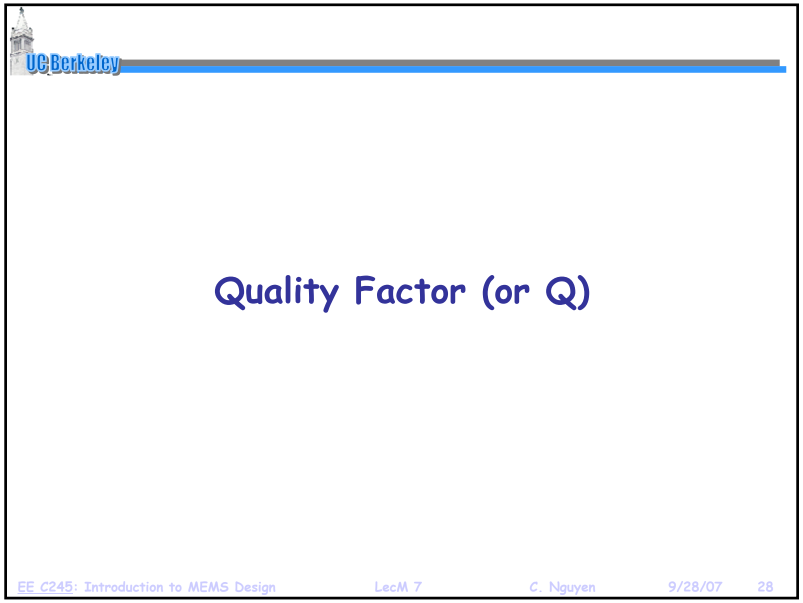
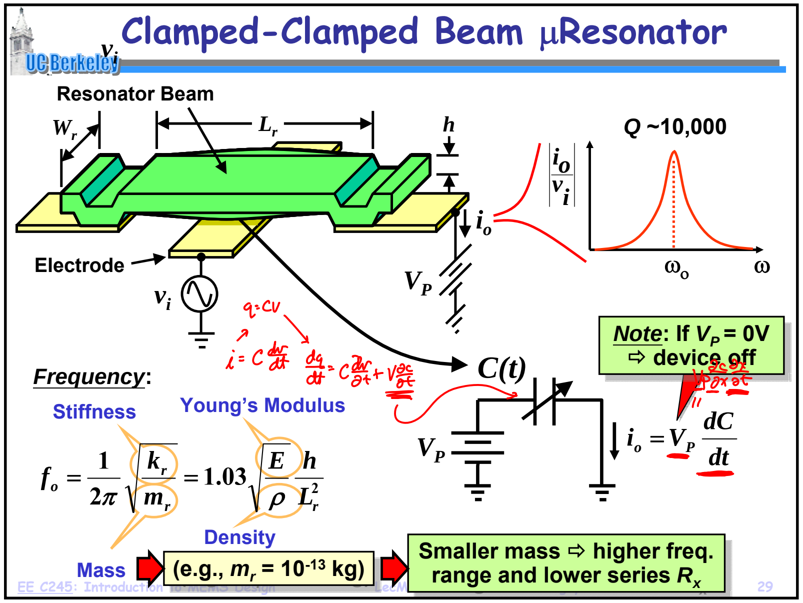


Quality Factor (or Q)



Clamped-Clamped Beam μResonator

$$i_o = V_P\frac{dC}{dt}$$

Note: If V_P = 0V ⇨ device off

$q = CV$

$i = C\frac{dv}{dt}$

$\frac{dq}{dt} = C\frac{\partial v}{\partial t} + V\frac{\partial c}{\partial t}$

$V_P\frac{\partial c}{\partial x}\frac{\partial x}{\partial t}$

Frequency:

