\frac{1}{L_xC_x}} \Rightarrow \frac{v_o}{v_i}(s) = \frac{R_D}{R_x+R_D}\,\frac{s(\omega_0/Q')}{s^2+s(\omega_0/Q')+\omega_0^2} = \frac{R_D}{R_x+R_D}\Theta(s,Q')$$

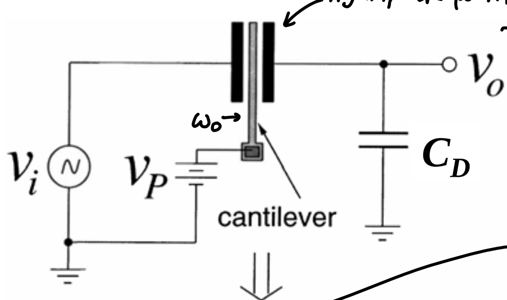
$$Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

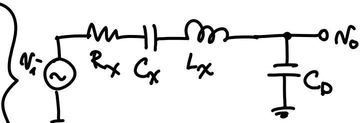


## Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:




Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\frac{v_o}{v_i}(s) = \frac{\frac{sC_x}{sC_D}}{sR_xC_x + 1 + s^2L_xC_x + \frac{sC_x}{sC_D}} = \frac{C_x/C_D}{1+C_x/C_D}\,\frac{1}{1 + s\frac{R_xC_x}{1+C_x/C_D} + s^2\frac{L_xC_x}{1+C_x/C_D}}$$

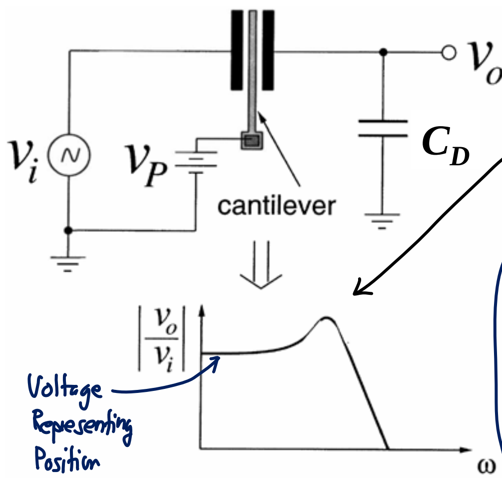
$$= \frac{C_x/C_D}{1+C_x/C_D}\,\frac{\frac{1+C_x/C_D}{L_xC_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1+C_x/C_D)}{L_xC_x}}$$

$$\left[\omega_0^2 = \frac{1}{L_xC_x} \to (\omega_0')^2 = \omega_0^2(1+C_x/C_D)\right]$$

$$\left[Q' = \frac{\omega_0' L_x}{R_x} \to \frac{R_x}{L_x} = \frac{\omega_0'}{Q'},\ Q' = Q\sqrt{1+C_x/C_D}\right]$$ over

## Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load




$$\frac{v_o}{v_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}$$

DC Gain Term — Low-Pass Biquad

To maximize gain → 1, need $C_D \ll C_x$. (must minimize $C_D$)

Note: Can use similar short-cut to the R case.

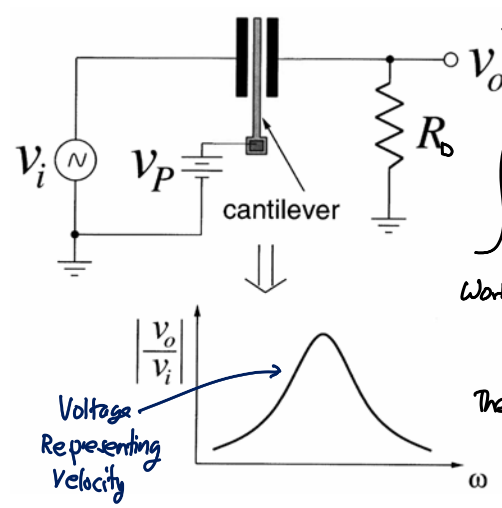
1. Get DC response → C's dominate.
2. Then: $\frac{v_o}{v_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, \omega_o', Q') \cdot \omega_o' Q'$



