EEC247B / ME C218 INTRODUCTION TO MEMS DESIGN SPRING 2015
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PROBLEM SET #7

Issued: *Monday, April 27, 2015*

Due (at 9 a.m.): Friday, May 8, 2015, in the EE C247B HW box near 125 Cory.

Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc.) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), cell phones, and game controllers (e.g., the Wii). Of these applications, the last four already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best explained via example.

This problem concerns the MEMS-based micro-gyroscope [by Acar & Shkel] pictured in the scanning electron micrograph (SEM) of Fig.PS7.1 and summarized in the figures that follow. This device is fabricated using a bulk-micromachining process, where the 100μm-thick silicon layer on a silicon-on-insulator (SOI) wafer is patterned and etched to the needed dimensions with perforations (i.e., etch holes), then a final HF wet release is used to remove oxide under the perforated structures, but not under the contiguous anchors (since this takes more time without perforations). Note that the perforations are not shown in Figs. 2-10, but they are shown in Fig.PS7.1.

In this device, momentum is generated by driving the proof mass into resonance vibration using the capacitive comb fingers along the x-axis. When the device is rotated about the z-axis (indicated in Fig.PS7.1 and Fig.PS7.2), the vibrating mass will attempt to preserve its momentum in the original x-direction, which will then make the mass appear to deflect in the y-direction. This y-directed motion is then sensed by parallel-plate capacitors to determine the rotation rate. In quantitative terms, the angular velocity Ω about the z-axis generates a *Coriolis Force* ($F = 2m_s\dot{x}_d \times \Omega$ where m_s is the equivalent mass in the sense mode, and $\dot{x}_d$ is the structure velocity in the drive mode) along the y-direction that drives the proof mass into y-directed vibration. The amplitude of the vibration is then picked up by the varying gap capacitances.

Several figures are provided to support the questions that follow. Table PS7.1 provides measured or target parameter values for the fabricated device. Figs. PS7.2-10 then identify different parts of the structure, indicate which portions are freely suspended and which are anchored (the black regions are anchored), and provide key dimensions. As indicated, the gaps of comb fingers are all 1 μm and the gaps of parallel-plate capacitive fingers are all 2 μm. The thicknesses of the structures are all 100 μm.

Fig.PS7.2 presents the structure together with a simple set of electronic circuits that sustain oscillation along the drive axis and sense Coriolis-induced motions along the sense axis. Finally, assume for this problem that a DC bias voltage of 20 V is applied to the movable structure.

(Note that in an actual implementation, the sense amplifier often provides a balanced differential input that senses the differential current from both sense electrodes to maximize performance. In other words, the grounded electrode in Fig.PS7.2 would not be grounded, but would also be directed to an amplifier input. The present problem uses a single-ended pick-off configuration only to simplify things.)

Answer the following questions regarding this gyroscope:

1. Use the surface area information given in the figures to determine the needed spring constants at the electrode locations for (i) the drive and (ii) the sense modes. Ignore the masses of the springs in this problem.
2. What suspension beam lengths, L_d and L_s, are required to achieve the needed spring constants for (i) the drive and (ii) the sense modes determined in 1?
3. Identify the electrodes for the (i) drive and (ii) sense modes and determine the capacitance and change in capacitance per unit displacement for each.
4. Draw and specify (numerically) all element values in the equivalent circuits (transformers + LCR) modeling the (i) drive mode and (ii) the sense mode.
5. Code the equivalent circuits in 4 into SPICE netlists and simulate Bode plots for the voltage-to-velocity transfer functions using SPICE that include the low frequency and resonance responses of the structures.
6. Assume that during steady-state oscillation along the drive axis the drive amplifier delivers an AC voltage v_d with an amplitude of 2.5 V and a frequency equal to the resonance frequency of the drive mode. Also, assume that the input of the drive amplifier detects velocity and that its input resistance is very small. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.
7. Determine the value of the output voltage noise spectral density of the gyro circuit (i.e., at the output node, where v_O is indicated in Fig.PS7.2) at the frequency of the drive mode. Data for the op-amp noise generators and value of the op-amp feedback resistor are given in Table PS7.2.
8. What is the minimum detectable angular rate of this gyro circuit if the output sense circuit is limited by a low-pass filter to a 1 kHz bandwidth?

