

EE C247B - ME C218

Introduction to MEMS Design


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Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 15: Gyros, Noise, & MDS

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Lecture Outline

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
 - ↗ Gyroscopes
 - ↗ Gyro Circuit Modeling
 - ↗ Minimum Detectable Signal (MDS)
 - Noise
 - Angle Random Walk (ARW)

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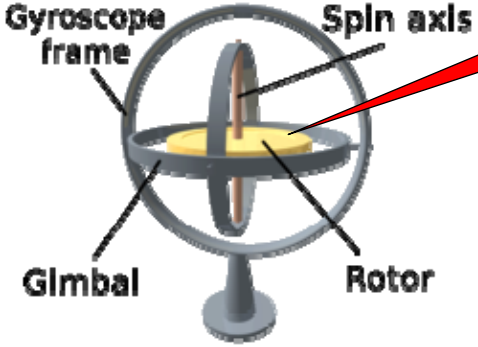
Gyroscopes

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Classic Spinning Gyroscope

- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope



Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbal chassis



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Vibratory Gyroscopes

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- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- **Example:** vibrating mass in a rotating frame

Mass at rest
 Driven into vibration along the y-axis
 y-displaced mass
 Capacitance between mass and frame = constant
 Rotate 30°
 Get an x' component of motion
 $C(t_2) > C(t_1)$

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Basic Vibratory Gyroscope Operation

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Principle of Operation

- Tuning Fork Gyroscope:

Input Rotation $\vec{\Omega}$
 Driven Vibration @ f_0
 Coriolis (Sense) Response
 Coriolis Torque
 Detect motion out-of-the plane of the tuning fork as rotation!
 Side View:
 not force on support
 support = 0
 very little anchor dissipation
 Top View:
 Post

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Basic Vibratory Gyroscope Operation

Principle of Operation

- Tuning Fork Gyroscope:

The diagram shows a tuning fork structure on a yellow base. A coordinate system (x, y, z) is centered at the base. **Input Rotation** is indicated by a curved arrow around the z-axis, labeled $\vec{\Omega}$. **Driven Vibration** is shown as a blue arrow along the y-axis, labeled \vec{v} at frequency f_o . **Coriolis (Sense) Response** is shown as a green arrow along the x-axis, labeled \vec{a}_c . **Coriolis Torque** is shown as a curved arrow around the y-axis. A green arrow labeled "sense direction" points from the response towards the diagram. A red arrow labeled "drive direction" points from the vibration towards the diagram. A red arrow labeled "same frequency" points from the vibration to the response.

Drive/Sense Response Spectra:

The graph plots **Amplitude** on the y-axis against **frequency ω** on the x-axis. Two curves are shown: a blue curve for **Drive Response** and a black curve for **Sense Response**. Both curves peak at the same frequency, f_o (labeled as T_1), indicated by a vertical dashed line and a red dot. A green arrow points to this frequency on the x-axis.

$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$ (Coriolis Acceleration)

 $\vec{F}_c = m\vec{a}_c$ (Coriolis Force)

 $\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$ (Coriolis Displacement)

Parameters: **Driven Velocity**, **Rotation Rate**, **Beam Mass**, **Beam Stiffness (in sense direction)**, **Sense Frequency**.

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Vibratory Gyroscope Performance

Principle of Operation

- Tuning Fork Gyroscope:

The diagram is identical to the one in slide 7, showing the tuning fork structure, input rotation $\vec{\Omega}$, driven vibration \vec{v} , and Coriolis response \vec{a}_c .

$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$

 $\vec{a}_c = 2\vec{v} \times \vec{\Omega}$

Parameters: **Beam Mass**, **Beam Stiffness**, **Sense Frequency**, **Driven Velocity**.

- To maximize the output signal x , need:
 - ↪ Large sense-axis mass
 - ↪ Small sense-axis stiffness
 - ↪ (Above together mean low resonance frequency)
 - ↪ Large drive amplitude for large driven velocity (so use comb-drive)
 - ↪ If can match drive freq. to sense freq., then can amplify output by Q times

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MEMS-Based Gyroscopes

The image shows three different MEMS gyroscope designs. On the left, a Tuning Fork Gyroscope (Ayazi, GA Tech.) is shown with labels for 'Central Post' and 'Proof Mass'. Below it is another Tuning Fork Gyroscope (Draper Labs.). On the right, a Vibrating Ring Gyroscope (Michigan) is shown as a circular ring with radial spokes. Below that is a Nuclear Magnetic Resonance Gyro (NIST) with labels for 'Laser', 'Polarizer', 'Rb/Xe Cell', 'Photodiode', and an angle θt . Dimensions of 3.2 mm, 1 mm, and 1 mm are indicated for the NIST gyro.

