Lecture 23: Noise

- **Announcements:**
- Module 17 on Noise & MDS online
- HW#7 online next week and due Friday morning, May 8
- Project slide #3 due Friday, May 1
- This is a pre-recorded video lecture
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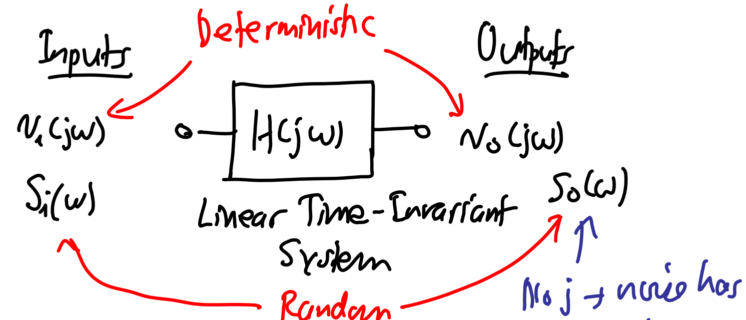
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- ------------------------------------
- **Last Time:**
- Went through Module 16 on sensing circuit non-idealities and integration
- Now continue with another important non-ideality: Noise
- Start with slide 20 of Module 16, then …
- Start into Module 17 on Noise & MDS, slides 1-7

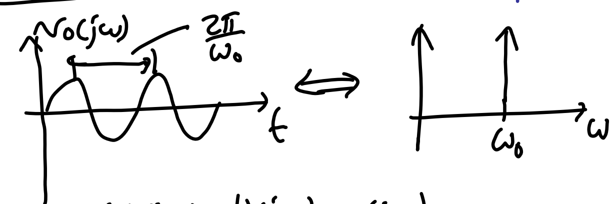
over

Circuit Noise Calculations



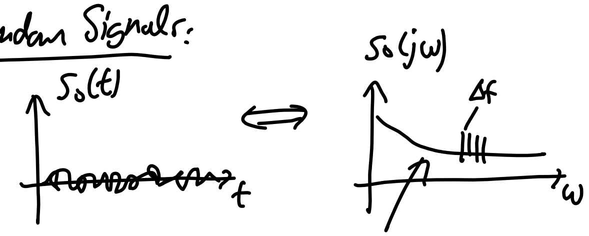
No j → noise has random phase, so j is pointless!

Deterministic Signals:



$$v_o(j\omega) = H(j\omega)\, v_i(j\omega)$$

Random Signals:



Mean-Square Spectral Density

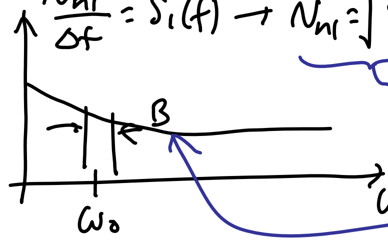
$$S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$$

$$\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$$ → How is it we can do this?

Root mean-square amplitudes

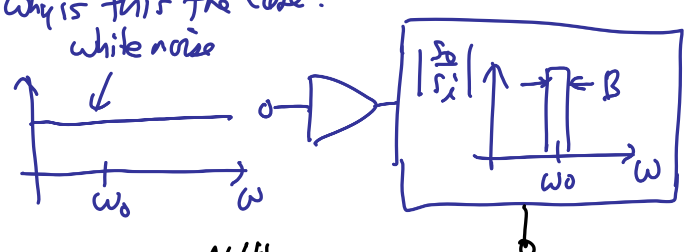
(Handling Noise Deterministically)

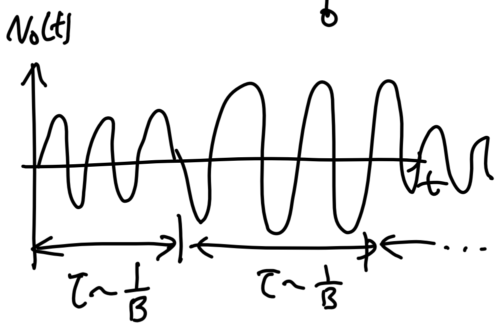
$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_1(f) \rightarrow v_{n1} = \sqrt{S_1(f)\,B}$$



Can approximate this by a sinusoidal voltage generator (esp. when B is small, say 1Hz)

Why is this the case?

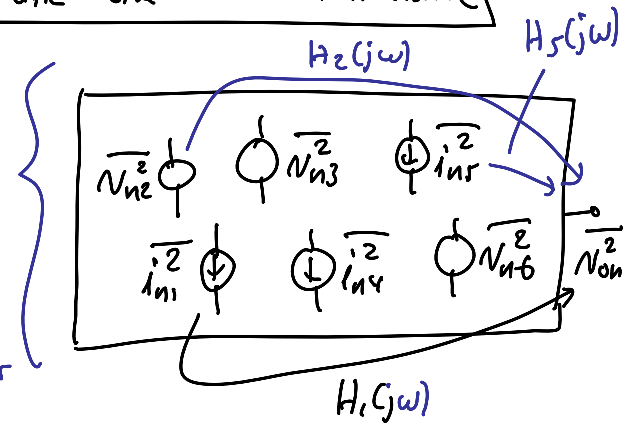




Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B!

