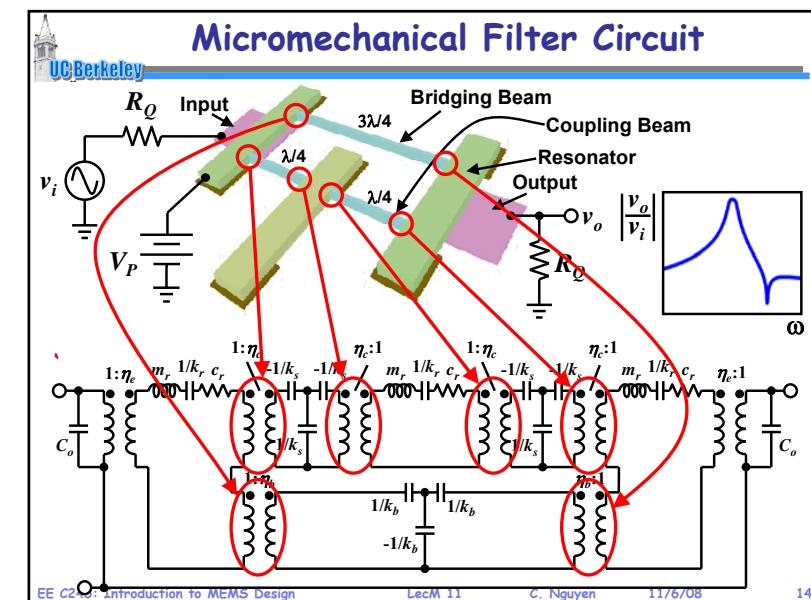
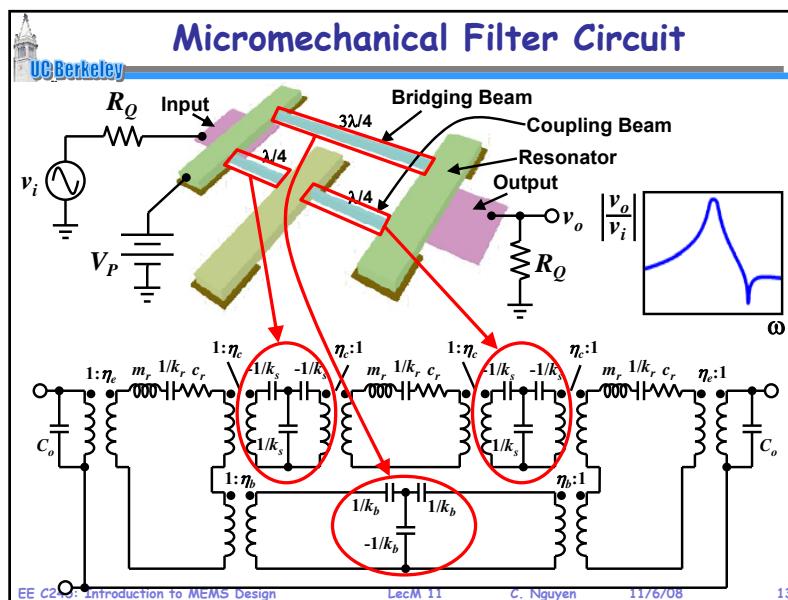
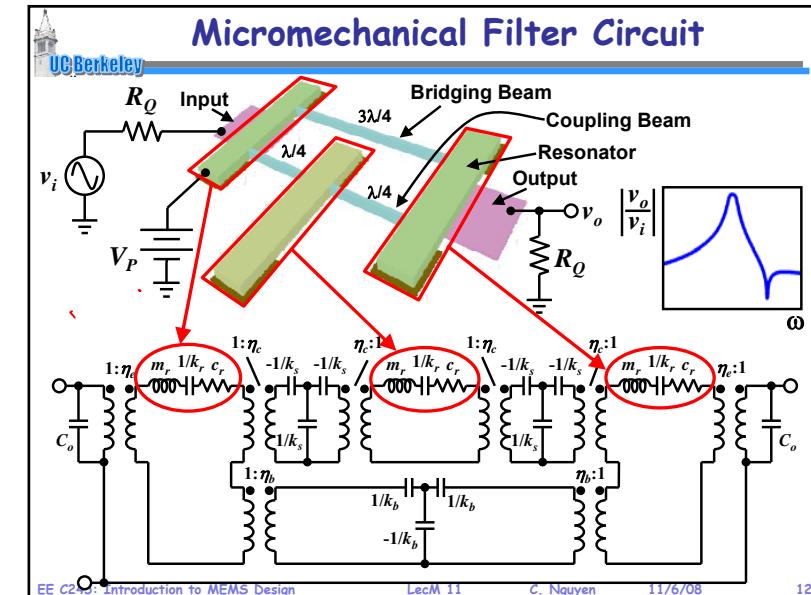
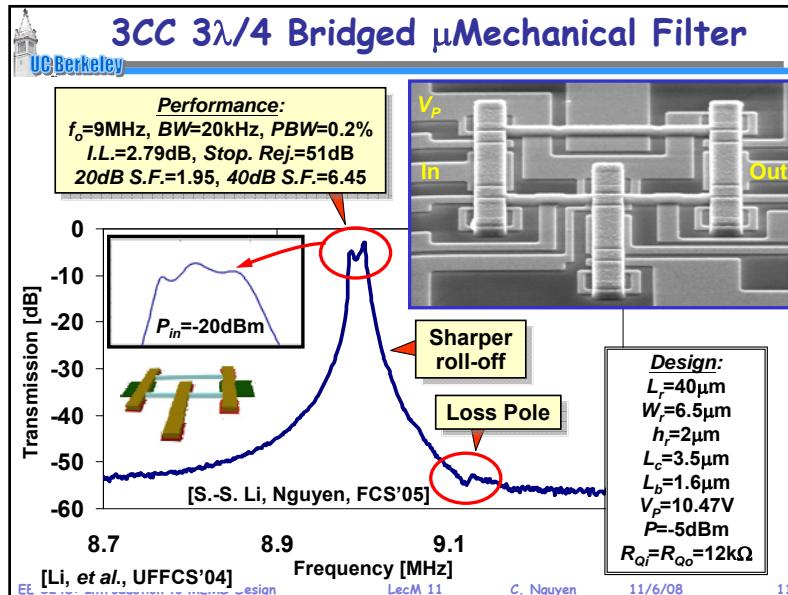