Table PS7.1

	Parameters	**Measured/Target Value**
	Young's Modulus (GPa)	150
	Density (kg/m^3)	2300
Drive Mode	Resonance Frequency (Hz)	3000
	Quality Factor	200
Sense Mode	Resonance Frequency (Hz)	3200
	Quality Factor	100

Table PS7.2

Electrical Circuit Data	Op-amp Feedback Resistor	$R_f = 10\ \mathrm{M}\Omega$
	Op-amp Input Referred Noise Voltage	$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12nV/\sqrt{Hz}$
	Op-amp Input Referred Noise Current	$\sqrt{\frac{\overline{i_{ia}^2}}{\Delta f}} = 0.01pA/\sqrt{Hz}$

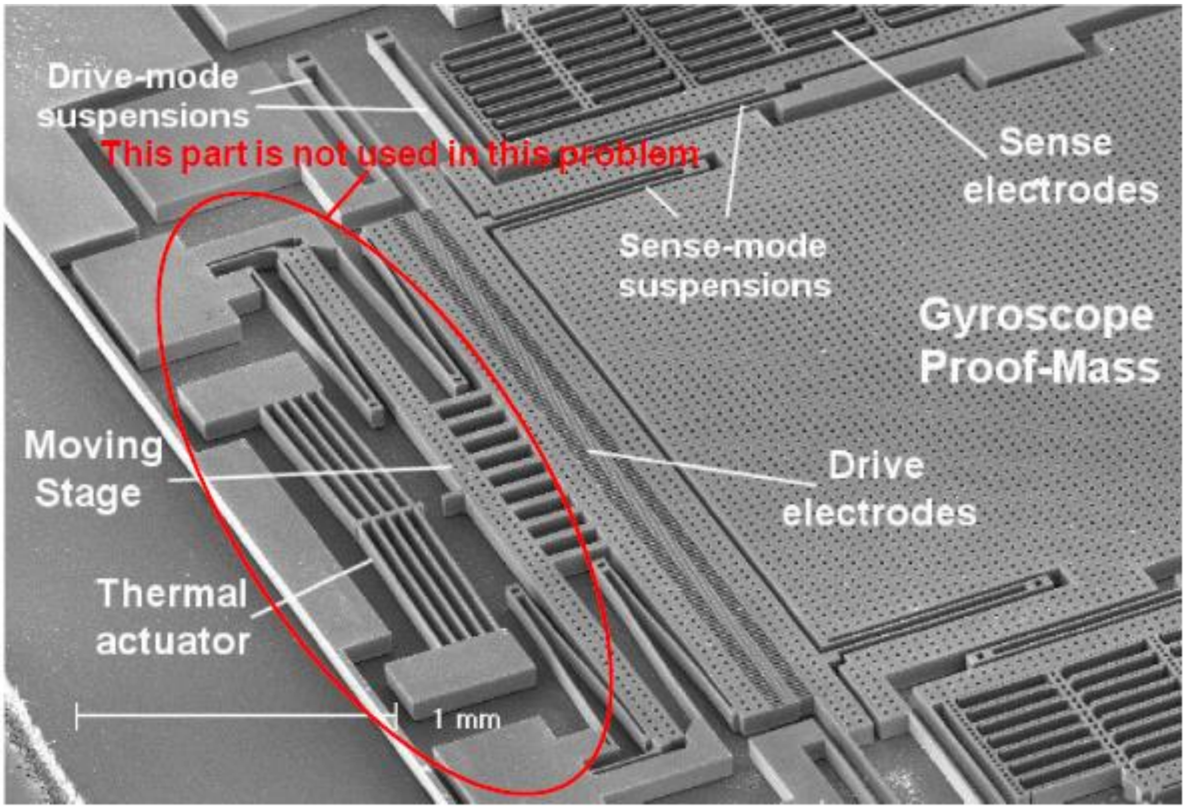