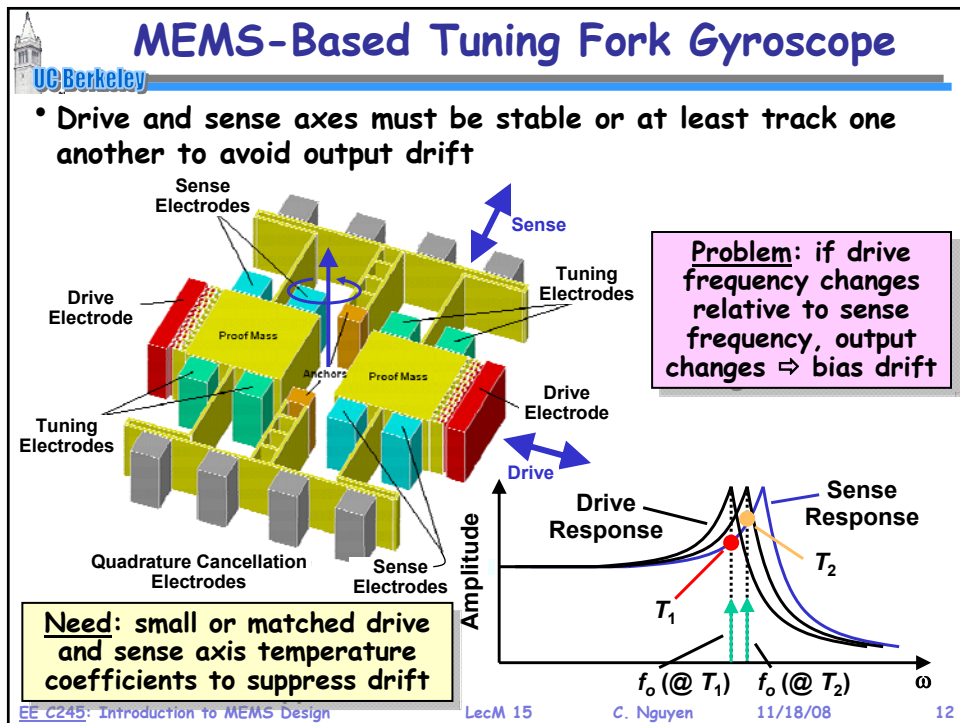
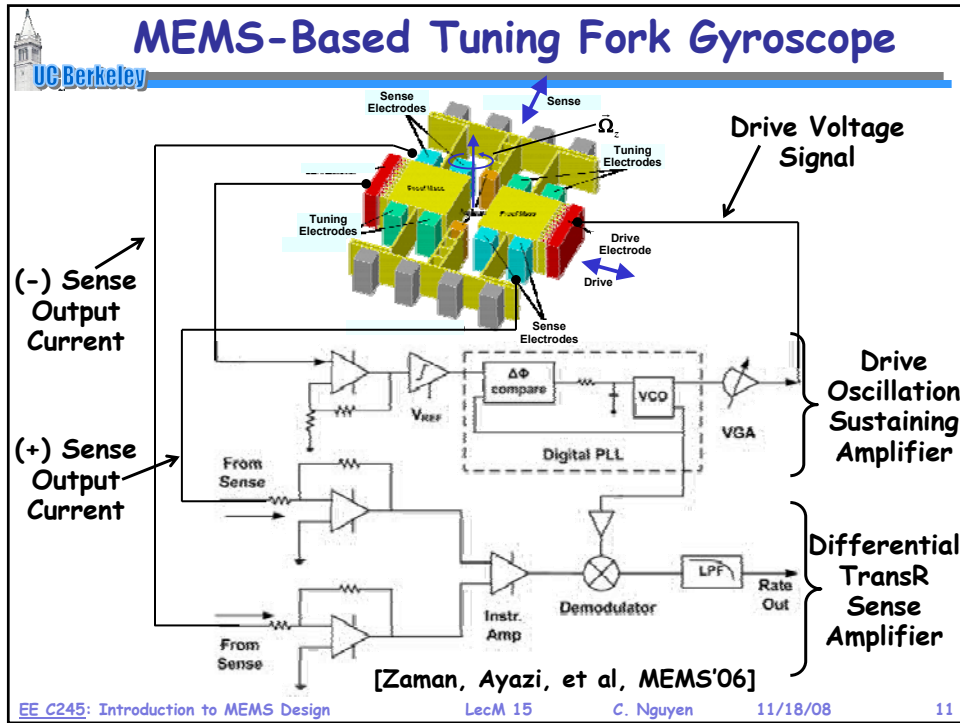
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MEMS-Based Tuning Fork Gyroscope