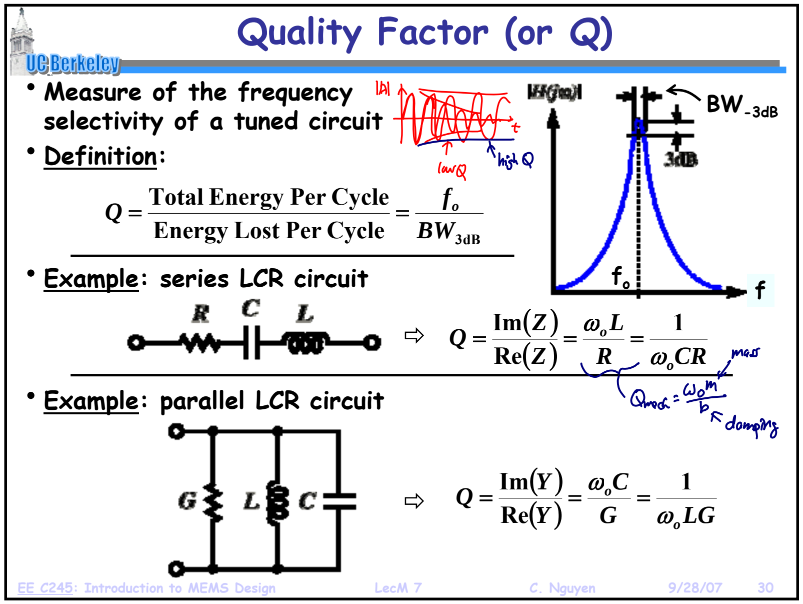
Stiffness

Young's Modulus

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

Density

Mass ➡ (e.g., $m_r = 10^{-13}$ kg) ➡ Smaller mass ⇨ higher freq. range and lower series R_x



Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

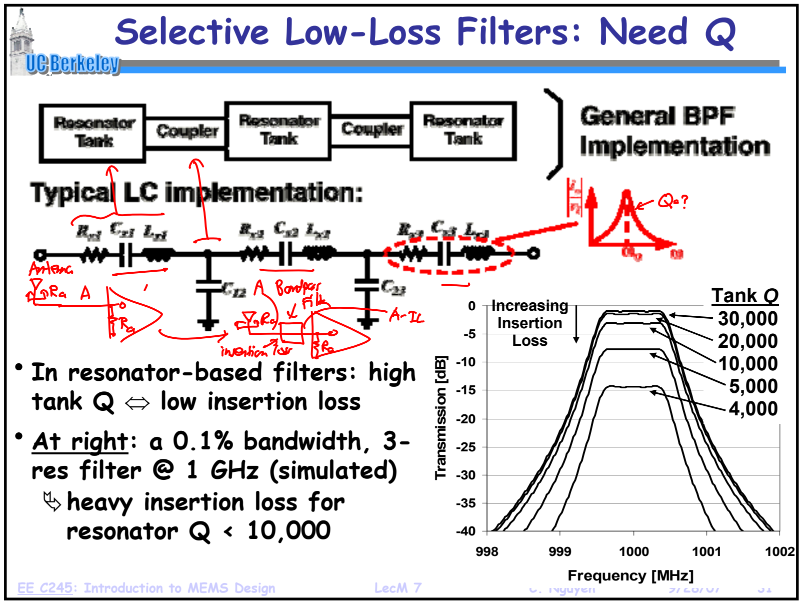
- Example: series LCR circuit

$$⇨ \quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$Q_{mech} = \frac{\omega_o m}{b}$ ← mass ← damping

- Example: parallel LCR circuit

$$⇨ \quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

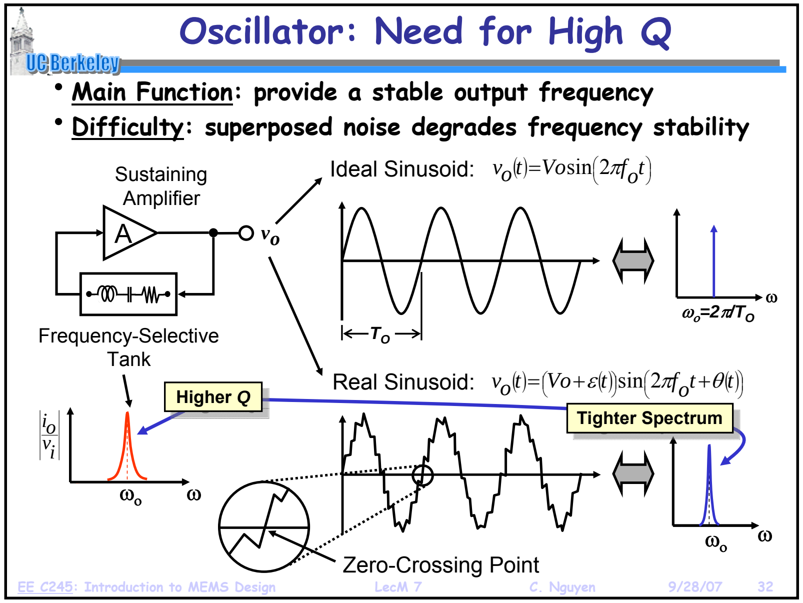


Selective Low-Loss Filters: Need Q

General BPF Implementation

Typical LC implementation:

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ➥ heavy insertion loss for resonator Q < 10,000

