## PROBLEM SET \#6

Issued: Friday, April 17, 2015
Due (at 9 a.m.): Friday, April 24, 2015, in the EE C247B HW box near 125 Cory.

1) Fig. PS6.1 presents the accelerometer structure of PS5-Q1 with capacitive electrodes. Here, red wires connect 3 top and 3 bottom electrode fingers to form a total capacitive electrode, $C_{1}$; and black wires connect other electrode fingers to form a total capacitive electrode $C_{2}$.


Fig. PS6.1: Accelerometer structure of PS5-Q1
Structural Material Properties:
$E=150 \mathrm{GPa}, \rho=2330 \mathrm{~kg} / \mathrm{m}^{3}$, Poisson ratio $=0.2$
Geometric Dimensions:
$L_{b 1}=150 \mu \mathrm{~m}, L_{b 2}=200 \mu \mathrm{~m}, L_{b 3}=250 \mu \mathrm{~m}, L_{m}=120 \mu \mathrm{~m}, L_{t}=20 \mu \mathrm{~m}$
$W_{b 1}=1 \mu \mathrm{~m}, W_{b 2}=2 \mu \mathrm{~m}, W_{b 3}=2 \mu \mathrm{~m}, W_{m}=500 \mu \mathrm{~m}, W_{t}=100 \mu \mathrm{~m}$

For this problem, assume that the mechanical equivalent circuit has the following values at the proof mass location: $m_{e q}=110 \mathrm{ng}, k_{e q}=0.4 \mathrm{~N} / \mathrm{m}, b_{e q}=6.7 \times 10^{-8} \mathrm{~kg} / \mathrm{s}$
a) Draw the electrical equivalent circuit with transformers assuming that a $V_{P}=30 \mathrm{~V}$ is applied to the proof mass and the electrode gap spacing at rest is $d_{o}=2 \mu \mathrm{~m}$.
b) Suppose $C_{1}$ is used an input electrode and $C_{2}$ as an output electrode. Derive an expression and calculate the numerical value of impedance looking into the input electrode when the output electrode is shorted to ground.
c) Assuming a 10 mV input sinusoidal (electrical) signal at the resonance frequency is applied to the input electrode, calculate the numerical value of the output current when $C_{2}$ is grounded.
2) Suppose the accelerometer structure of PS5-Q2 is hooked up as shown in Fig. PS6.2, with a DC bias $V_{\text {bias }}$ of 10 V applied to the structure. Assume the following values for the mechanical circuit elements:

In $x$-direction: $m_{x}=1.95 \times 10^{-9} \mathrm{~kg}, k_{x}=12.6 \mathrm{~N} / \mathrm{m}, b_{x}=31.4 \times 10^{-6} \mathrm{~kg} / \mathrm{s}$
In $y$-direction: $m_{y}=1.91 \times 10^{-9} \mathrm{~kg}, k_{y}=16.1 \mathrm{~N} / \mathrm{m}, b_{y}=35.1 \times 10^{-6} \mathrm{~kg} / \mathrm{s}$
a) If a sinusoidal acceleration acts on the structure in the $x$-direction with a magnitude of 1 g , what will be the magnitude and phase of the resulting output current $i_{B}$ as a function of frequency? Plot the magnitude and phase from 1 kHz to 1 MHz .
b) If a sinusoidal acceleration acts on the structure in the $y$-direction with a magnitude of 1 g , what will be the magnitude and phase of the resulting output current $i_{C}$ as a function of frequency? Plot the magnitude and phase from 1 kHz to 1 MHz . Do not forget to include the effect of electrical stiffness in this part. You can assume the value of electrical stiffness is $5.8 \mathrm{~N} / \mathrm{m}$ for the given $V_{\text {bias }}$.
c) Derive an expression and calculate the numerical value of impedance looking into port $D$.

You can use the results of PS5 for your calculations.


Fig. PS6.2: Top view of the accelerometer with circuit connections
3) Fig. PS6.3 presents the top view of a micromechanical device constructed in a $2 \mu$ m-thick structural layer with numerous ports. The mechanical structure is electrically connected to the ground plane (since it is anchored to it) and the ground plane is accessible via terminal 4. Assume that terminals bearing the same number are electrically connected.


Fig. PS6. 3

## Structural Material Properties:

$E=150 \mathrm{GPa}, \rho=2300 \mathrm{~kg} / \mathrm{m}^{3}$, Poisson ratio $=0.226, Q=80,000$
Geometric Dimensions:
$L=50 \mu \mathrm{~m}$, Finger Overlaps $=L_{o}=10 \mu \mathrm{~m}, W=2 \mu \mathrm{~m}$, Thickness $=H=2 \mu \mathrm{~m}$, $d_{o}=1 \mu \mathrm{~m}$, Folding Truss Area $=200 \mu \mathrm{~m}^{2}$, Shuttle Area $=3,000 \mu \mathrm{~m}^{2}$
a) Calculate the $x$-directed resonance frequency of this structure when all ports are grounded.
b) Draw a transformer-based equivalent circuit modeling the electrical behavior of this device with ports 1, 2 and 3 as terminals, with port 4 held at 60 V , and all others assumed at DC ground. (No need to show port 4 in this circuit.) Calculate the numerical values for all of its elements.
c) Suppose the device is hooked up as shown in Fig. PS6.4, where $v_{i}$ is a sinusoidal voltage at the resonance frequency with an amplitude of 10 mV . What is the output current io? Provide an expression and a numerical value with units.


Fig. PS6.4: Ports with electrical connections for part c)
d) Now suppose a resistor $R_{P}=5 \mathrm{M} \Omega$ is placed in series with the DC-bias voltage $V_{P}$ at port 4, as shown in Fig. PS6.5, with the same input voltage $v_{i}$. Now, what is the output current $i o$ ? Provide an expression and a numerical value with units.


Fig. PS6.5: Ports with electrical connections for part d)