The diagram illustrates the structure and operation of a MEMS-based tuning fork gyroscope. It shows two proof masses connected by anchors, with various electrodes (Drive, Tuning, Sense, and Quadrature Cancellation) positioned around them. Blue arrows indicate the 'Drive Mode' (in-plane motion) and red arrows indicate the 'Sense Mode' (z-axis rotation).

- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]

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Mode Matching for Higher Resolution

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- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification

Problem: mismatch between drive and sense frequencies \Rightarrow even larger drift!

Need: small or matched drive and sense axis temperature coefficients to make this work

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Issue: Zero Rate Bias Error

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- Imbalances in the system can lead to zero rate bias error

Drive imbalance \Rightarrow off-axis motion of the proof mass

Output signal in phase with the Coriolis acceleration

Mass imbalance \Rightarrow off-axis motion of the proof mass

Quadrature output signal that can be confused with the Coriolis acceleration

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Nuclear Magnetic Res. Gyroscope

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- The ultimate in miniaturized spinning gyroscopes?
 - ↳ from CSAC, we may now have the technology to do this

-20°
0°
20°

Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier ⇒ less susceptible to B field

-20°
0°
20°

Atoms Aligned Nuclear Spins

Soln: Spin polarize Xe¹²⁹ nuclei by first polarizing e- of Rb⁸⁷ (a la CSAC), then allowing spin exchange

Challenge: suppressing the effects of B field

Laser
Polarizer
Rb/Xe Cell
Photodiode
3.2 mm
1 mm
1 mm
 $\dot{\theta}$

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MEMS-Based Tuning Fork Gyroscope

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[Zaman, Ayazi, et al, MEMS'06]

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Determining Sensor Resolution

