Lecture 16: Capacitive Transducers

- Announcements:
- HW#5 will be posted soon
- Module 12 titled Capacitive Transducers is online
- Project Definition described today
- Graded midterm coming back at end of class today
- -----------------------------------
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
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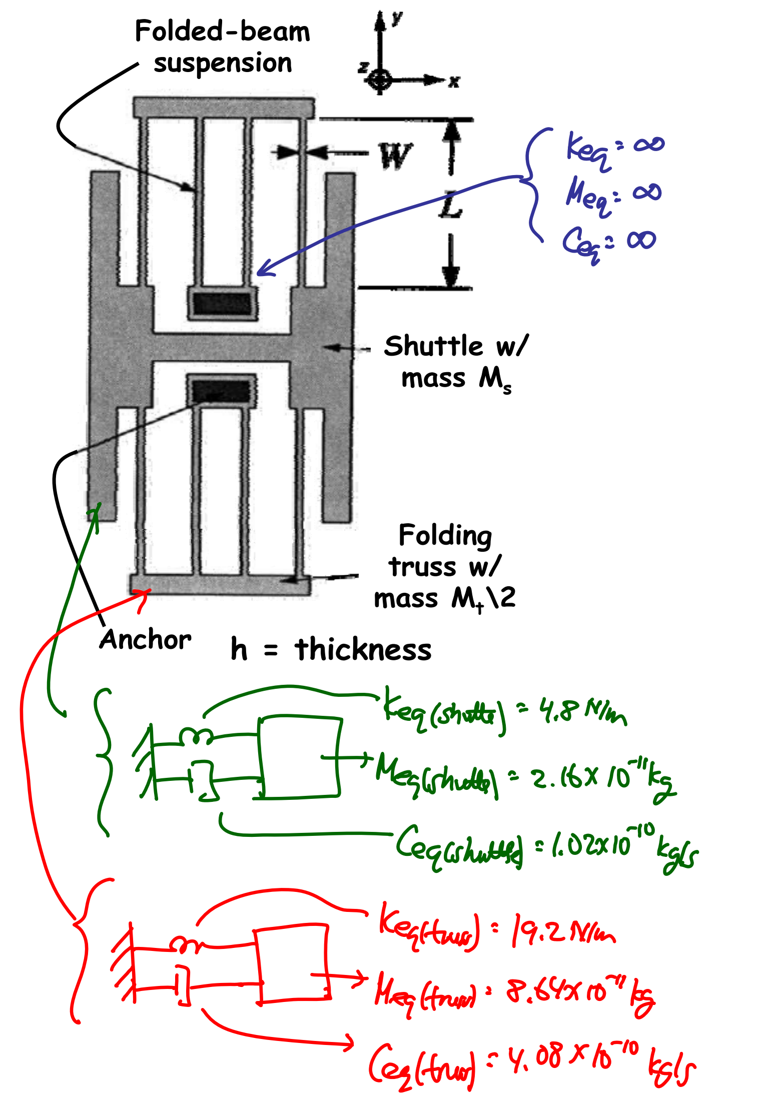
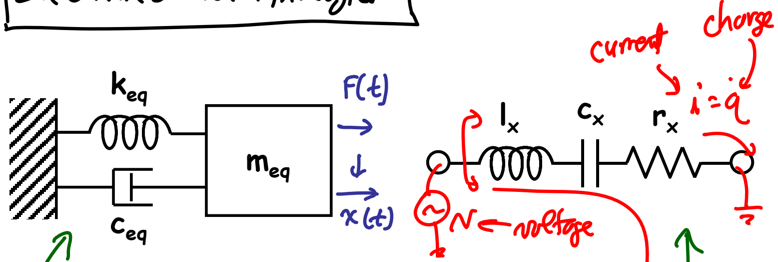
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- -----------------------------------
- Last Time:
- Looking at electromechanical analogies



Electromechanical Analogies



$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos(\omega t)$

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

→ using phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

Impedance looking into:

$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

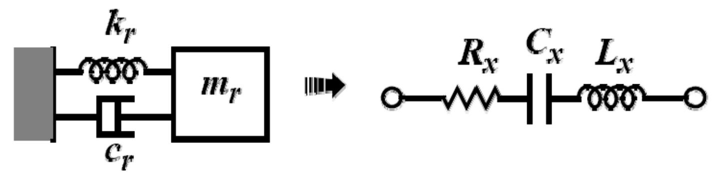
$$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$$

⇒ by analogy:

$F \rightarrow v$ $m_{eq} \rightarrow l_x$ $c_{eq} \rightarrow r_x$

$\dot{x} \rightarrow i$ $k_{eq} \rightarrow \frac{1}{c_x}$

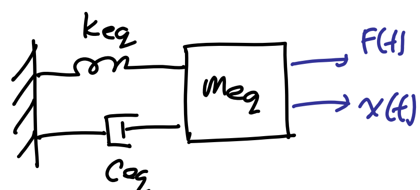
[Parameter Relationships in the Current analogy]



- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

Lowpass Biquad Transfer Function



$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ convert to full phasor form:

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$$

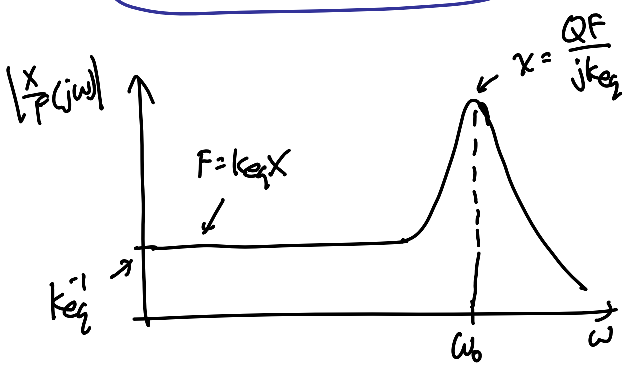
$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\omega\frac{c_{eq}}{k_{eq}}\right]^{-1}$$