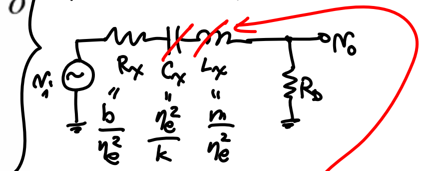
## Velocity Sensing Circuits

## Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load




Since this structure has completely symmetrical I/O ports:




Work @ resonance: (to simplify the analysis) → $C_x$ & $L_x$ cancel

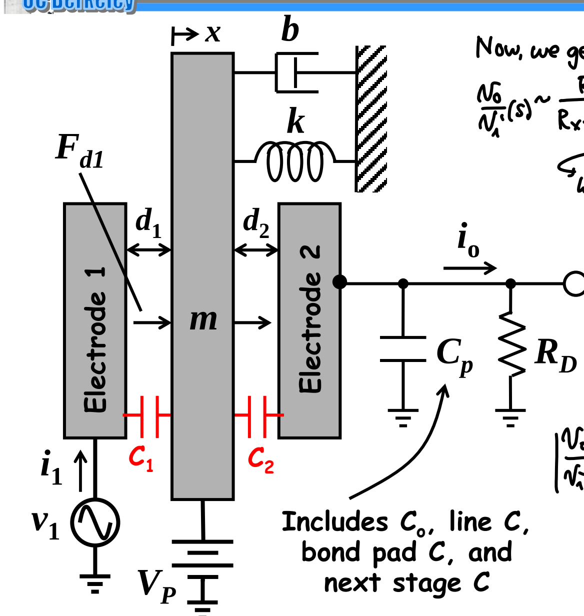
$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D} \quad \text{(@ resonance)}$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x + R_D} \, \text{Ⓗ}(s, Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x + R_D}\right)$$



## Problems With Purely Resistive Sensing




Includes $C_o$, line C, bond pad C, and next stage C

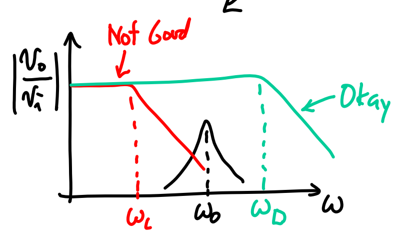
Now, we get: (approximately)

$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot \text{Ⓗ}(s, \omega_o', Q')$$

$\omega_o', Q'$: Depend on both $R_D$ & $C_p$.

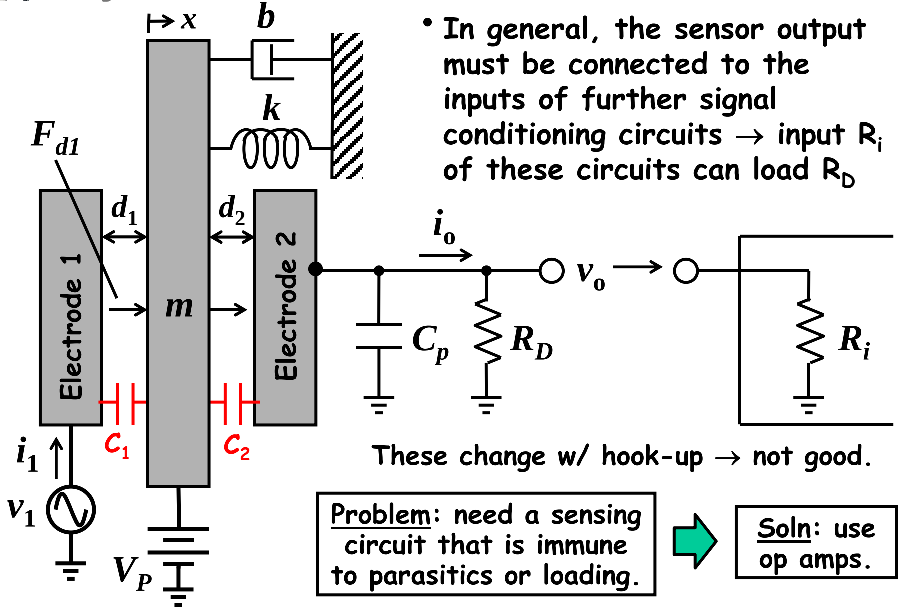
$$\omega_p = \frac{1}{(R_x \| R_D) C_p}$$