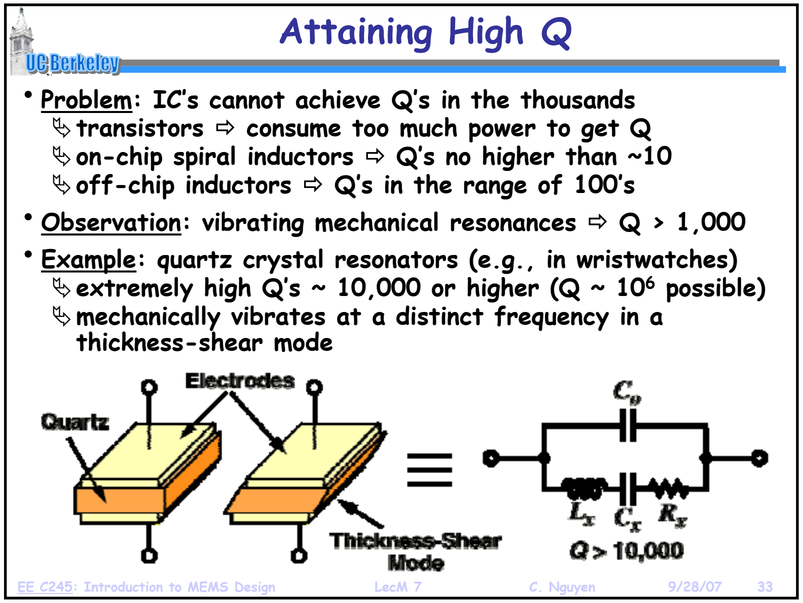


Oscillator: Need for High Q

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability

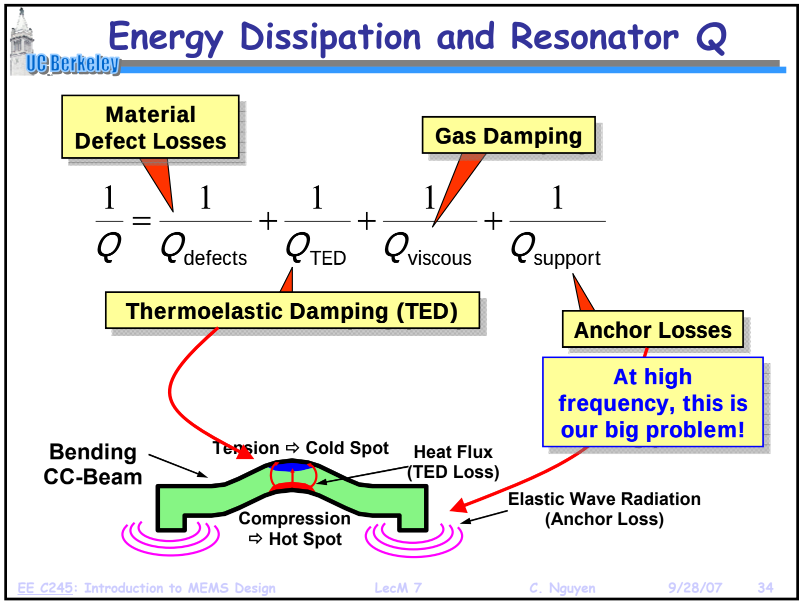
Ideal Sinusoid: $v_o(t)=V_o\sin(2\pi f_o t)$

Real Sinusoid: $v_o(t)=(V_o+\varepsilon(t))\sin(2\pi f_o t+\theta(t))$



Attaining High Q

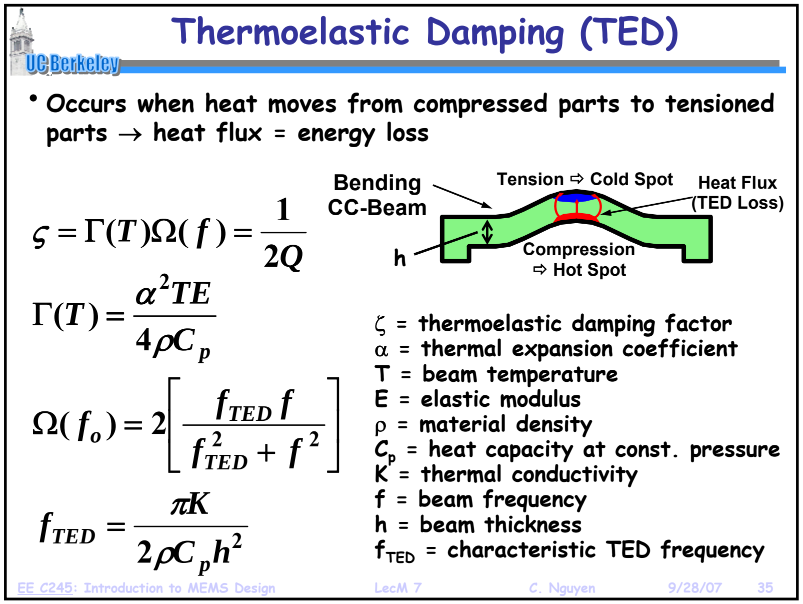
- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode