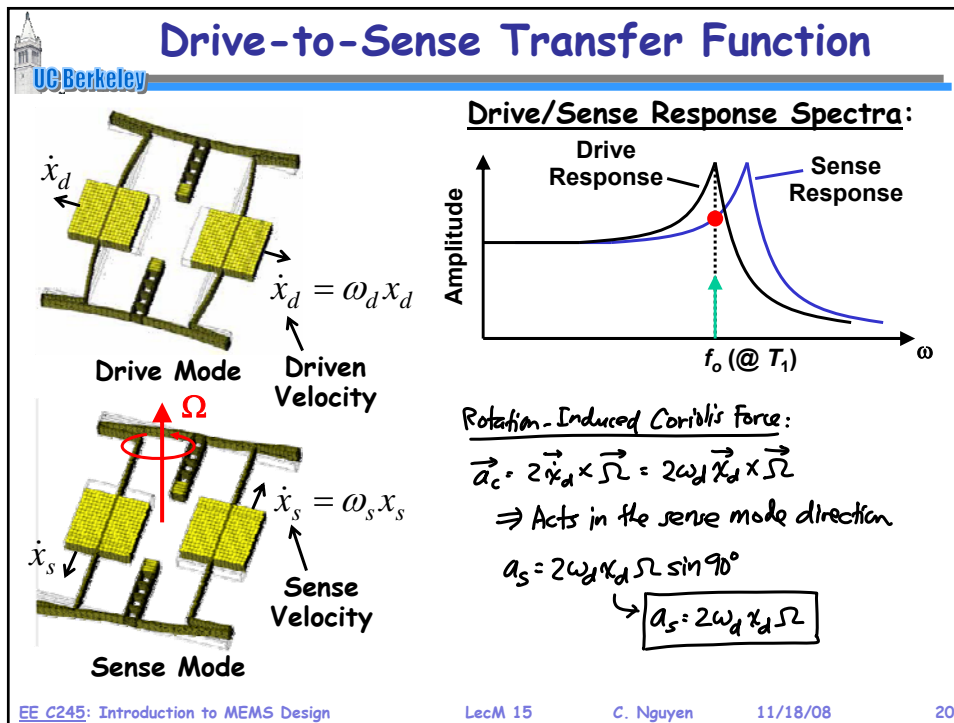
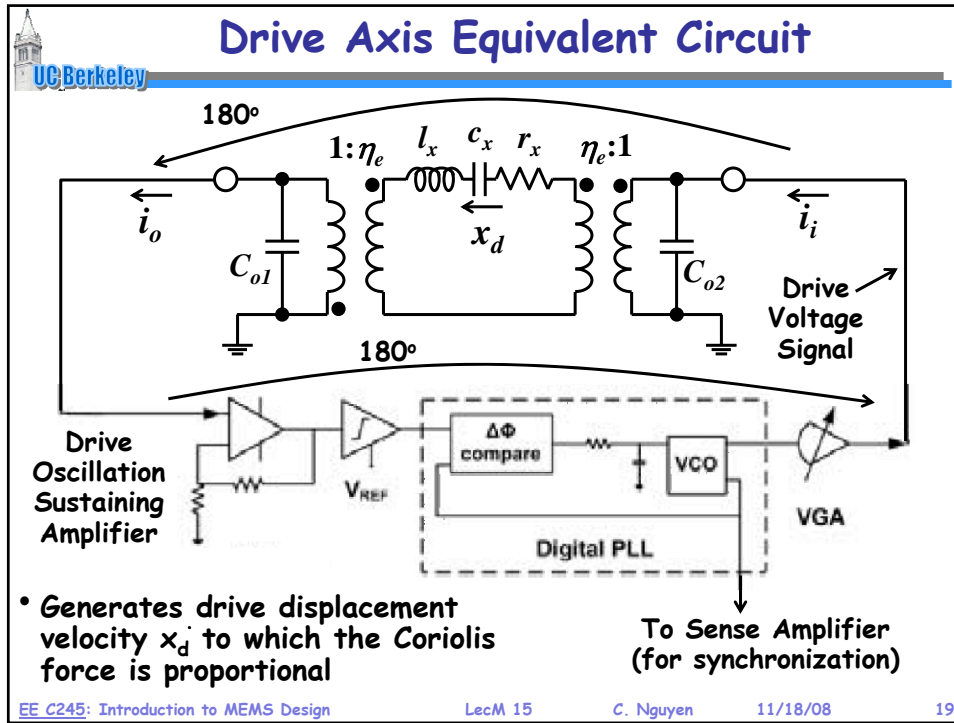
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MEMS-Based Tuning Fork Gyroscope

[Zaman, Ayazi, et al, MEMS'06]

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Gyro Readout Equivalent Circuit (for a single tine)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

Noise Sources

Gyro Sense Element Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

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Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal (MDS):** Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensor

Signal Conditioning Circuit

Output
Includes desired output plus noise

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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Move Noise Sources to a Common Point

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- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

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Gyro Readout Equivalent Circuit (for a single tone)

