Lecture 20: Circuit Analysis & Gyros

- Announcements:
- I am out-of-town; this is a pre-recorded lecture
- HW#5 due tomorrow; HW#6 online soon
- Module 14 on Sensing Circuits online
- Module 15 on Gyros, Noise, & MDS online
- Project slide #2 due Friday, April 17
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- Reading: Senturia, Chpt. 6, Chpt. 14
- Equivalent Circuits II Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
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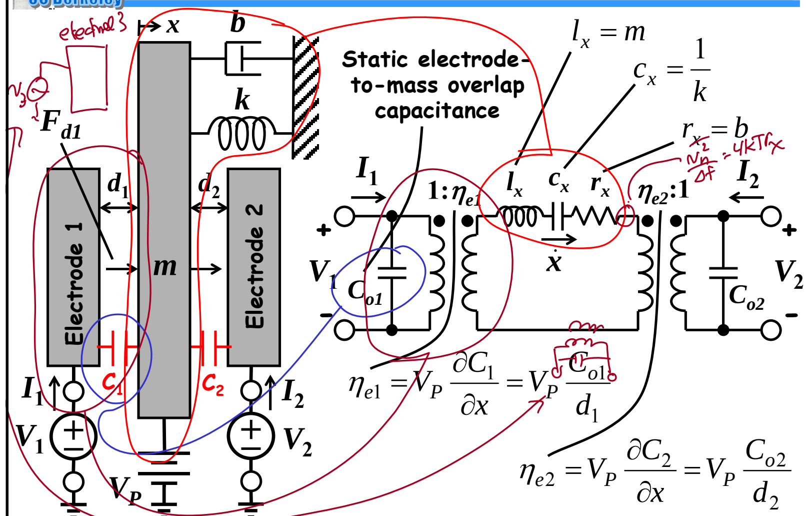
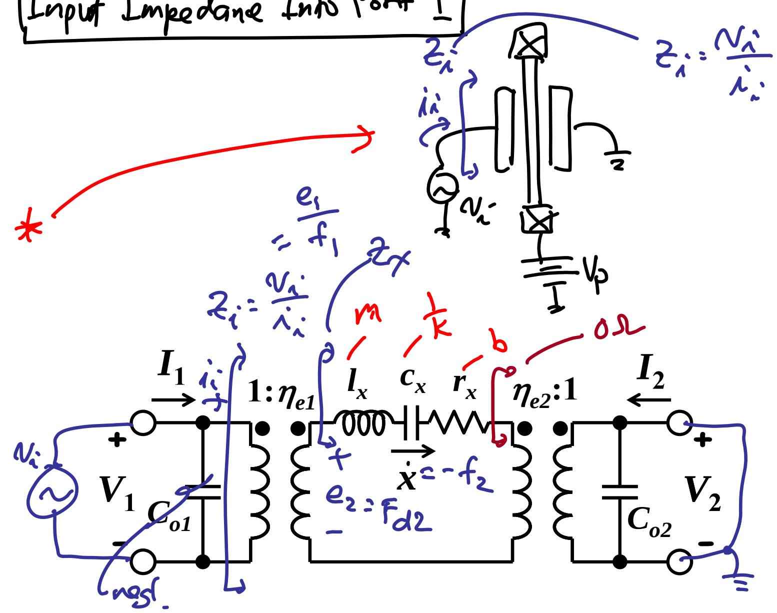
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
- --
- Last Time: Presented the complete electrical equivalent circuit for a general MEMS structure

Complete Electrical-Port Equiv. Circuit



Input Impedance Into Port I



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \quad e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta}, \quad f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2$$

$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \longrightarrow \frac{v_i}{i_i} = Z_i = -\frac{1}{\eta^2}\frac{F_{d2}}{(-\dot{x}_2)}$$

$$Z_i = \frac{1}{\eta_{e1}^2} Z_x$$

$$Z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

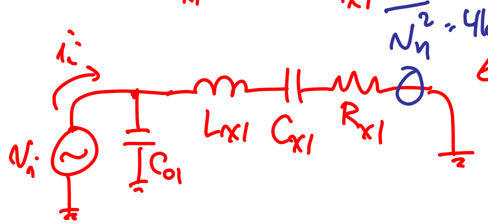
$$= \underbrace{j\omega\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

m, k, b

to model noise!