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources



Assume noise sources are uncorrelated.

1. For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value $i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f} \cdot (1\text{Hz})}$
2. Calculate $v_{on1}(\omega) = i_{n1}(\omega)\, H_1(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H_1(j\omega)|^2$
4. Repeat for each noise source: $\overline{v_{n2}^2}, \overline{v_{n3}^2}, \overline{i_{n4}^2}, \ldots \rightarrow$ output

⑤ Add noise powers (mean-square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

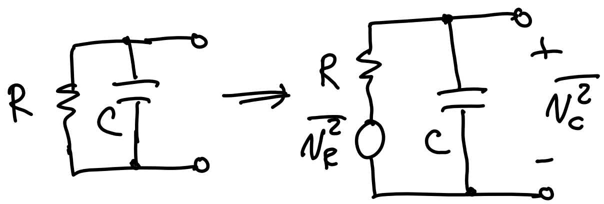
$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

↑ total rms value

- Go through Module 17, slides 12-16

$\boxed{\text{Why } \frac{\overline{v_{nR}^2}}{\Delta f} = 4kTR?}$ (a heuristic argument)

Consider an RC ckt:



$$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_c^2}$$

$\therefore \boxed{\overline{v_c^2} = \frac{kT}{C}}$ ← integrated noise over all freqs. (total mean-square voltage integrated over all freqs.)

*

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ gives us this (*) (assuming white noise)

$$\overline{v_c^2} = \int_0^\infty \left|\frac{1}{1+j\omega RC}\right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df$$

[noise is white] [$\omega_b = \frac{1}{RC}$] $\rightarrow = \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega}\, d\omega$

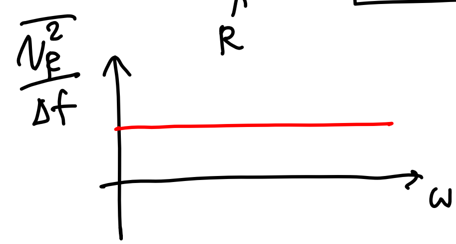
$$\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^2}{\omega_b}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Bigg|_0^\infty$$

$$= \frac{1}{2\cancel{\pi}}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kT\left(\frac{\omega_b}{C}\right) \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR} \checkmark$$

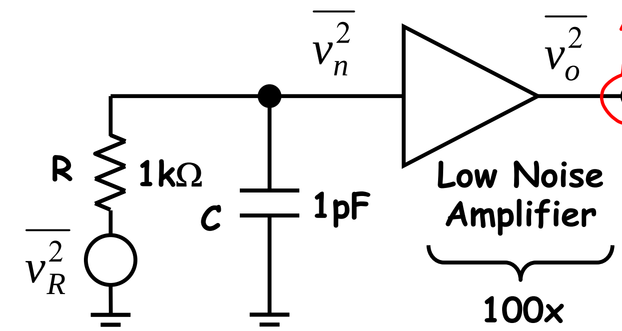
↑ R



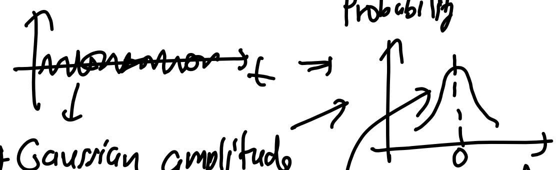
- Go through Module 17, slides 19-20

$\boxed{\text{Example. Typical Noise Numbers}}$

Measure w/ AC voltmeter



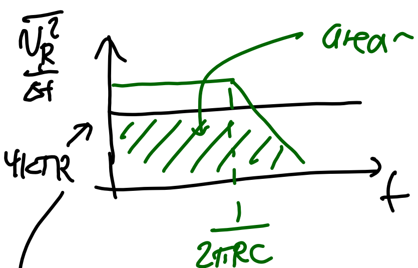
Measure on a Spectrum Analyzer



Get Gaussian amplitude distribution

68% within $\pm\sigma$

99.7% within $\pm 3\sigma$



$R = 1k\Omega \rightarrow \sqrt{(1.66\times10^{-20})(1K)}$ *

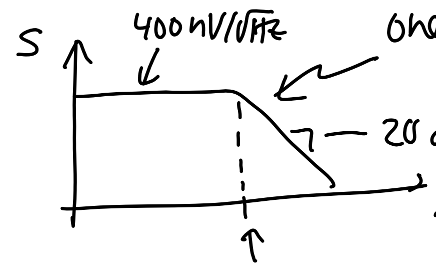
* — $1k\Omega$: $4nV/\sqrt{Hz}$ (for every 1K of R)

$1pF$: $\sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

Case: AC Voltmeter

$\sqrt{\overline{N_o^2}} = (100)(64\mu V_{rms}) = \underline{\underline{6.4mV\ rms}}$

Case: Spectrum Analyzer



$\frac{1}{2\pi(1K)(1p)} = 60MHz$

- Go through Module 17, slides 23-29