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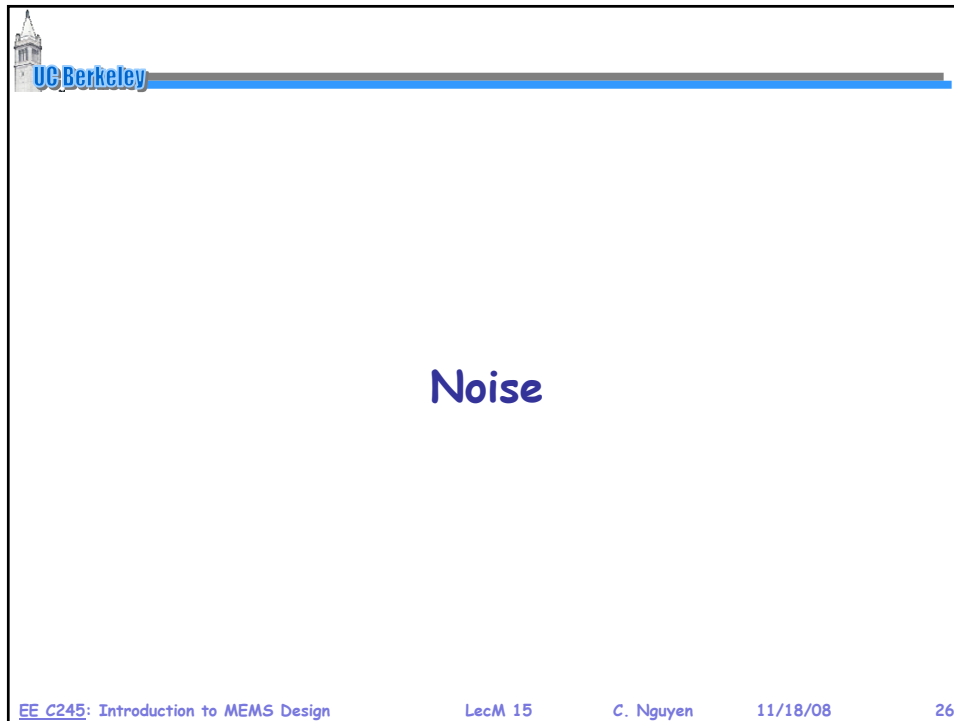
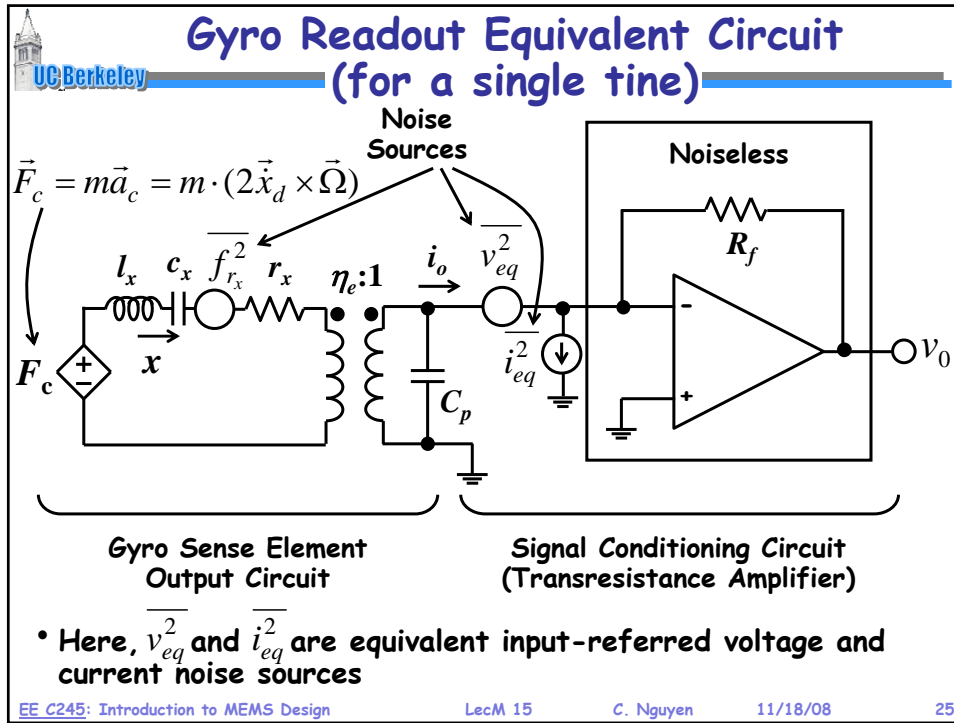
Noise Sources


$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

Gyro Sense Element Output Circuit Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

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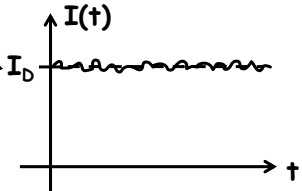




Noise

- **Noise:** Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value

Avg. value
(e.g. could be
DC current)




- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

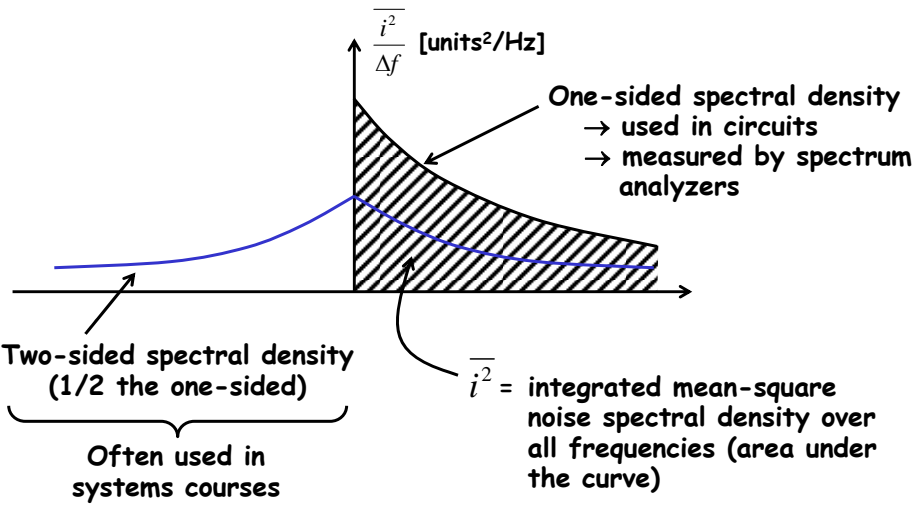
Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |I - I_D|^2 dt$

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Noise Spectral Density

- We can plot the spectral density of this mean-square value:



One-sided spectral density
 → used in circuits
 → measured by spectrum analyzers

Two-sided spectral density
 (1/2 the one-sided)
 Often used in systems courses

$\overline{i^2} =$ integrated mean-square noise spectral density over all frequencies (area under the curve)

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Circuit Noise Calculations

The diagram shows a block labeled "Linear Time-Invariant System" with transfer function $H(j\omega)$. It has two input paths: a "Deterministic" path with input $v_i(j\omega)$ and a "Random" path with input $S_i(\omega)$. The outputs are $v_o(j\omega)$ (deterministic) and $S_o(\omega)$ (random). To the right, two pairs of plots show the time-domain and frequency-domain representations: a sinusoidal wave $v_o(t)$ with period $2\pi/\omega_o$ and its spectral density $v_o(j\omega)$ with a peak at ω_o ; and a noisy signal $S_o(t)$ and its mean square spectral density $S_o(j\omega)$ with a peak at ω_o .

No j → noise has random phase, so j is pointless!

- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$
 $\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$ → How is it we can do this?
 Root mean square amplitudes

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Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

The diagram shows a noise spectral density plot $\frac{v_{n1}^2}{\Delta f} = S_1(f)$ with a narrow bandwidth B centered at ω_o . This is approximated by a sinusoidal voltage generator $v_o(t) = |A| \cos \omega_o t$ with amplitude $|A|$ and period $\tau \sim 1/B$. A block diagram shows the noise $S_n(j\omega)$ passing through a filter with gain $|S_o/S_i|$ and bandwidth B to produce the sinusoidal output.

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

$\tau \sim \frac{1}{B}$

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period $1/B$.

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Systematic Noise Calculation Procedure

General Circuit With Several Noise Sources

- Assume noise sources are uncorrelated
- 1. For $\overline{i_{n1}^2}$ replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}} \cdot (1 \text{ Hz})$$

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Systematic Noise Calculation Procedure

- Calculate $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$ (treating it like a deterministic signal)
- Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
- Repeat for each noise source: $\overline{i_{n1}^2}, \overline{v_{n2}^2}, \overline{v_{n3}^2}$
- Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$$

↑
Total rms value

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Determining Sensor Resolution

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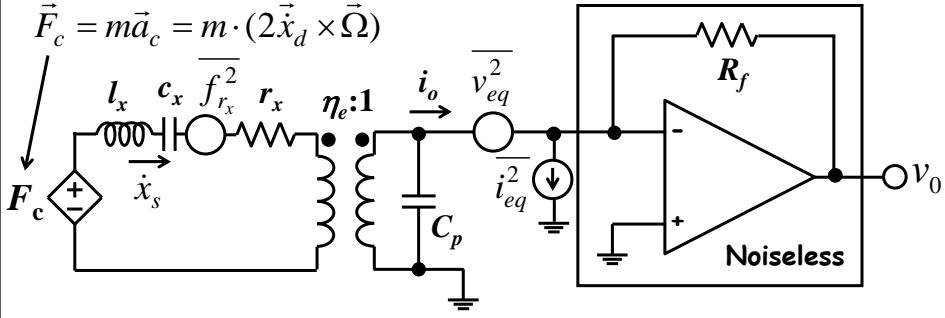
Example: Gyro MDS Calculation

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

- The gyro sense presents a large effective source impedance
 - ↳ Currents are the important variable; voltages are "opened" out
 - ↳ Must compare i_o with the total current noise i_{eqTOT} going into the amplifier circuit

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Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$

