

### Circuit Noise Calculations

**Inputs:**  $v_i(j\omega)$  (Deterministic),  $S_i(\omega)$  (Random)

**Outputs:**  $v_o(j\omega)$  (Deterministic),  $S_o(\omega)$  (Random)

**System:** Linear Time-Invariant System  $H(j\omega)$

**Mean square spectral density:**  $S_o(\omega)$

**Root mean square amplitudes:**  $\sqrt{S_o(\omega)} = |H(j\omega)| \sqrt{S_i(\omega)}$

**Handwritten notes:** "No  $j$  noise near random phase, so  $j$  is pointless!"

- Deterministic:**  $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- Random:**  $S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$

How is it we can do this?

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### Handling Noise Deterministically

• Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

$\frac{v_{n1}^2}{\Delta f} = S_1(f) \rightarrow v_{n1} = \sqrt{S_1(f) \cdot B}$

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

**Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.**

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

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### Systematic Noise Calculation Procedure

**General Circuit With Several Noise Sources**

- Assume noise sources are uncorrelated
- 1. For  $i_{n1}^2$ , replace w/ a deterministic source of value  $i_{n1} = \sqrt{\frac{i_{n1}^2}{\Delta f}} \cdot (1 \text{ Hz})$

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### Systematic Noise Calculation Procedure

- Calculate  $v_{on1}(\omega) = i_{n1}(\omega)H_1(j\omega)$  (treating it like a deterministic signal)
- Determine  $v_{on1}^2 = i_{n1}^2 \cdot |H_1(j\omega)|^2$
- Repeat for each noise source:  $i_{n1}^2, v_{n2}^2, v_{n3}^2$
- Add noise power (mean square values)

$$v_{onTOT}^2 = v_{on1}^2 + v_{on2}^2 + v_{on3}^2 + v_{on4}^2 + \dots$$

$$v_{onTOT} = \sqrt{v_{on1}^2 + v_{on2}^2 + v_{on3}^2 + v_{on4}^2 + \dots}$$

Total rms value

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### Example: Typical Noise Numbers

Hookup the circuit below and make some measurements

Measure w/ AC voltmeter  
Measure w/ spectrum analyzer  
Get Gaussian amplitude distribution

Probability vs Amplitude  
68% within  $\pm\sigma$   
99.7% within  $\pm 3\sigma$

$4kTR$   
 $\frac{1}{2\pi RC}$   
 $1k\Omega: 4nV/\sqrt{Hz}$  (for every 1k of R)  
 $1pF: \sqrt{\frac{kT}{C}} = 64\mu V_{rms}$

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### Example: Typical Noise Numbers

Hookup the circuit below and make some measurements

Measure w/ AC voltmeter  
Measure w/ spectrum analyzer

AC Voltmeter  
 $\sqrt{N_o^2} = (100)(64\mu V_{rms}) = 6.4mV_{rms}$

Spectrum Analyzer  
 $\frac{1}{(2\pi)(1k)(1p)} = 60MHz$   
400 nV/ $\sqrt{Hz}$   
20dB/dec  
one-sided spectral density

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### Back to Determining Sensor Resolution

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### MEMS-Based Tuning Fork Gyroscope

Sense Electrodes, Tuning Electrodes, Drive Electrodes  
Drive Voltage Signal  
Drive Oscillation Sustaining Amplifier  
Differential TransR Sense Amplifier  
[Zaman, Ayazi, et al, MEMS'06]

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### Drive Axis Equivalent Circuit

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180°

180°

Drive Voltage Signal

Drive Oscillation Sustaining Amplifier

Digital PLL

To Sense Amplifier (for synchronization)

- Generates drive displacement velocity  $\dot{x}_d$  to which the Coriolis force is proportional

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### Drive-to-Sense Transfer Function

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Drive Mode

Sense Mode

Drive/Sense Response Spectra:

Amplitude

Drive Response

Sense Response

$f_o (@ T_1)$

Rotation-Induced Coriolis Force:

$$\vec{a}_c = 2\vec{\omega}_d \times \vec{v}_s = 2\omega_d \dot{x}_s \hat{z} \times \hat{y}$$

$$\Rightarrow \text{Acts in the sense mode direction}$$

$$a_s = 2\omega_d \dot{x}_d \Omega \sin 90^\circ$$

$$a_s = 2\omega_d \dot{x}_d \Omega$$

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### Gyro Readout Equivalent Circuit (for a single tine)

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Noise Sources

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$

parallel-plate sense electrodes

noises associated w/

Coriolis force

Gyro Sense Element Output Circuit

Signal Conditioning Circuit (Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

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### Minimum Detectable Signal (MDS)

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Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

Sensed Signal

MDS

Sensor

Signal Conditioning Circuit

Output

Noise dominates the minimum detectable signal

Includes desired output plus noise

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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### Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

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### Equivalent Input-Referred Voltage and Current Noise Sources

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### Equivalent Input $v$ , $i$ Noise Generators

- Take a noisy 2-port network and represent it by a noiseless network with input  $v$  and  $i$  noise generators that generate the same total output noise

- **Remarks:**
  1. Works for linear time-invariant networks
  2.  $v_{eq}$  and  $i_{eq}$  are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of  $v_{eq}$  and  $i_{eq}$  dominates, which removes the need to address correlation
  4. If correlation is important  $\rightarrow$  easier to return to original network with internal noise sources

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### Calculation of $v_{eq}^2$ and $i_{eq}^2$

a) To get  $v_{eq}^2$  for a two-port:

Case I

Case II

- 1) Short input, find  $v_{0I}^2$  (or  $i_{0I}^2$ )
- 2) For eq. network, short input, find  $v_{0II}^2$  (or  $i_{0II}^2$ )

$$\begin{matrix} \parallel & \parallel \\ f(v_{eq}^2) & f(i_{eq}^2) \end{matrix}$$

- 3) Set  $v_{0I}^2 = v_{0II}^2 \rightarrow$  solve for  $v_{eq}^2$  (or  $i_{eq}^2 = i_{0II}^2$ )

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### Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get  $\overline{i_{eq}^2}$  for a 2-port:

- 1) Open input, find  $\overline{v_{0I}^2}$  (or  $\overline{i_{0I}^2}$ )
- 2) Open input for eq. circuit, find  $\overline{v_{0II}^2}$  (or  $\overline{i_{0II}^2}$ )
- 3) Set  $\overline{v_{0I}^2} = \overline{v_{0II}^2} \left( \overline{i_{eq}^2} \right) \rightarrow$  solve for  $\overline{i_{eq}^2}$  (or  $\overline{i_{0I}^2} = \overline{i_{0II}^2} \left( \overline{i_{eq}^2} \right)$ )

• Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated

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### Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance  $R_s$  is **small** compared to input resistance  $R_i \rightarrow$  i.e., voltage source input
  2. Source resistance  $R_s$  is **large** compared to input resistance  $R_i \rightarrow$  i.e., current source input

1)  $R_s = \text{small}$  (ideally = 0 for an ideal voltage source):

$\therefore$  For  $R_s = \text{small}$ ,  $\overline{i_{eq}^2}$  can be neglected  $\rightarrow$  only  $\overline{v_{eq}^2}$  is important!  
(Thus, we need not deal with correlation)

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### Cases Where Correlation Is Not Important

2)  $R_s = \text{large}$  (Ideally =  $\infty$  for an ideal current source)

$R_s \gg R_i$

Voltage  $\overline{v_{eq}^2}$  effectively "opened" out!

$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$

$\therefore$  For  $R_s = \text{large}$ ,  $\overline{v_{eq}^2}$  can be neglected!  
 $\rightarrow$  only  $\overline{i_{eq}^2}$  is important!  
(... and again, we need not deal with correlation)

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### Example: TransR Amplifier Noise (cont)

- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

$$\overline{v_{eq}^2} = \overline{v_{ia}^2}$$

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