Energy Dissipation and Resonator Q

$$\frac{1}{Q}=\frac{1}{Q_{\text{defects}}}+\frac{1}{Q_{\text{TED}}}+\frac{1}{Q_{\text{viscous}}}+\frac{1}{Q_{\text{support}}}$$

At high frequency, this is our big problem!



Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

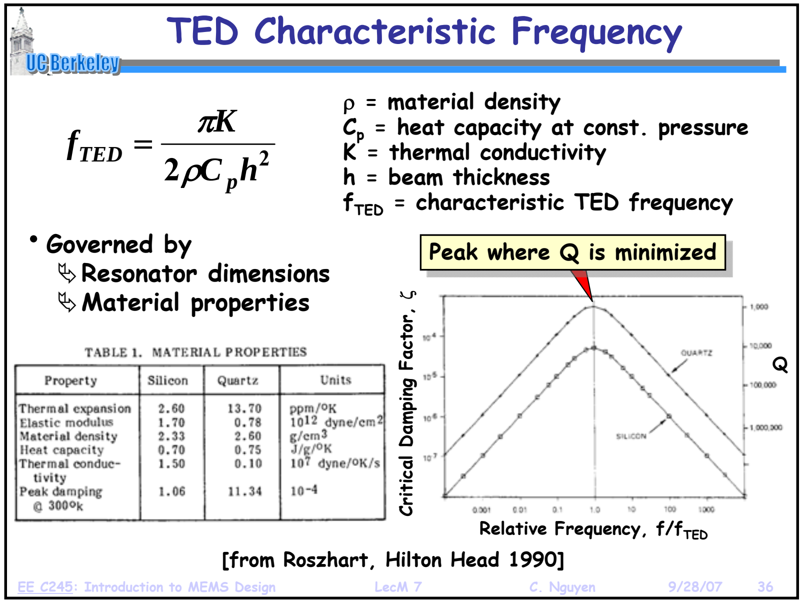
$$\varsigma=\Gamma(T)\Omega(f)=\frac{1}{2Q}$$

$$\Gamma(T)=\frac{\alpha^2TE}{4\rho C_p}$$

$$\Omega(f_o)=2\left[\frac{f_{TED}\,f}{f_{TED}^2+f^2}\right]$$

$$f_{TED}=\frac{\pi K}{2\rho C_p h^2}$$

ζ = thermoelastic damping factor
α = thermal expansion coefficient
T = beam temperature
E = elastic modulus
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
f_{TED} = characteristic TED frequency



TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

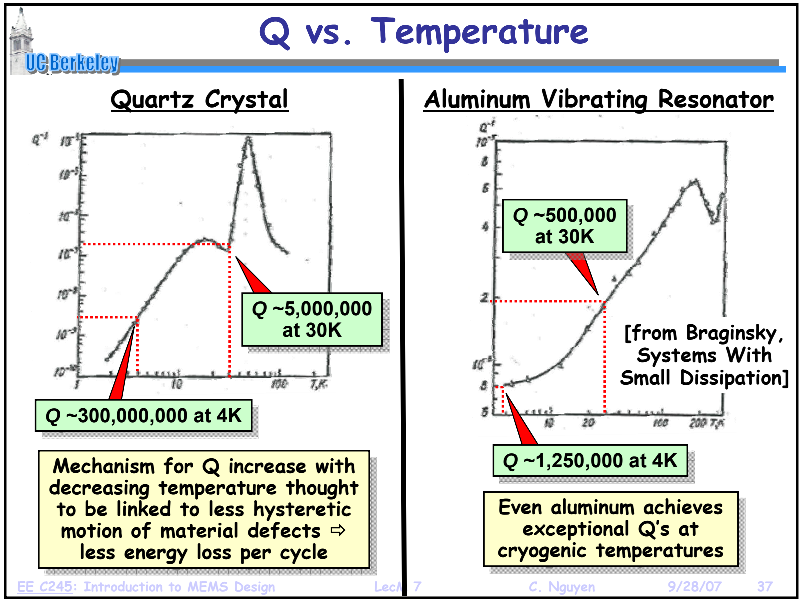
- Governed by
  - Resonator dimensions
  - Material properties

TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm^2
Material density	2.33	2.60	g/cm^3
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

Peak where Q is minimized

[from Roszhart, Hilton Head 1990]



Q vs. Temperature

Quartz Crystal

Q ~5,000,000 at 30K

Q ~300,000,000 at 4K

Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle

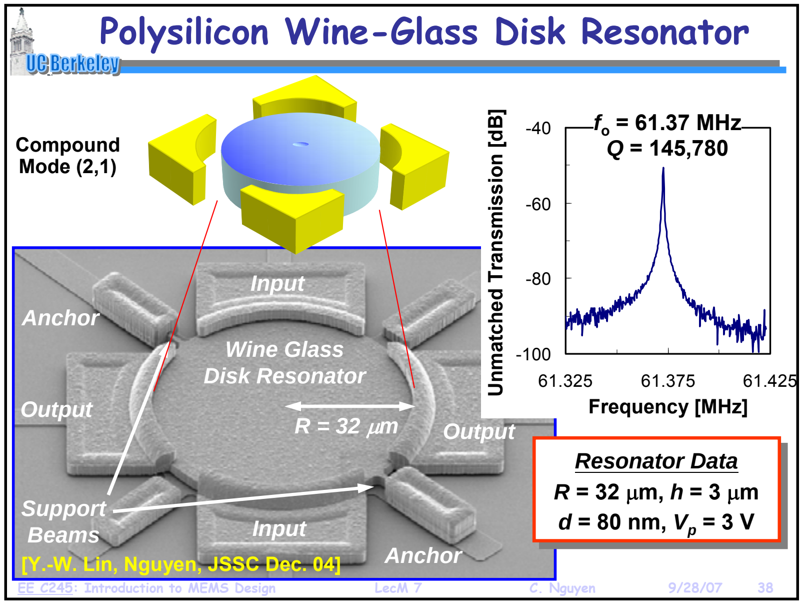
Aluminum Vibrating Resonator

Q ~500,000 at 30K

[from Braginsky, Systems With Small Dissipation]

Q ~1,250,000 at 4K

Even aluminum achieves exceptional Q's at cryogenic temperatures



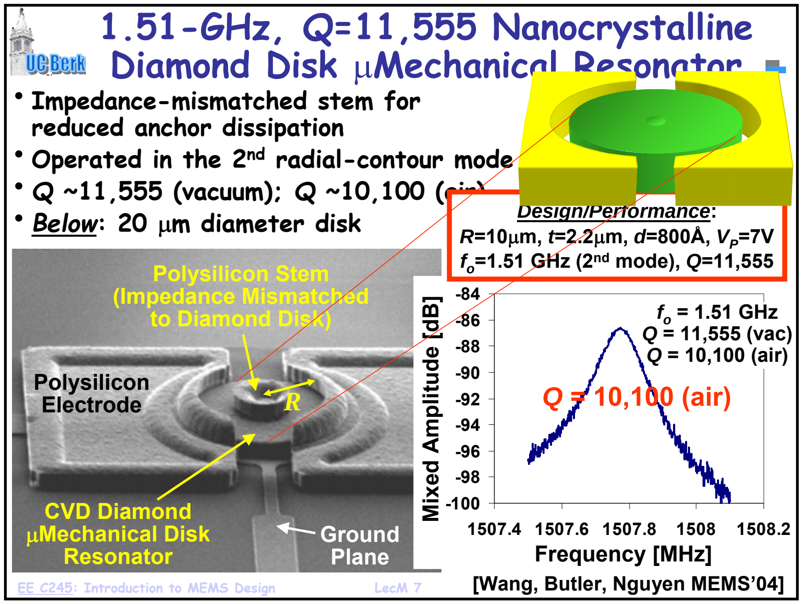
Polysilicon Wine-Glass Disk Resonator

Compound Mode (2,1)

[Y.-W. Lin, Nguyen, JSSC Dec. 04]

f_o = 61.37 MHz
Q = 145,780

Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V



1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- *Below*: 20 μm diameter disk