Impact depends on where $\omega_p$ is relative to $\omega_o$



## Problems With Purely Resistive Sensing

- In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input $R_i$ of these circuits can load $R_D$



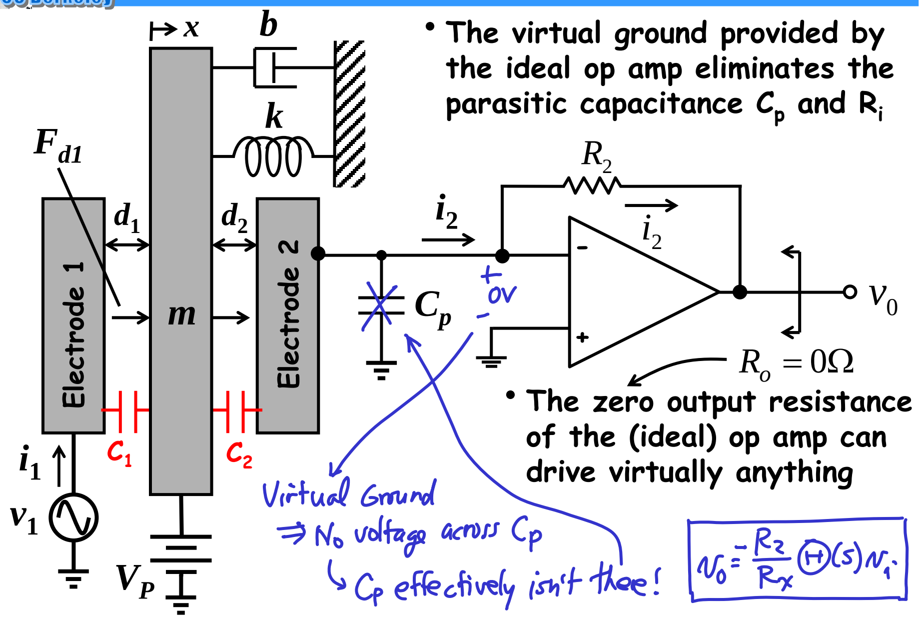

These change w/ hook-up → not good.

**Problem**: need a sensing circuit that is immune to parasitics or loading. → **Soln**: use op amps.



## The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$ and $R_i$



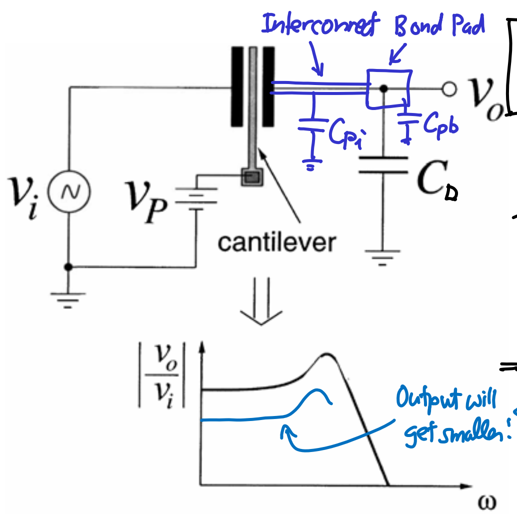

- The zero output resistance of the (ideal) op amp can drive virtually anything

## Position Sensing Circuits



## Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load




$$\frac{v_o}{v_i}(s) = \frac{C_x/C_D}{1+C_x/C_D} \cdot \frac{1}{s} \cdot \Theta(s, \omega_o', Q') \cdot \omega_o' Q'$$

Integration yields displacement.

To maximize gain, minimize $C_D$.