Fig.PS7.1: Scanning electron micrograph of the MEMS-based micro-gyroscope

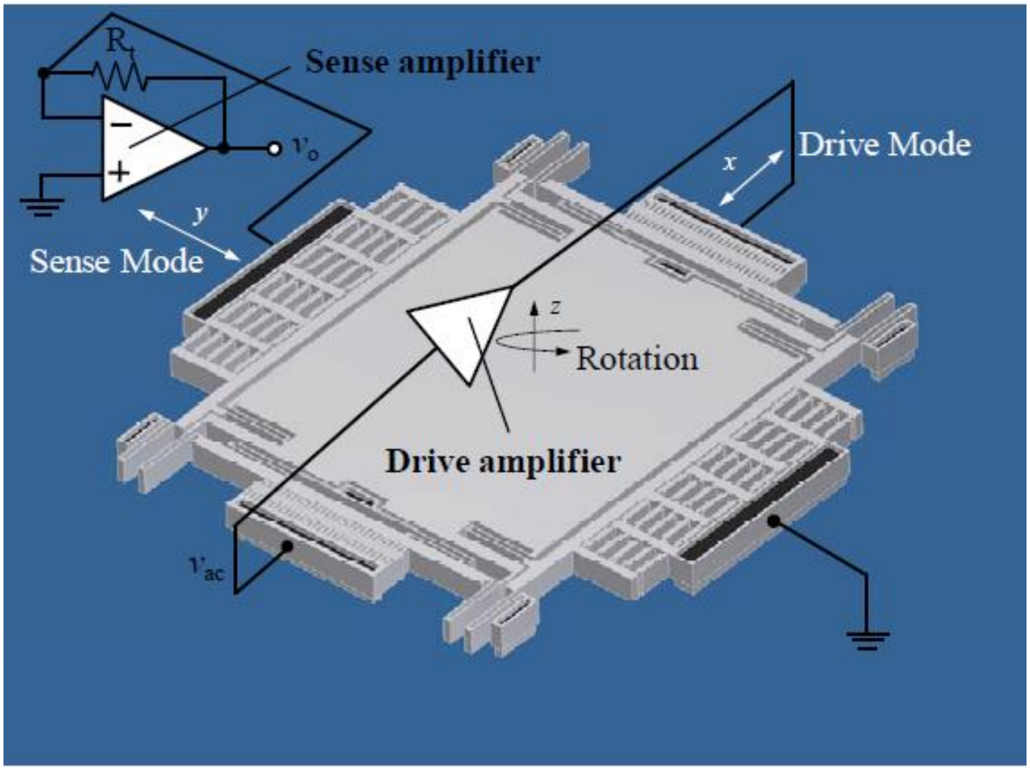

Fig.PS7.2: Isometric View

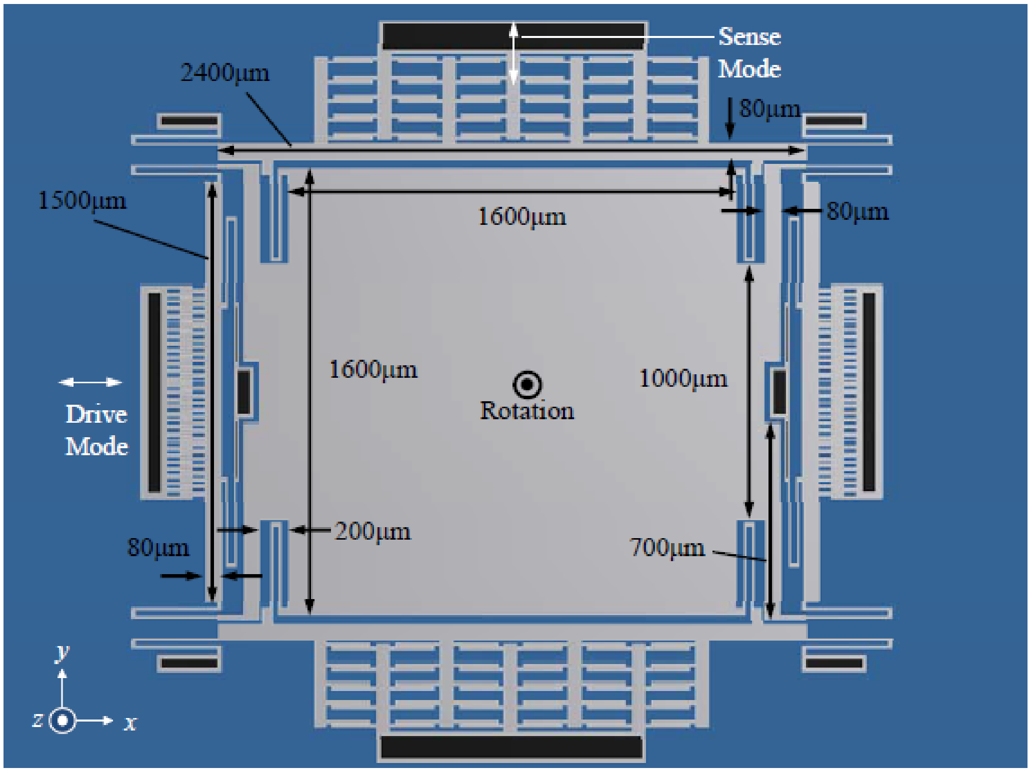