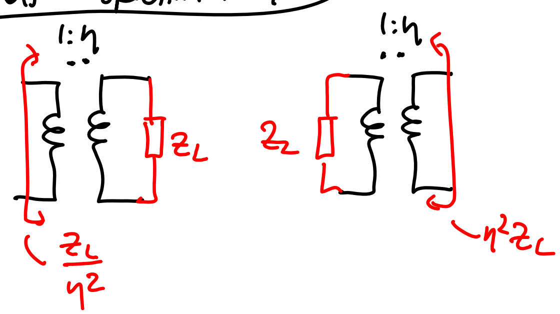
$$\overline{v_n^2} = 4kTR_x\Delta f$$

*

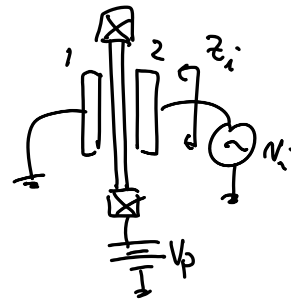


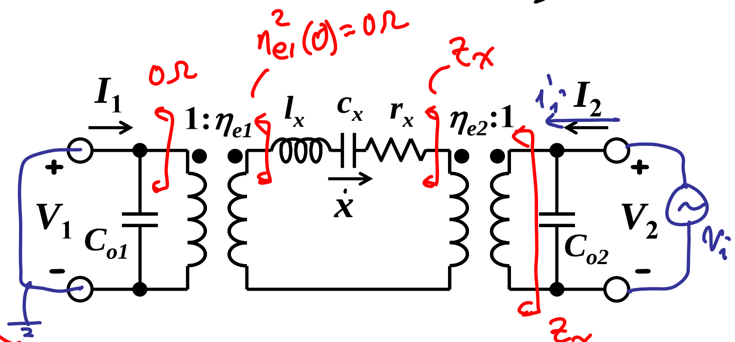
Purely Electrical Equiv. Ckt.

Xformer Inspection Analysis



Input Impedance Into Port 2

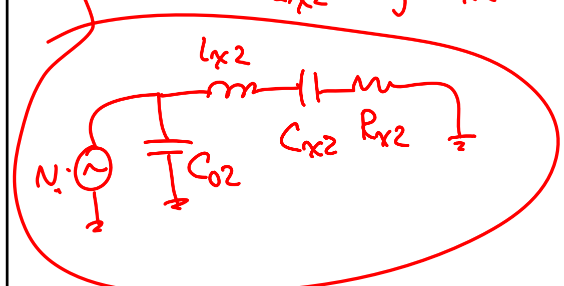




$$\frac{Z_x}{\eta_{e2}^2}$$

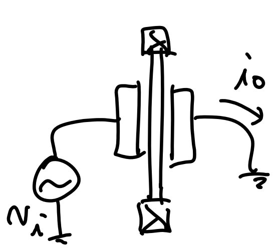
$$Z_i = \frac{v_i}{i_i} = \frac{Z_x}{\eta_{e2}^2}$$

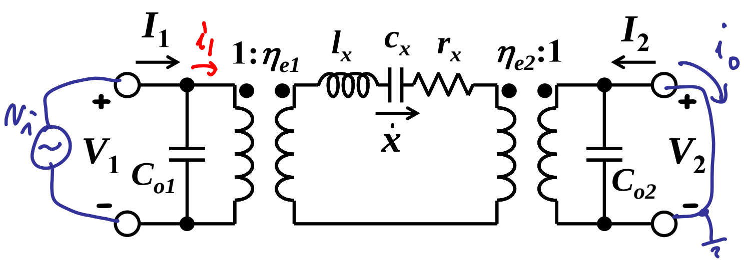
$$= \underbrace{j\omega\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \underbrace{\frac{1}{j\omega(\eta_{e2}^2 c_x)}}_{\frac{1}{j\omega C_{x2}}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1 to Port 2 Transconductance

$$G = \frac{\dot{i}_o}{v_i}$$





$$\dot{x} = \frac{1}{\eta_{e1}} \dot{i}_1 \qquad \dot{i}_o = \eta_{e2}\dot{x}$$

$$\rightarrow \dot{i}_o = \frac{\eta_{e2}}{\eta_{e1}} \dot{i}_1 = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{Z_i}\right)$$

$$= \frac{\eta_{e2}}{\eta_{e1}} v_i \left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$$

$$\frac{\dot{i}_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}$$

$$\frac{\dot{i}_o}{v_i}(j\omega) = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1}$$

$$L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}, \quad C_{x12} = \eta_{e1}\eta_{e2} c_x, \quad R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$$

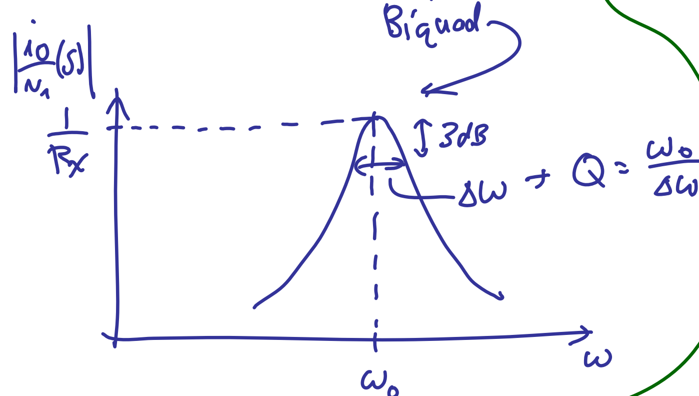
Separate freq. response & magnitude:

$$\frac{\dot{i}_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_xC_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left[\frac{1}{L_xC_x} = \omega_o^2, \quad Q = \frac{\omega_o L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o}{Q}\right]$$

$$\frac{\dot{i}_o}{v_i}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain Term}} \underbrace{\frac{s\left(\frac{\omega_o}{Q}\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2}}_{\text{Freq. Shaping Term}} = \underbrace{\frac{1}{R_x}}_{\text{resonance magnitude}} H(s)$$