⇒ Problem: parasitic capacitance

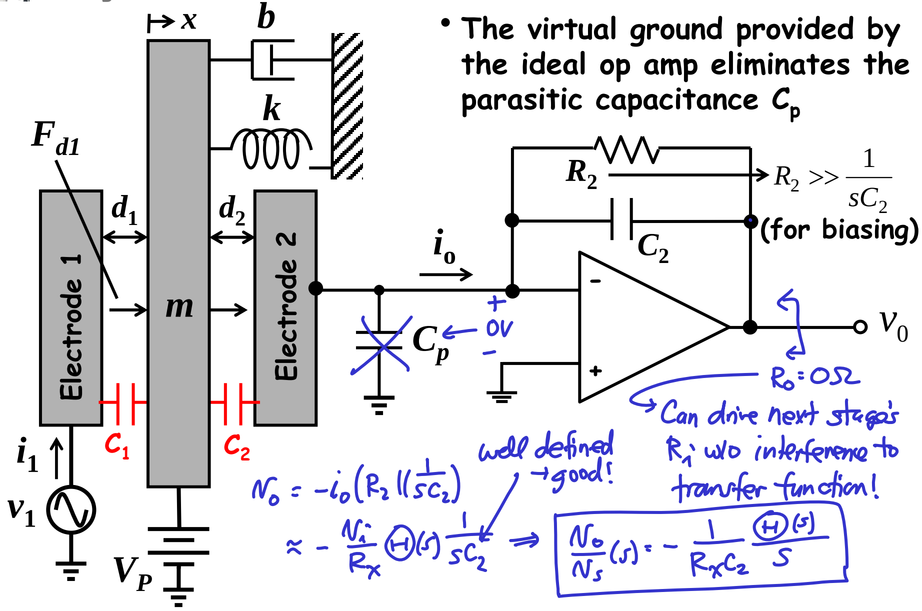
$C_D \rightarrow C_D + C_{pi} + C_{pb}$

⇒ DC Gain: $\dfrac{C_x/(C_D+C_{pi}+C_{pb})}{1+C_x/(C_D+C_{pi}+C_{pb})}$

Remedy: Suppress $C_p$ via use of op amps.

## The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance $C_p$

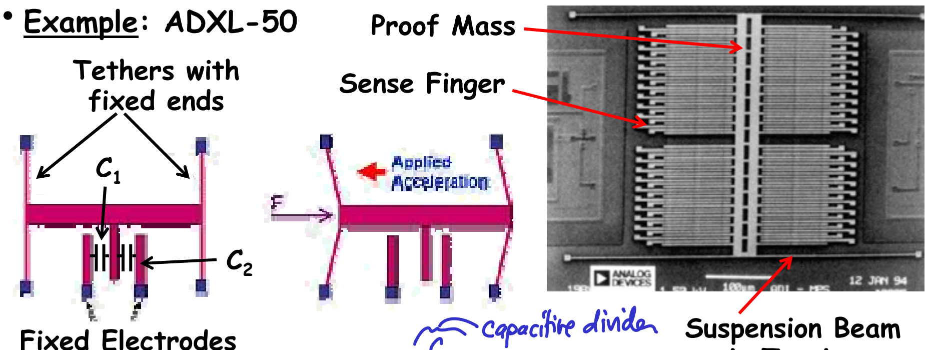


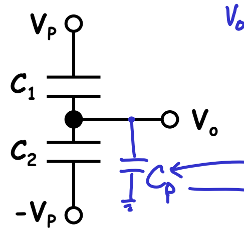



## Differential Position Sensing

## Differential Position Sensing

- Example: ADXL-50




$$V_o = -V_P + (2V_P)\frac{C_1}{C_1+C_2} \quad \text{capacitive divider}$$

$$= \frac{-V_PC_1 - V_PC_2 + 2V_PC_1}{C_1+C_2} = \boxed{V_P\left(\frac{C_1-C_2}{C_1+C_2}\right) = V_o}$$

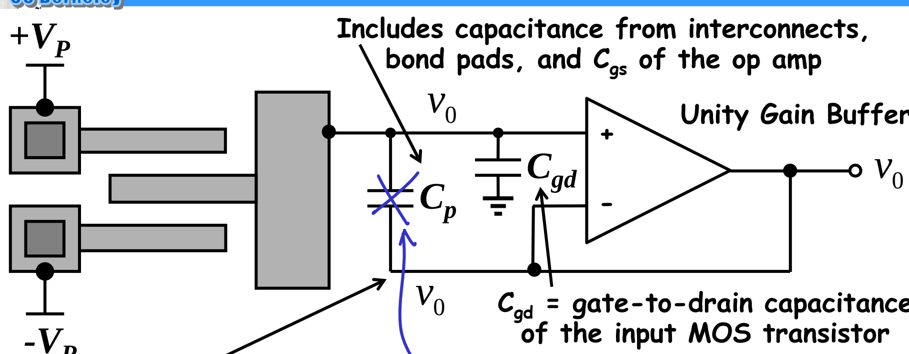
Issue: Parasitic Capacitance

$$\boxed{V_o = \frac{C_1-C_2}{C_1+C_2+C_p}V_P}$$

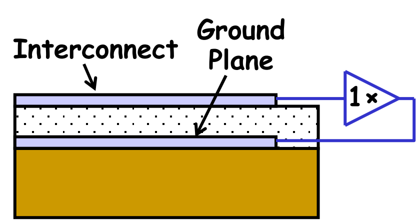
As before, $C_p$ reduces gain → Soln: use op amp!