Fig.PS7.3: Top View

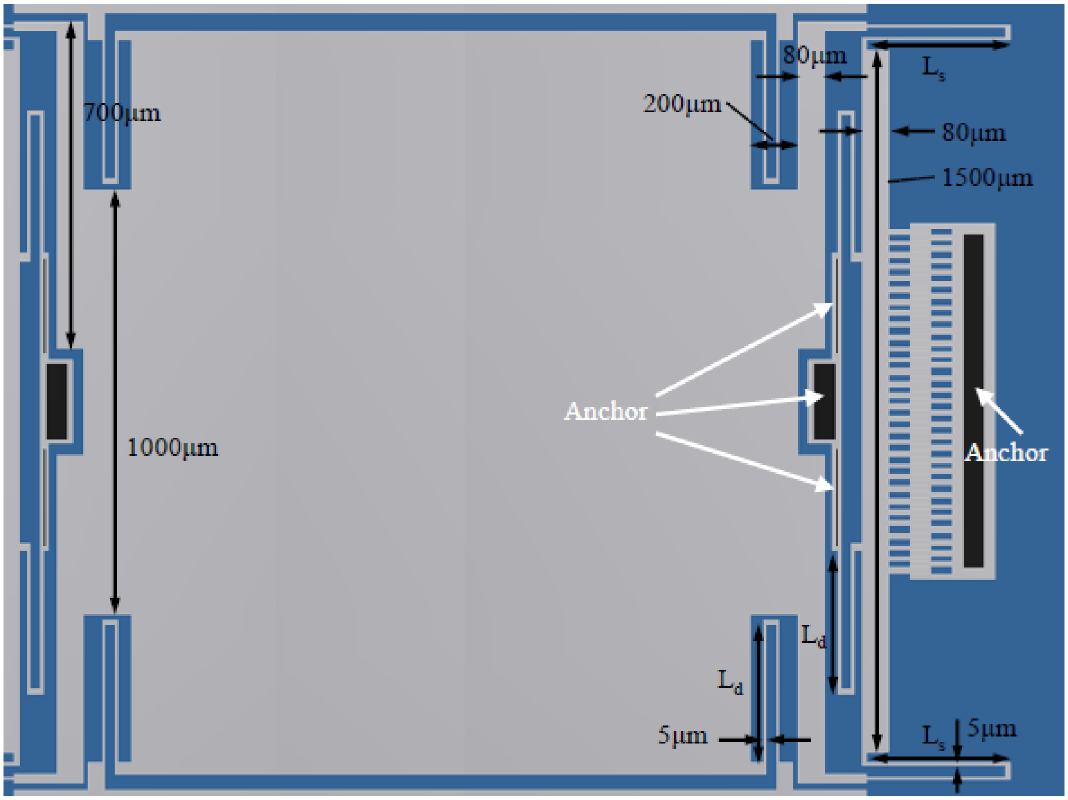

Fig.PS7.4: Gyroscope dimensions-1

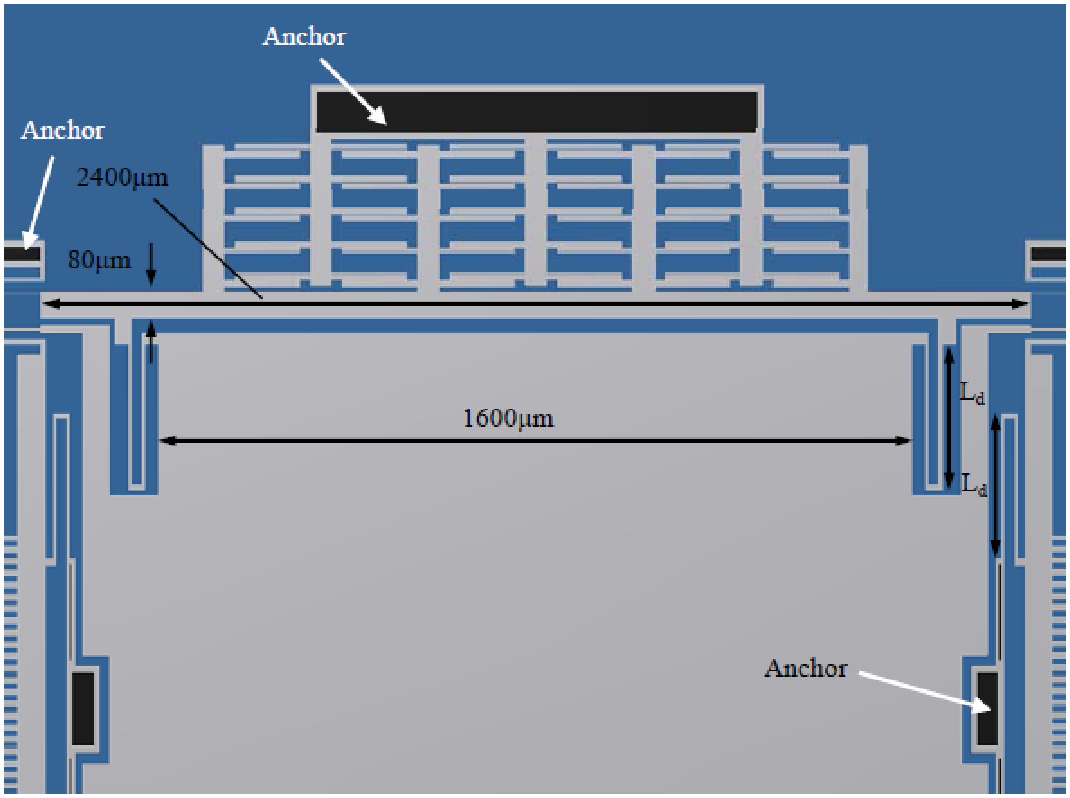

Fig.PS7.5: Gyroscope dimensions-2