Freq. Shaping Term → Bandpass Biquad

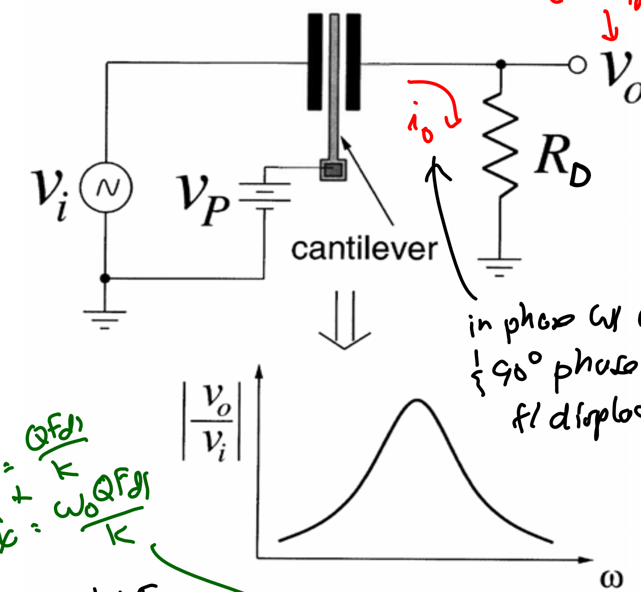


significance: Can just solve the ckt. @ resonance, then multiply by a proper $H(s)$!

- Go through Module 15 slides 1-12

Velocity-to-Voltage Conversion

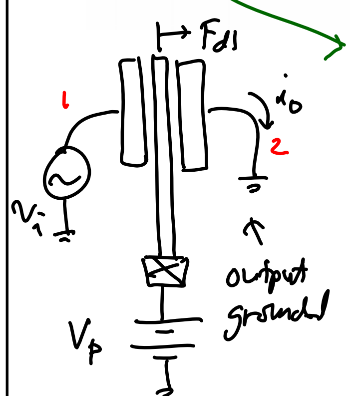
represent velocity



in phase w/ velocity & 90° phase shift f/ displacement

$x = \frac{QF_{d1}}{k}$

$\dot{x} = \frac{\omega_0 QF_{d1}}{k}$



$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_0 Q}{k} H(s)$$

$$\left[F_{d1} = \eta_{e1} v_i\right]$$

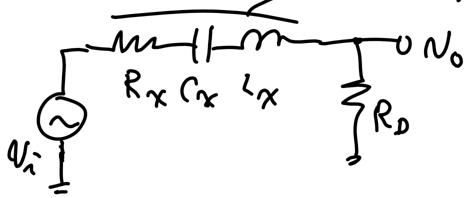
$$\frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_0 Q}{k} H(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_0 Q}{k} H(s)$$

$$= \frac{\eta_{e1}\eta_{e2} Q}{m\omega_0} H(s)$$

$\frac{1}{R_{x12}}$

Now, include R_D: $\quad Q = \frac{\omega_o L_x}{R_x}$



$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_D + R_x + \frac{1}{sC_x} + sL_x} = \cdots \text{math} \cdots$$

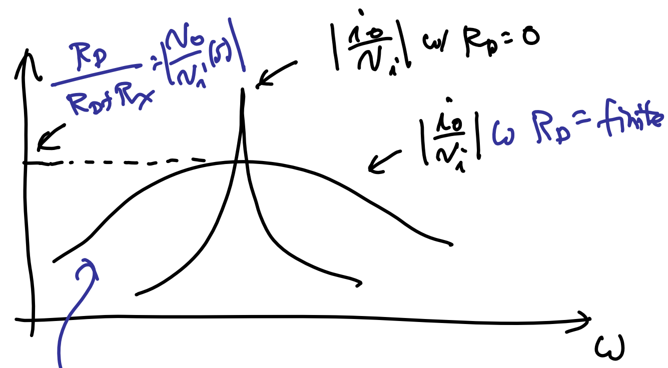
$$= \underbrace{\frac{R_D}{R_D+R_x}}_{\text{Gain Term}} \; \underbrace{\frac{s\left(\frac{R_x+R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x+R_D}{L_x}\right) + \frac{1}{L_xC_x}}}_{\text{Freq. Shaping Term}}$$

$$\left[Q = \frac{\omega_o L_x}{R_x} \rightarrow Q' = \frac{\omega_o L_x}{R_x + R_D} \rightarrow \frac{R_x+R_D}{L_x} = \frac{\omega_o}{Q'} \right]$$

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_D+R_x} \; \frac{s(\omega_o/Q')}{s^2 + s(\omega_o/Q') + \omega_o^2}$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_D}{R_D+R_x} \cdot H(s, Q') \qquad Q' = Q\left(\frac{R_x}{R_x+R_D}\right)}$$

proportional to velocity



$\left|\frac{v_o}{v_i}(s)\right| \Rightarrow Q'$ = lower than Q $\Big\}$ Big Problem!

$= Q\underbrace{\left(\frac{R_x}{R_x+R_D}\right)}_{<1}$