- First, find the rotation to i_o transfer function:


$$\dot{x}_s = \frac{\omega_s Q}{k_s} \Theta_s(j\omega_d) F_s = \frac{\omega_s Q}{k_s} \cdot 2\omega_d \kappa_d \Omega m \cdot \Theta(j\omega_d)$$

$$[F_s = F_c = 2\omega_d \kappa_d \Omega m] \quad \downarrow \quad \frac{1}{\omega_s^2}$$

$$\dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \Theta(j\omega_d) \cdot \Omega$$

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Example: Gyro MDS Calculation (cont)



$$i_o = \eta_e \dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \eta_e \Theta(j\omega_d) \cdot \Omega \rightarrow i_o = A \Omega$$

$A \triangleq \text{scale factor}$

Where $A = 2 \frac{\omega_d}{\omega_s} Q \kappa_d \eta_e \Theta(j\omega_d)$

When $\Omega = \Omega_{\min} \triangleq \text{MDS}$, $i_o = i_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/√Hz

$$\therefore i_{eqTOT} = A \Omega_{\min} \rightarrow \Omega_{\min} = \frac{i_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) \left[\left(\frac{\%}{hr} \right) / \sqrt{Hz} \right]$$

Angle Random Walk: $ARW = \frac{1}{60} \Omega_{\min} \left[\frac{\circ}{\sqrt{hr}} \right]$

↪ Easier to determine directional error as a function of elapsed time.

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Example: Gyro MDS Calculation (cont)

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$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

i_{eq}^2

v_{eq}^2

i_{eq}^2

$R_s: \text{large} \therefore N_{eq}^2 \text{ "opened" out}$

• Now, find the i_{eqTOT} entering the amplifier input:

$$i_{eqTOT}^2 = i_s^2 + i_{eq}^2 \rightarrow i_{eqTOT}^2 = i_s^2 + i_f^2 + i_{ia}^2 + \frac{N_{ia}^2}{R_f^2}$$

$\frac{f_{rx}^2}{\Delta f} = 4kTR_x$
 ↗ Brownian motion noise of the sense element → determined entirely by the noise in $r_x \rightarrow f_{rx}^2$
 ↘ easiest to convert to an all electrical equiv. ckt.

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Example: Gyro MDS Calculation (cont)

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To Amplifier Input

i_{eqTOT}^2

Where $L_x = \frac{R_x}{\eta_e^2}$, $C_x = \eta_e^2 C_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore i_s^2 = N_{R_x} \left(\frac{1}{R_x} \right) |H(j\omega_d)|^2 \rightarrow \frac{i_s^2}{\Delta f} = 4kTR_x \left(\frac{1}{R_x^2} \right) |H(j\omega_d)|^2$$

$$\Rightarrow \frac{i_s^2}{\Delta f} = \frac{4kT}{R_x} |H(j\omega_d)|^2$$

Thus:

$$\frac{i_{eqTOT}^2}{\Delta f} = \frac{4kT}{R_x} |H(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{\Delta f} \left(\frac{1}{R_f^2} \right)$$

Learn to get there from EE240.
 ↘ or just get them from a data sheet ...

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LF356 Op Amp Data Sheet

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LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers

General Description
 These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Common Features

- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits
- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01 \text{ pA}/\sqrt{\text{Hz}}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers

Uncommon Features

	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 ($A_v=5$)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/ μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/ $\sqrt{\text{Hz}}$

Handwritten notes:
 $\sqrt{\frac{0.2 \text{ pA}}{\Delta f}} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
 $\sqrt{\frac{12 \text{ nV}}{\Delta f}} = 12 \text{ nV}/\sqrt{\text{Hz}}$

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Example ARW Calculation

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Example Design:

Sensor Element:

- $m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg}/\text{m}^3) = 4.6 \times 10^{-10} \text{ kg}$
- $\omega_s = 2\pi(15\text{kHz})$
- $\omega_d = 2\pi(10\text{kHz})$
- $k_s = \omega_s^2 m = 4.09 \text{ N/m}$
- $x_d = 20 \mu\text{m}$
- $Q_s = 50,000$
- $V_p = 5\text{V}$
- $h = 20 \mu\text{m}$
- $d = 1 \mu\text{m}$

Sensing Circuitry:

- $R_f = 100\text{k}\Omega$
- $i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
- $v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$

The diagram shows a 3D perspective of a MEMS gyroscope. It features a central proof mass (yellow) with sense electrodes (red) and tuning electrodes (green) around it. Drive electrodes (blue) are also visible. The diagram is annotated with labels for Sense Electrodes, Proof Mass, Tuning Electrodes, Drive Electrodes, and Sense Electrodes. A coordinate system with Ω_z is shown.