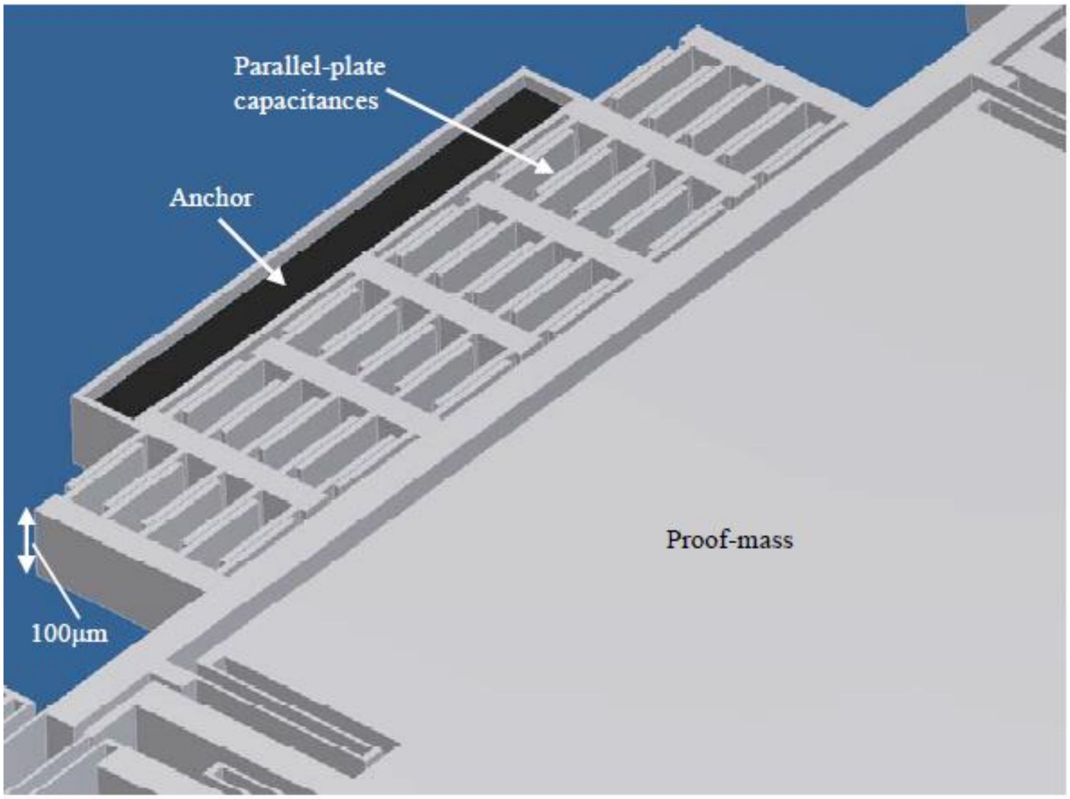

Fig.PS7.6: Sense electrodes

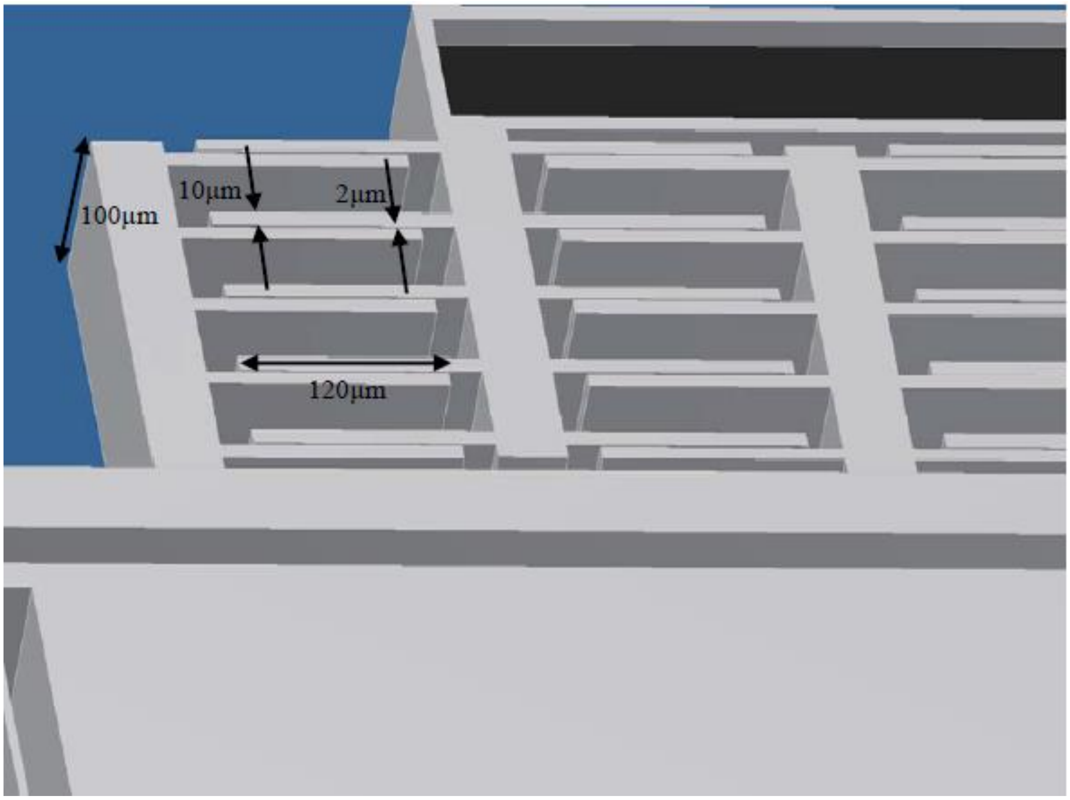

Fig.PS7.7: Sense electrodes details

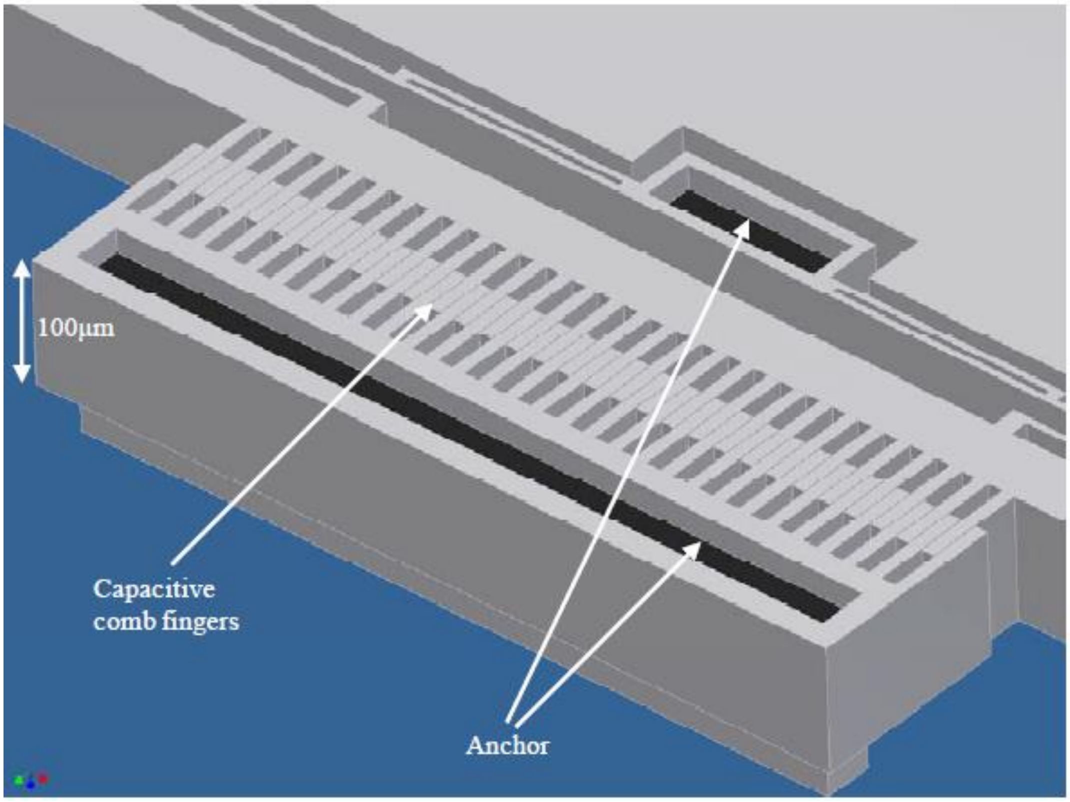

Fig.PS7.8: Drive electrodes