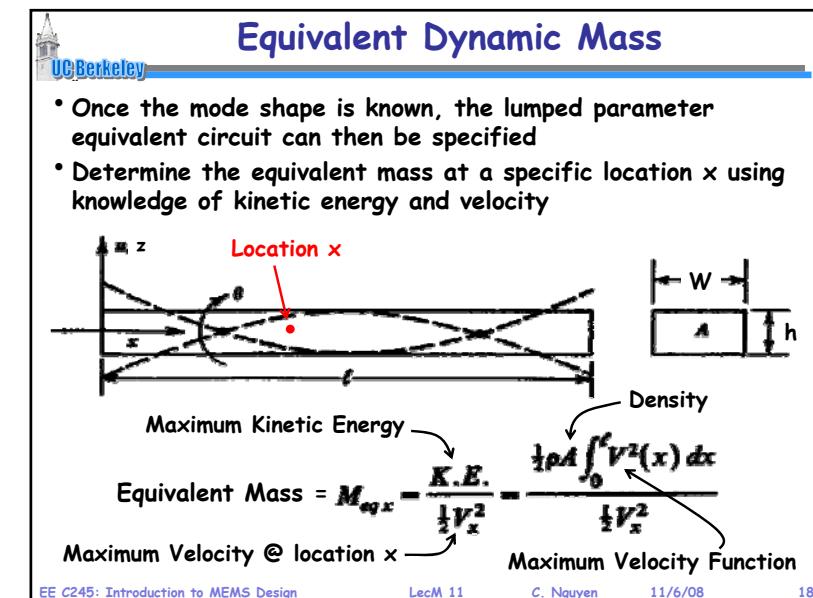
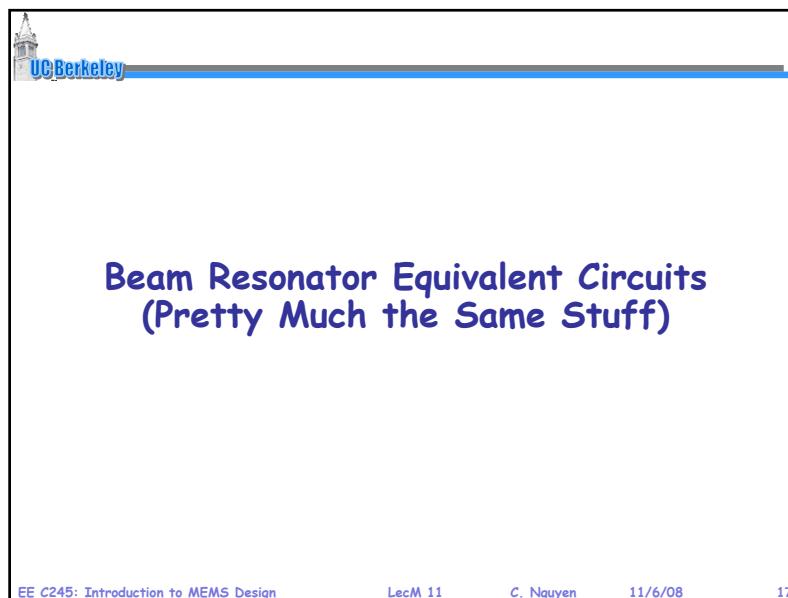
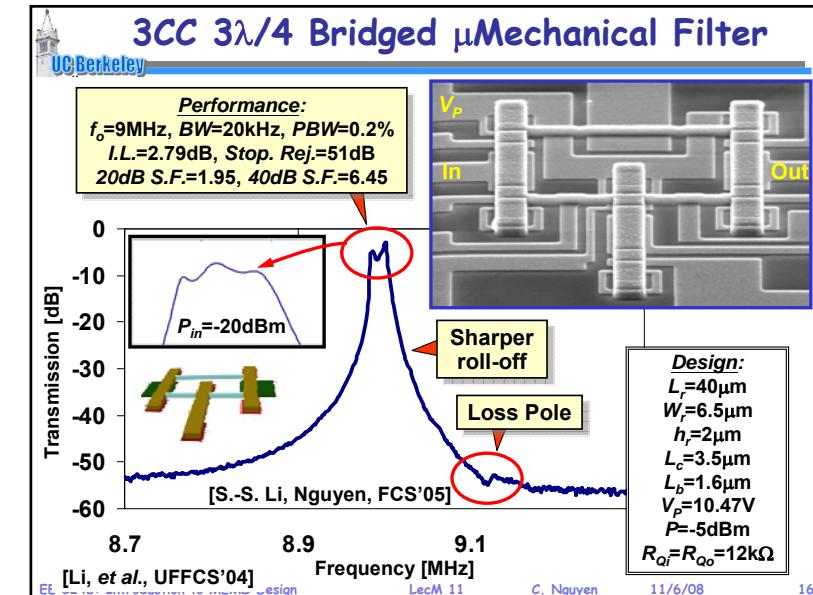
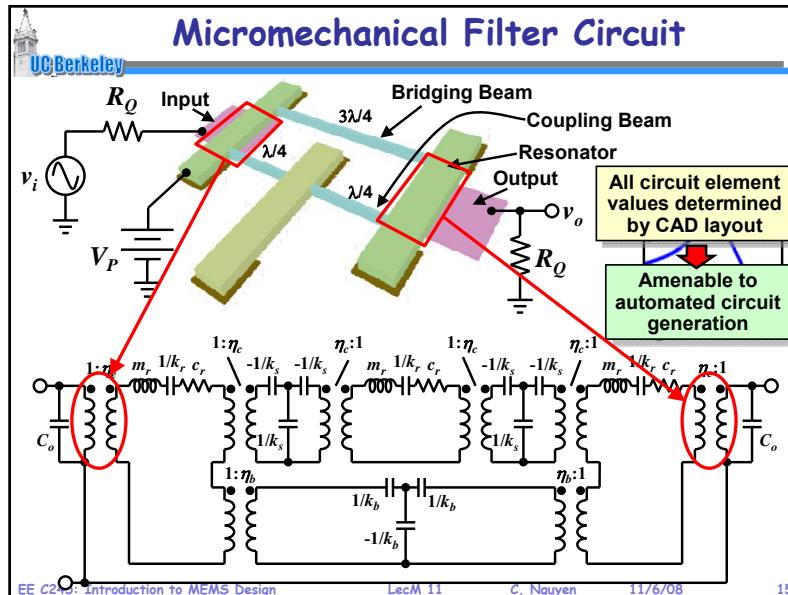