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Example ARW Calculation (cont)

Get rotation rate to output current scale factor:

$$A = z \frac{\omega_d}{\omega_s} Q_s \kappa_d \eta_e |\mathcal{H}(j\omega_d)| = z \left(\frac{10k}{15k} \right) (50k) (20\mu) (5) (2000\epsilon_0) (0.000024) = \underline{2.83 \times 10^{-12} C}$$

$$\mathcal{H}(j\omega_d) = \frac{(j\omega_d)(\omega_s/\Omega_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10k)(15k)/(50k)}{(15k)^2 - (10k)^2 + \frac{j(10k)(15k)}{50k}} = \frac{j(3k)}{1.25 \times 10^8 + j(3k)}$$

$$\Rightarrow |\mathcal{H}(j\omega_d)| = \frac{3k}{\sqrt{(1.25 \times 10^8)^2 + (3k)^2}} = 0.000024 \quad 8.854 \times 10^{-8} F/m$$

$$\frac{\partial C}{\partial x} = \frac{C_0}{d} = \frac{\epsilon_0 h \omega_p}{d} = \frac{\epsilon_0 (20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = 5(2000\epsilon_0) = 8.854 \times 10^{-12} F/m$$

Assume electrode covers the whole sidewall.

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x} |\mathcal{H}(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_a^2}}{\Delta f} + \frac{\overline{N_{i_a}^2}}{\Delta f} \left(\frac{1}{R_f} \right)$$

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Example ARW Calculation (cont)

$$R_x = \frac{\omega_s m}{Q_s \eta_e^2} = \frac{2\pi(15k)(9.6 \times 10^{-10})}{(50k)(8.854 \times 10^{-8})^2} = 110.6 k\Omega$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6k)} (0.000024)^2 + \frac{(1.66 \times 10^{-29})}{1M} + (0.01p)^2 + \frac{(12n)^2}{(1M)^2}$$

$\rightarrow 8.64 \times 10^{-35} A^2/Hz$ $1.66 \times 10^{-26} A^2/Hz$ $1 \times 10^{-28} A^2/Hz$ $1.44 \times 10^{-28} A^2/Hz$
 sensor element noise insignificant Noise from R_f dominates!


$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68 \times 10^{-26} A^2/Hz \rightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30 \times 10^{-13} A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left(\frac{180}{\pi} \right) = 9448 (\%hr)/\sqrt{Hz}$$

And finally:

$$ARW = \frac{1}{60} \Omega_{min} = \frac{1}{60} (9448) = \underline{157 \%hr} = ARW \Rightarrow \text{Almost turned around in 1 hour!}$$

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What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15\text{kHz}$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2 \frac{\omega_d}{\omega_s} Q_s X_d \eta_e |\Theta(j\omega_d)| = 2 Q_s X_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_0) = 1.77 \times 10^{-7} \text{C}$$

$$\frac{\dot{i}_{eqTOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})^2}{(110.6k)^2} + \frac{(1.66 \times 10^{-29})^2}{1M} + (0.01p)^2 + \frac{(12n)^2}{(1M)^2}$$

$\swarrow 1.51 \times 10^{-25} \text{ A}^2/\text{Hz}$
 $\swarrow 1.66 \times 10^{-26} \text{ A}^2/\text{Hz}$
 $\swarrow 1 \times 10^{-28} \text{ A}^2/\text{Hz}$
 $\swarrow 1.44 \times 10^{-28} \text{ A}^2/\text{Hz}$

Now, the sensor element dominates!

$$\therefore \frac{\dot{i}_{eqTOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{ A}^2/\text{Hz} \rightarrow \dot{i}_{eqTOT} = \sqrt{\frac{\dot{i}_{eqTOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{ A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{\dot{i}_{eqTOT}}{A} \left(\frac{3600s}{hr} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left(\frac{180}{\pi} \right) = 0.476 \text{ } (\%hr)/\sqrt{\text{Hz}}$$

And finally:

$$ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \text{ } (\%hr) = ARW \Rightarrow \text{Navigation grade!}$$

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