## Buffer-Bootstrapped Position Sensing



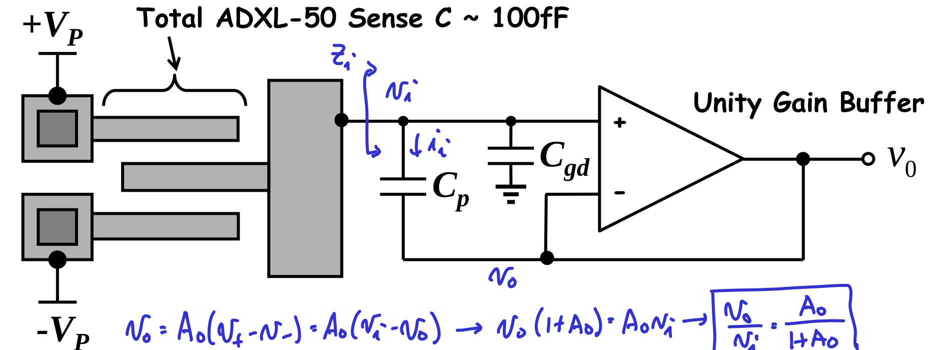

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across $C_p$
  - It's effectively not there!





## Effect of Finite Op Amp Gain




$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\frac{v_o}{v_i} = \frac{A_o}{1+A_o}}$

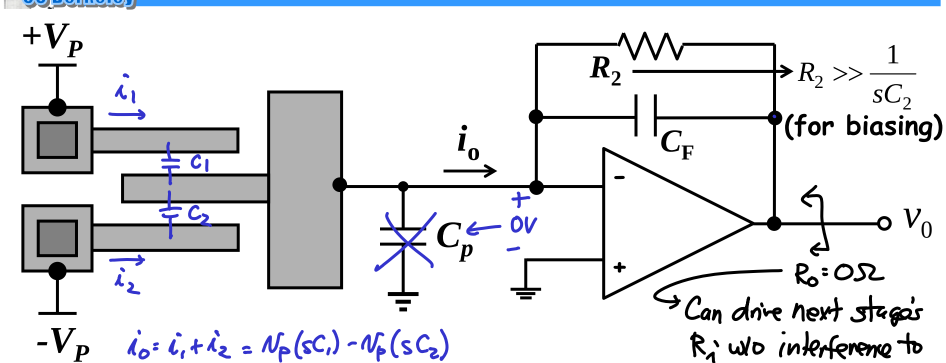
Get $Z_i = \frac{v_i}{i_i}$: $\quad i_i = (v_i - v_o)sC_p = v_i\left(1 - \frac{A_o}{1+A_o}\right)sC_p = v_i \frac{1}{1+A_o} sC_p$

$\therefore \frac{v_i}{i_i} = z_i = \frac{1}{s\left[\frac{C_p}{1+A_o}\right]} \rightarrow \boxed{C_{eff} = \frac{C_p}{1+A_o}}$ No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$
$\Rightarrow C_{eff} = \frac{2p}{101} = 20fF$
Not negligible compared w/ ADXL-50 $C_{tot} \sim 100fF$!



## Integrator-Based Diff. Position Sensing




Can drive next stage's $R_i$ w/o interference to transfer function!

$i_o = i_1 + i_2 = v_P(sC_1) - v_P(sC_2)$
$= v_P s(C_1 - C_2)$

$\therefore v_o = -i_o\left(\frac{1}{sC_F}\right) = -v_P\left(\frac{C_1 - C_2}{C_F}\right)$

$\boxed{\frac{v_o}{v_P} = -\frac{C_1 - C_2}{C_F}}$ $\Rightarrow$ A seemingly perfect differential sensor/amplifier output! ...but only when the op amp is ideal...