Lecture 16m1: Equivalent Circuits I



Lecture 16m1: Equivalent Circuits I



Lecture 16m1: Equivalent Circuits I

Equivalent Dynamic Mass

- We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B \left[S(\cosh kx + \cos kx') + (\sinh kx + \sin kx') \right], S = \frac{A}{B}$

$$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\cancel{K} \rho A}{\cancel{\frac{1}{2}}} \int_0^l \omega_0^2 [u(x')]^2 dx'$$

$$M_{eq}(x) = \rho A \int_0^l \cancel{B^2} \left[S(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx') \right]^2 dx'$$

$$\cancel{B^2} \left[S(\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]^2$$

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Equivalent Dynamic Stiffness & Damping

- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

- And damping also follows readily

$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

cancel damping

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Equivalent Lumped Mechanical Circuit

$K_{eq}(x) = \omega_0^2 M_{eq}(x)$

$M_{eq}(x) = \frac{\rho A \int_0^l [u(x')]^2 dx'}{[u(x)]^2}$

$C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q}$

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Equivalent Lumped Mechanical Circuit

Example: Polysilicon w/ $\ell=14.9\mu\text{m}$, $W=6\mu\text{m}$, $h=2\mu\text{m} \rightarrow 70 \text{ MHz}$

$K_{eq}(0) = 19,927 \text{ N/m}$
 $M_{eq}(0) = 1.03 \times 10^{-13} \text{ kg}$
 $C_{eq}(0) = 5.66 \times 10^{-9} \text{ kg/s}$

$K_{eq}(\ell/2) = 53,938 \text{ N/m}$
 $M_{eq}(\ell/2) = 2.78 \times 10^{-13} \text{ kg}$
 $C_{eq}(\ell/2) = 1.53 \times 10^{-8} \text{ kg/s}$

$K_{eq}(\text{node}) = \infty$
 $M_{eq}(\text{node}) = \infty$
 $C_{eq}(\text{node}) = \infty$

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