Lecture 18: ke and Comb Drive

- Announcements:
- HW#5 online; due Wednesday, April 15
- Project slide #1 due Friday, April 10
- I will be on travel next week; will either record lectures or do make-up lectures next Friday
- Will announce via Piazza
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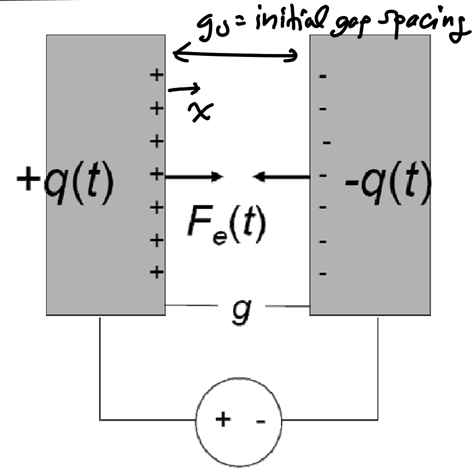
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- -----------------------------------
- Last Time:
- Looking at methods for linearizing the voltage-to-force transfer function of a parallel-plate capacitive-gap transducer

over

Linearizing the Voltage-to-Force Transfer Function



$$v(t) = \underbrace{V_P}_{\text{Bias (DC)}} + \underbrace{v_i(t)}_{\text{Signal (AC)}}$$

$$F_e(t) = \frac{\partial W'}{\partial x} = \frac{\partial}{\partial x}\left[\frac{1}{2} C [v(t)]^2\right]$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}\frac{\partial C}{\partial x}\left(V_P + v_i(t)\right)^2$$

$$= \frac{1}{2}\left[V_P^2 + 2V_P v_i(t) + \cancel{[v_i(t)]^2}^{\,0}\right]\frac{\partial C}{\partial x}$$

$$[V_P \gg v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2}V_P^2\frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_P\frac{\partial C}{\partial x}v_i(t)}_{\text{AC drive signal}}$$

What's $\frac{\partial C}{\partial x}$?

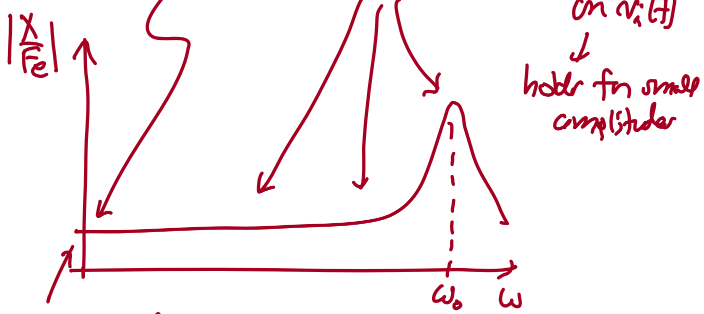
$$C_0 = \frac{\epsilon A}{g_0} \rightarrow C(x) = \frac{\epsilon A}{g_0 - x} = C_0\left(1 - \frac{x}{g_0}\right)^{-1}$$

$$[x \ll g_0] \Rightarrow \approx C_0\left(1 + \frac{x}{g_0}\right) = C(x)$$

$$\therefore \frac{\partial C}{\partial x} = \frac{C_o}{g_o} = \frac{\epsilon A}{g_o^2}$$

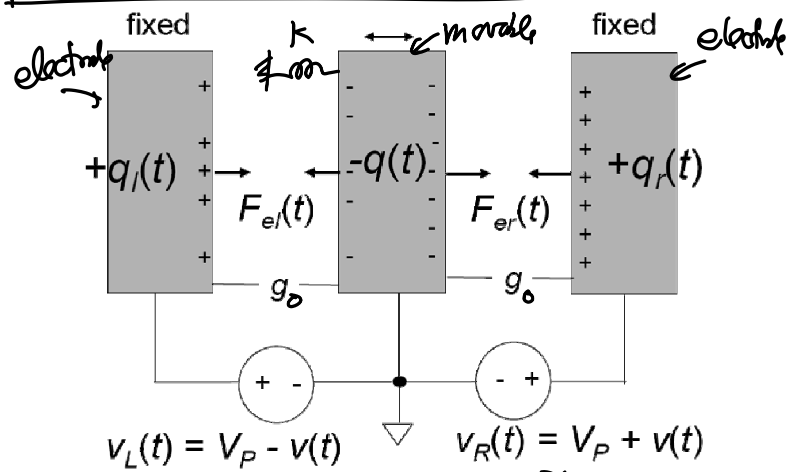
$$\Rightarrow F_e(t) = \underbrace{\frac{1}{2}\frac{C_o}{g_o}V_P^2}_{\text{DC Offset}} + \underbrace{V_P\frac{C_o}{g_o}v_i(t)}_{\text{Signal Component}} \qquad v_i(t) = V_i\cos\omega t$$

this is a linear dependence on $v_i(t)$ → holds for small amplitudes



very small response → but must still worry about pull in V_{PI}

$\boxed{\text{Cancel the DC Offset Using Differential Symmetry}}$



differential input into balanced structure

$$F_{net} = F_{er}(t) - F_{el}(t)$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$$

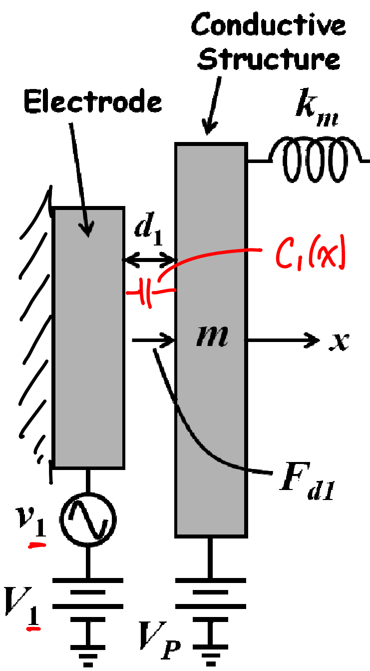
$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\cancel{V_P^2} + 2V_P v(t) + \cancel{[v(t)]^2} - \left(\cancel{V_P^2} - 2V_P v(t) + \cancel{[v(t)]^2}\right)\right\}$$

