## PROBLEM SET \#5

Issued: Tuesday, March 31, 2015
Due (at 9 a.m.): Wednesday, April 15, 2015, in the EE C247B HW box near 125 Cory.

1) Fig. PS5.1 presents a simple accelerometer structure with capacitive electrodes for motion sensing. The polysilicon structure is $2 \mu \mathrm{~m}$ thick and suspended $3 \mu \mathrm{~m}$ above the substrate except for the anchoring locations indicated as the darkly shaded regions. Key dimensions for the beams and data on the structural material used in this problem are given in the box below the figure. Assume that all folding trusses and shuttles are rigid in all directions.


Fig. PS5. 1
Structural Material Properties:
$E=150 \mathrm{GPa}, \rho=2330 \mathrm{~kg} / \mathrm{m}^{3}$, Poisson ratio $=0.2$
$\underline{\text { Geometric Dimensions: }}$
$L_{b l}=150 \mu \mathrm{~m}, L_{b 2}=200 \mu \mathrm{~m}, L_{b 3}=250 \mu \mathrm{~m}, L_{m}=120 \mu \mathrm{~m}$
$W_{m}=500 \mu \mathrm{~m}, W_{b 1}=1 \mu \mathrm{~m}, W_{b 2}=2 \mu \mathrm{~m}$
a) Calculate the spring constant at a proof mass location.
b) Calculate the resonance frequency of the accelerometer assuming no voltage is applied to the structure or the capacitive electrodes.
c) Draw the mechanical equivalent circuit of this accelerometer and calculate numerical values of the circuit elements at:
i) A proof mass location
ii) A truss location

Assume the $Q$ of the resonance is 100 .
2) Fig. PS5.2-7 show a dual-axis $x-y$ accelerometer manufactured via the following 3-mask surface micromachining process:
(1) Deposit $1.0 \mu \mathrm{~m}$ of $\mathrm{SiO}_{2}$ on a Silicon wafer
(2) Deposit 300 nm of $\mathrm{SiN}_{3}$ via LPCVD
(3) Deposit 200 nm of in-situ doped polysilicon
(4) Lithographically define and etch polysilicon interconnects (anisotropic). Mask: POLY1
(5) Deposit $2.0 \mu \mathrm{~m}$ of $\mathrm{SiO}_{2}$ (LTO)
(6) Lithographically define and then etch anchor openings in $\mathrm{SiO}_{2}$ (anisotropic). Mask: ANCHOR
(7) Deposit $3.0 \mu \mathrm{~m}$ of in-situ doped polysilicon
(8) Lithographically define and then etch the polysilicon structure (anisotropic). Mask: POLY2
(9) Etch $\mathrm{SiO}_{2}$ completely using an HF etch, releasing structure (isotropic)

Assume all materials are stress-free at room temperature and have the material properties listed in Table PS6.1.

Table PS6. 1

| Material | $\rho_{m}$ <br> $\mathrm{~kg} / \mathrm{m}^{3}$ | $E$ <br> GPa | $v$ | $\alpha_{T}$ <br> $\mu \mathrm{strain} / \mathrm{K}$ | $\sigma_{0}$ <br> MPa | Comment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polysilicon | 2300 | 150 | 0.2 | 2.8 | Varies | Random Grains |
| Silicon Dioxide | 2200 | 69 | 0.17 | 0.7 | -300 | Amorphous |
| Silicon Nitride | 3170 | 270 | 0.27 | 2.3 | 1100 | Stoichiometric |



Fig. PS5.2: Perspective view of the accelerometer


Fig. PS5.3: POLY1, ANCHOR and POLY2 masks


Fig. PS5.4: Top view of the accelerometer


Fig. PS5.5: Top view of accelerometer showing only the interconnect layer


Fig. PS5.6: Zoom-in view of the $\boldsymbol{x}$-direction interdigital comb finger structure. There are 71 fingers on each side of the proof mass


Fig. PS5.7: Top view showing the $y$-direction capacitor structure


Fig. PS5.8: Zoom-in view of the $\boldsymbol{y}$-direction capacitor structure


Fig. PS5.9: Top view of the accelerometer with circuit connections for part b), c) and d)
a) Calculate the $x$ - and $y$-directed resonance frequencies of the accelerometer structure with no applied DC bias, making simplifying assumptions as necessary. Do NOT neglect the beam masses in these calculations.
b) Derive an expression for the capacitance between port $A$ and $B$ shown in Fig. PS5.8 as a function of displacement $x$ using a parallel-plate approximation. Also, calculate the overlap capacitance between these two ports at rest.
c) Derive an expression for the capacitance between port $A$ and $C$ shown in Fig. PS5.8 as a function of displacement $y$ using a parallel-plate approximation. Also calculate the overlap capacitance between these two ports at rest.
d) Suppose the accelerometer is now hooked up as shown in Fig. PS5.8, with a DC bias $V_{\text {bias }}$ of 10 V applied to the structure. Calculate the new $x$ - and $y$-directed resonance frequencies with this DC bias.
e) Referring to Fig. PS5.2-5.7, generate a 3-mask layout for this device using Cadence. You should use the technology (.tf) and display (.drf) files given in the class website to specify the names and colors of the mask. Export your layout as a .gds file titled: "ee247bHW5_yourlastname.gds". Note that ANCHOR is a dark field mask.
3) Fig. PS5.9 shows schematic of a system of parallel electrodes where the outer plates are fixed and the inner plate is movable. The inner plate is kept at ground potential and outer plates are biased with equal DC voltages to generate equal potential across each electrode gap. The inner plate is also attached to a spring that provides restoring force to keep the plate stable. Note that although the inner plate experiences equal forces on both sides, it would snap into one of the electrodes without a spring attachment since a tiny motion (e.g. due to thermal noise) would generate a force imbalance.
Assume that you fix the inner plate while ramping up the DC voltage applied to outer electrodes to a certain value and then release the inner electrode. Derive an expression for the DC voltage that will cause pull-in of the inner plate once released. Again, keep in mind that force imbalance will always occur under real-world conditions (in the presence of noise, finite asymmetry etc.).


Fig. PS5.10