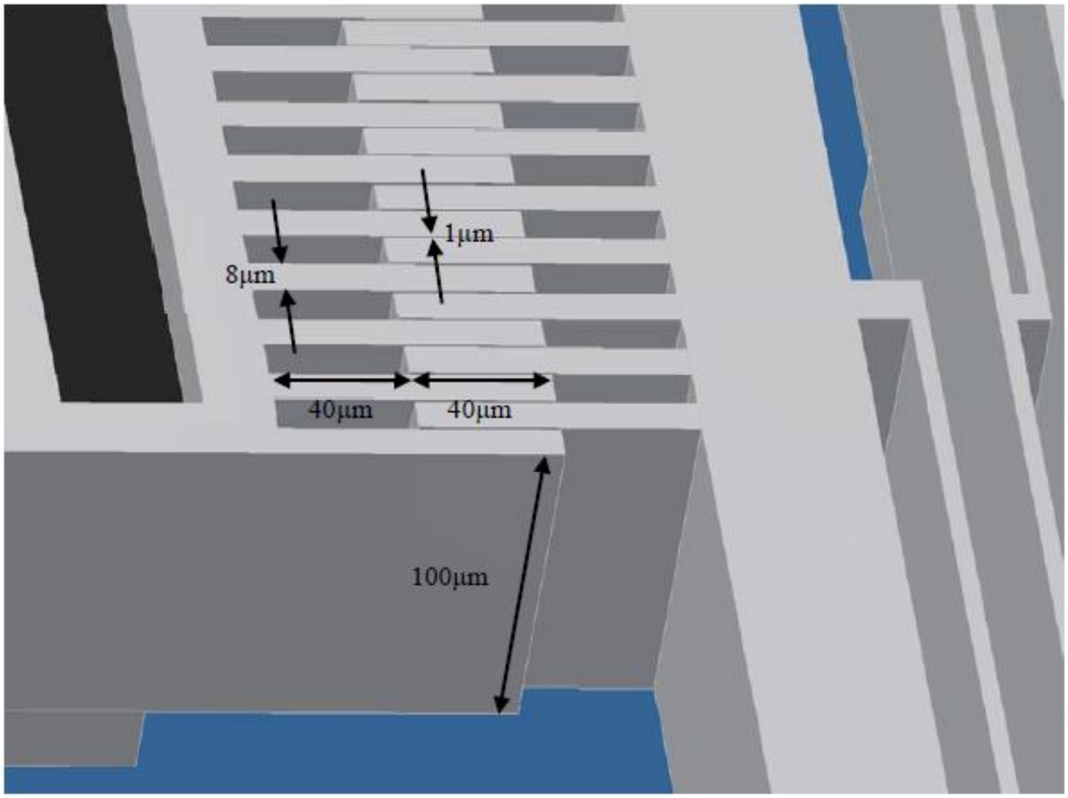

Fig.PS7.9: Drive electrodes details

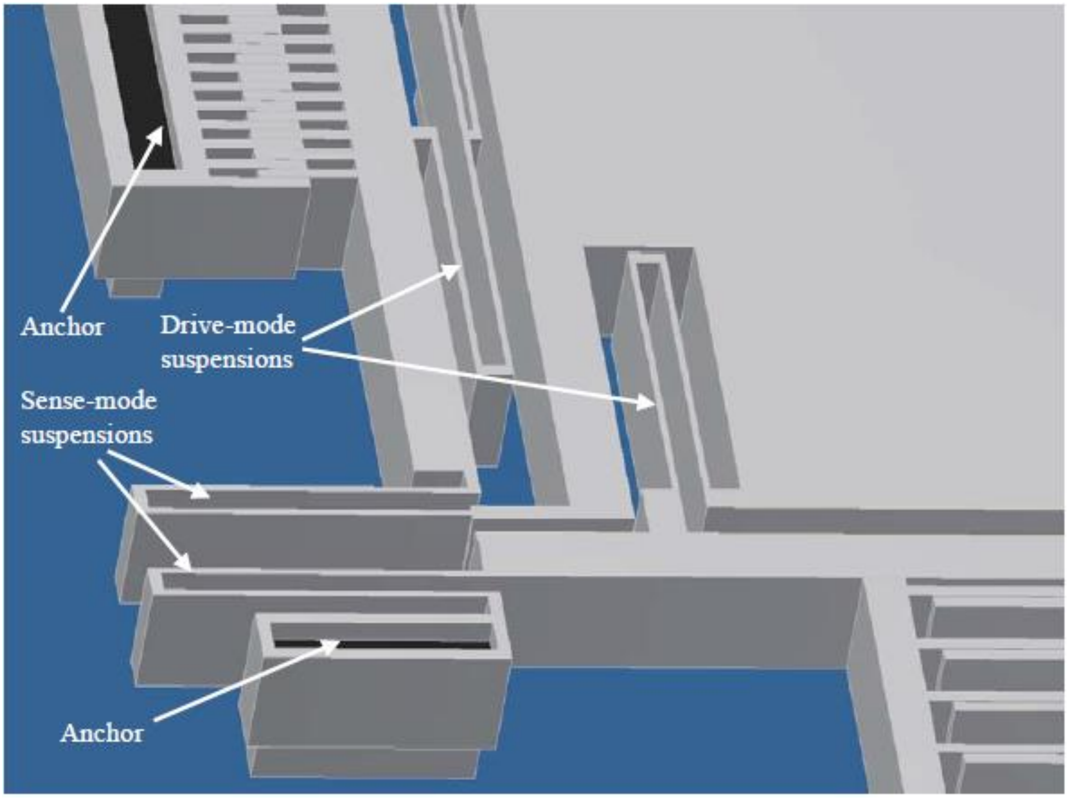

Fig.PS7.10: Suspension beams for drive-mode and sense-mode