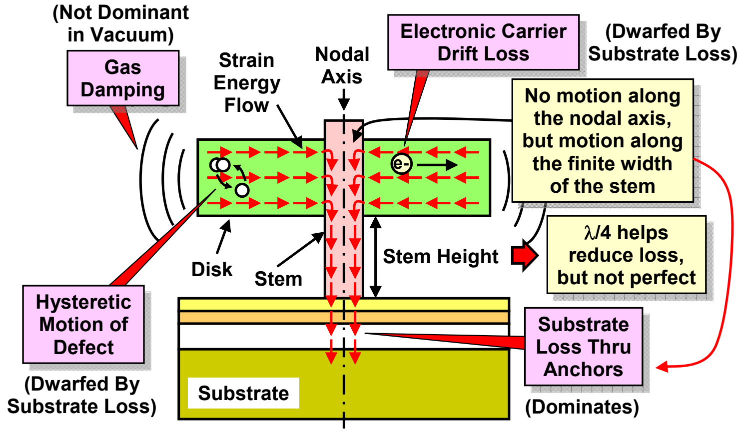
Design/Performance:
R=10μm, t=2.2μm, d=800Å, V_P=7V
f_o=1.51 GHz (2nd mode), Q=11,555

f_o = 1.51 GHz
Q = 11,555 (vac)
Q = 10,100 (air)

Q = 10,100 (air)

[Wang, Butler, Nguyen MEMS'04]

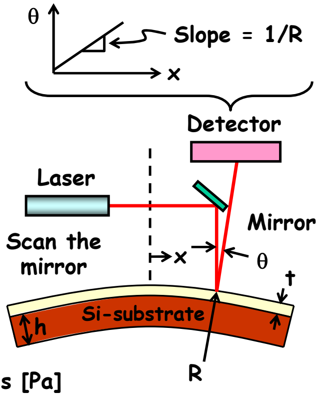
Disk Resonator Loss Mechanisms



MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:

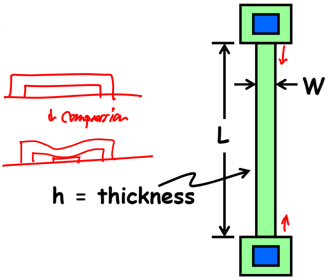


$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]

MEMS Stress Test Structure

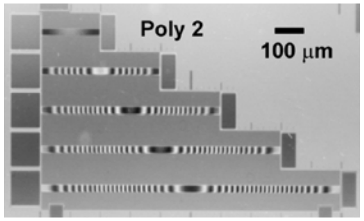
- **Simple Approach**: use a clamped-clamped beam
  - ↳ Compressive stress causes buckling
  - ↳ Arrays with increasing length are used to determine the critical buckling load, where

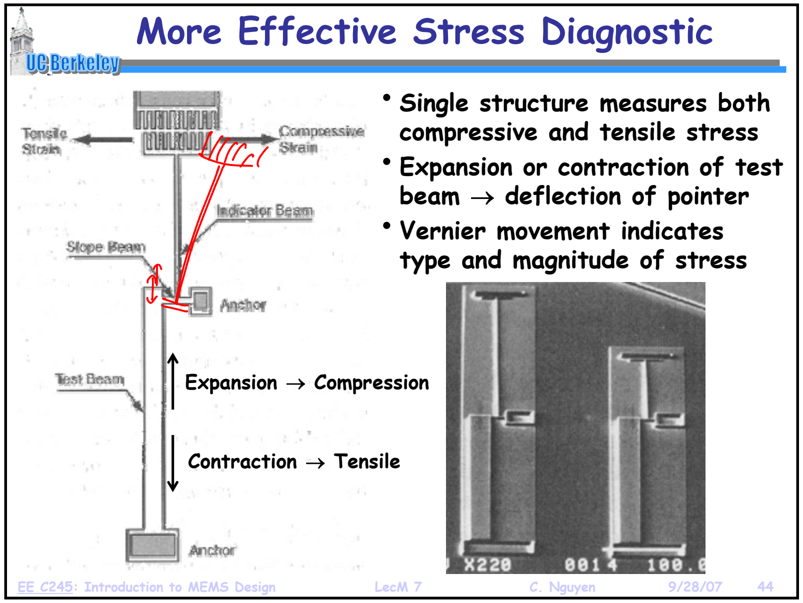


$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

E = Young's modulus [Pa]
I = (1/12)Wh³ = moment of inertia
L, W, h indicated in the figure