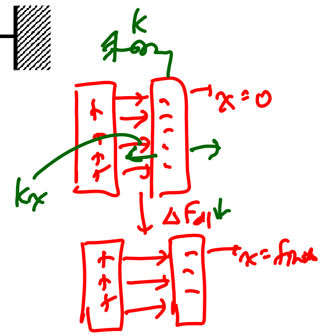
$$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_o}{g_o}v(t)}$$

→ No DC component → less pull-in problems, but only to the extent of matching

→ Linear w/ $v(t)$!

Nonlinearity Still Effects ω_r





More Complete Expressions

$$C_1(x) = \frac{\epsilon A}{d_1+x} = C_{01}\left(1+\frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1+\frac{x}{d_1}\right)^{-2}$$

[Expand into Taylor series]

$$\frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$, $A_2 = \frac{3}{d_1^2}$, $A_3 = -\frac{4}{d_1^3}$, ...

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(V_P - V_1 - v_1)^2 = \frac{1}{2}\frac{\partial C_1}{\partial x}(V_{P1} - v_1)^2$$

$$V_{P1} = V_P - V_1$$

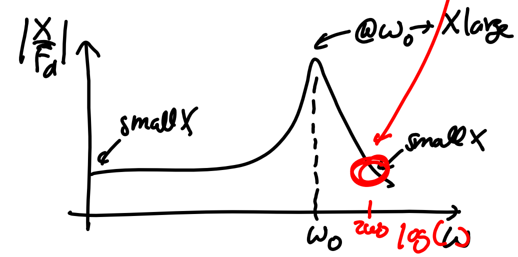
[small displacements: $x \ll d_1$]

$$F_{d1} = \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)(1+A_1x)(V_{P1}^2 - 2V_{P1}v_1 + v_1^2)$$

$$= \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)\left\{V_{P1}^2 - 2V_{P1}v_1 + v_1^2 + A_1V_{P1}^2x - 2A_1V_{P1}xv_1 + A_1xv_1^2\right\}$$

$$\frac{|v_1|}{2}(1+\cos 2\omega_0 t) = |v_1|^2\cos^2\omega_0 t$$

Resonance:



@ resonance:

$$x = \frac{QF_{d1}}{jk} \approx \frac{Q}{jk}\underbrace{\frac{\partial C_1}{\partial x}V_{P1}v_1}_{\text{approx. } F_{d1}}$$

approx. F_{d1} ← linear term

jk ↑ 90° phase shift

$$v_1 = |v_1|\cos\omega_0 t \rightarrow x = |x|\sin\omega_0 t$$

↑ 90° phase-shifted

Pick out force terms @ ω_0

$k_e \rightarrow$ electrical stiffness

$$F_{d1}\Big|_{\omega_0} = \underbrace{V_{P1}\frac{C_{01}}{d_1}|v_i|\cos\omega_0 t}_{\text{drive force term}} + V_{P1}^2\frac{C_{01}}{d_1^2}|x|\sin\omega_0 t$$

proportional to x

90° phase-shifted from $\therefore$ in phase w/ displacement!

Force term in phase as displacement → it's a stiffness!

Electrical Stiffness:

① A negative spring constant!

② Derived from V_P:

$$\boxed{k_e = V_{P1}^2\frac{C_{01}}{d_0^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

overlap area of C

DC Bias

gap → 3rd order dependence on gap!

$k_e \rightarrow$ can affect the resonance freq., f_0!

$\omega_0 \triangleq$ radian resonance freq. w/ no V_P applied (i.e., $V_P = 0V$)

$$\omega_0' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

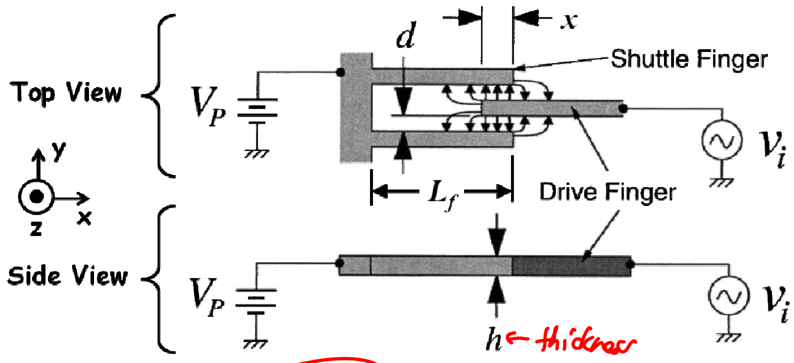
$$= \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_0' = \omega_0\left[1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right]^{1/2}}$$

now a fcn of dc-bias voltage! (voltage-controllable!)

- Go through Module 12 slides 26-35

Electrostatic Comb-Drive



$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$ Need $C(x)$.

$\left[C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$ ← Not a fcn of x!

↓ $k_e = 0$!

$F_d = \frac{1}{\cancel{2}}\frac{\cancel{2}\epsilon_0 h}{d}\left(\cancel{V_P^2} - 2V_P v_i + \cancel{v_i^2}\right)$

can balance out by symmetrically placed electrodes

$v_i \ll V_P$

$$\boxed{F_d = -2V_P \frac{\epsilon_0 h}{d} v_i}$$

- Go through remaning comb-drive slides in Module 12