*

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, \quad Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

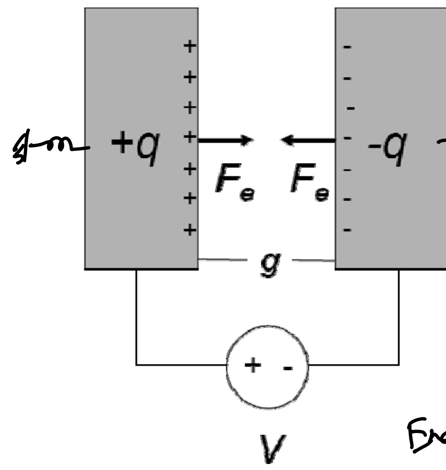
$$* \rightarrow \frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]^{-1}$$

$$\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}$$



- Go through pages 11-22 of Module 11
- Then, start into Module 12

Basic Physics of Electrostatic Actuation



Note: Assume the plates are supported elastically

Goal: Determine gap spacing g as a function of input variables.

Energy defines the needed relationships.

1st: Determine the energy of the system.

2nd: Ask, what can be done to Δ the energy of the system?

① Change the charge q.

② Change the separation g.

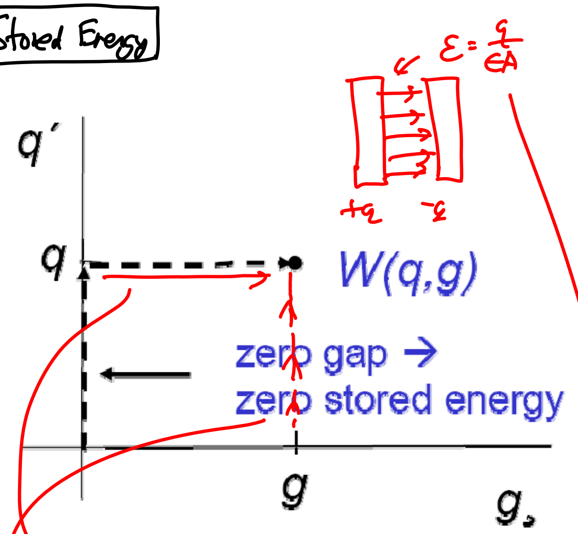
$$\Delta W(q,g) = V\Delta q + F_e \Delta g$$

$$dW = Vdq + F_e\,dg \leftarrow$$

hold q = const. → $Vdq \rightarrow 0$

$$dW = F_e\,dg \rightarrow F_e = \left.\frac{dW}{dg}\right|_{q = \text{const.}}$$

Stored Energy



No change in charge: $dq = 0$

$$W = 0 + \int_0^g F_e\, dg'$$

$$F_e = \left(\frac{q}{2}\right)\varepsilon = \frac{1}{2}\frac{q^2}{\epsilon A} \quad \text{(independent of } g\text{)}$$

$$\therefore W = \int_0^g F_e\, dg' = F_e g' \Big|_0^g = F_e g$$

$$\boxed{W(g) = \frac{1}{2}\frac{q^2}{\epsilon A} g}$$

Energy change needed to charge a C to q at a fixed gap g.

$$dW = V dq + F_e \cancel{dg}$$

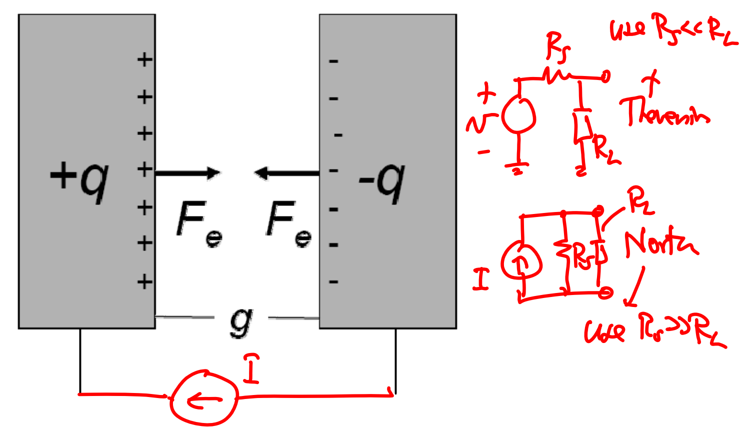
For a capacitor:

$$q = CV \rightarrow V = \frac{q}{C}$$

$$\therefore W(q) = \int_0^q V\, dq = \int_0^q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{q^2}{C}$$

$$\boxed{W(q) = \frac{1}{2}\frac{q^2}{\epsilon A} g}$$

Charge Control Case



From $dW = V dq + F_e dg$

⇒ Force is given by:

$$F_e = \frac{\partial W(q,g)}{\partial g}\Big|_{q = \text{const.}} = \frac{\partial}{\partial g}\left(\frac{1}{2}\frac{q^2}{\epsilon A} g\right)$$

$$\therefore \boxed{F_e = \frac{1}{2}\frac{q^2}{\epsilon A}} \quad \text{(indep. of gap spacing!)}$$

⇒ voltage is given by:

$$V = \frac{\partial \mathcal{W}(q,g)}{\partial q}\bigg|_{g=\text{const.}} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{\epsilon A}g\right)$$

$$= \frac{qg}{\epsilon A} \Rightarrow \boxed{V = \frac{q}{C}} \checkmark$$

(consistent w/ what we already know)