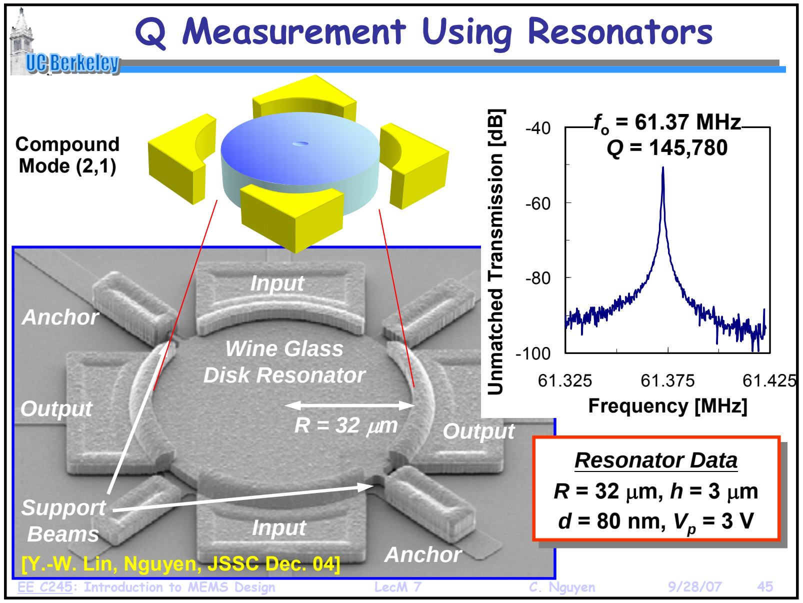
- ↳ **Limitation**: Only compressive stress is measurable





More Effective Stress Diagnostic

- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress

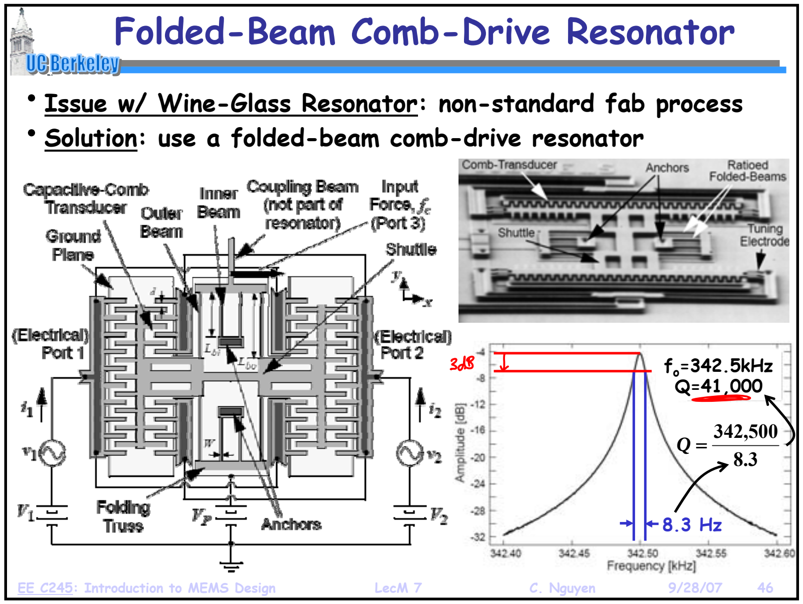


Q Measurement Using Resonators

Resonator Data

R = 32 μm, h = 3 μm

d = 80 nm, V_p = 3 V

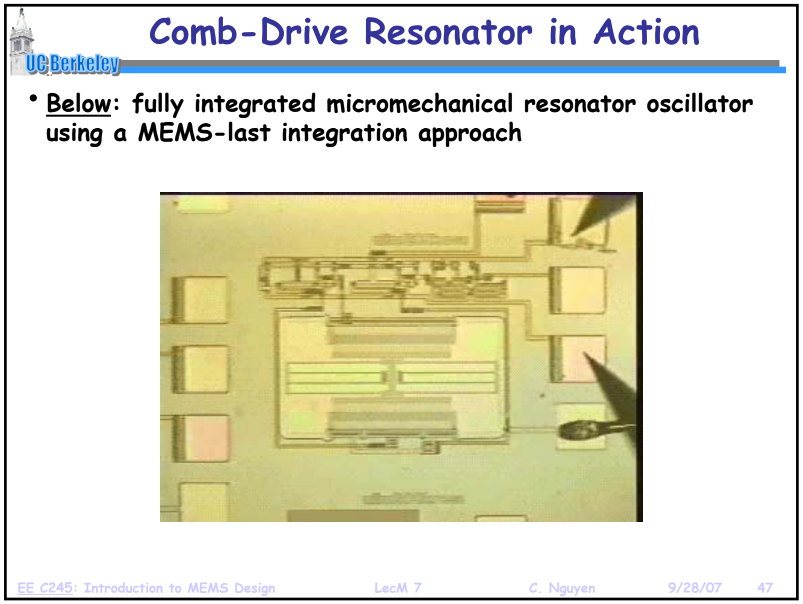


Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator

f$_o$=342.5kHz
Q=41,000

$$Q = \frac{342{,}500}{8.3}$$

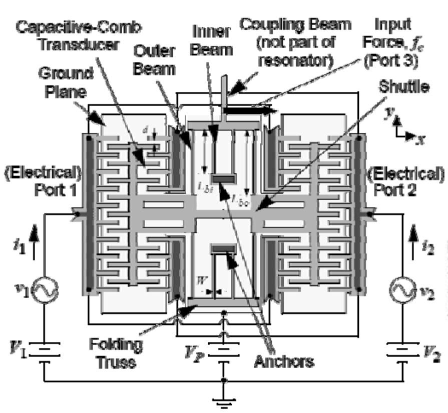


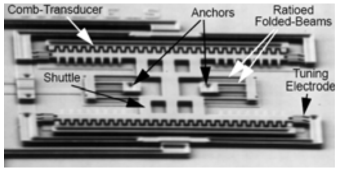
Comb-Drive Resonator in Action

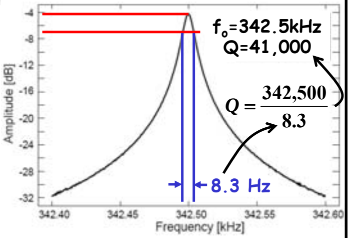
- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator







Measurement of Young's Modulus

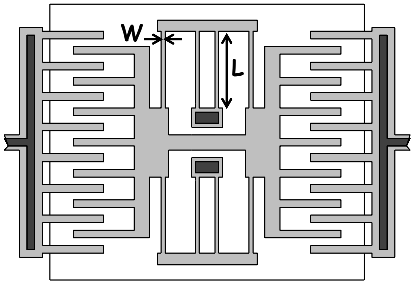
- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

Resonance Frequency = $$f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Young's modulus

Equivalent mass

h = thickness



- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\underset{\text{Stresses}}{\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}} = \underbrace{\begin{bmatrix}C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix}}_{\text{Stiffness Coefficients}} \underset{\text{Strains}}{\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}}$$

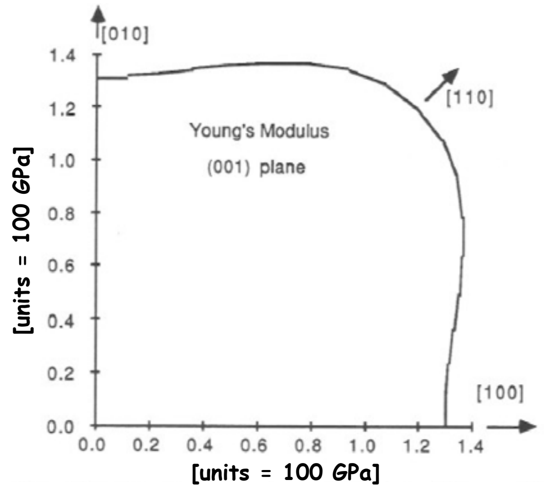
Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

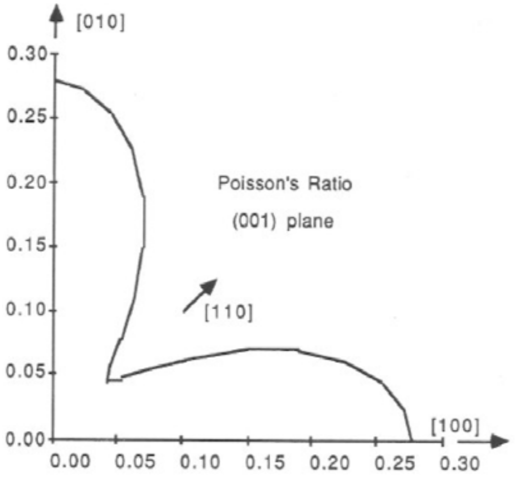
$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

$$\text{where}\ \begin{cases}C_{11} = 165.7\ \text{GPa}\\ C_{12} = 63.9\ \text{GPa}\\ C_{44} = 79.6\ \text{GPa}\end{cases}$$

Young's Modulus in the (001) Plane

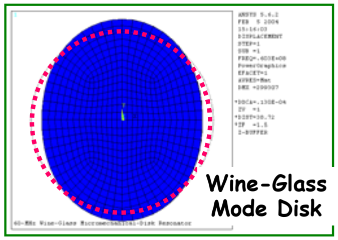


Poisson Ratio in (001) Plane

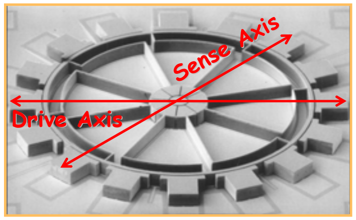


Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here



Wine-Glass Mode Disk



Ring Gyroscope