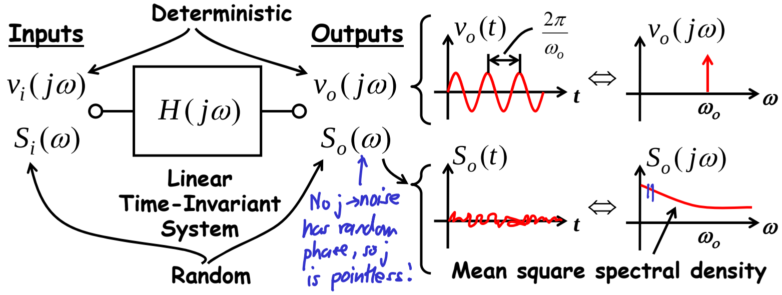
Circuit Noise Calculations



No j → noise has random phase, so j is pointless!

- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = \left|H(j\omega)\right|^2 S_i(\omega)$

$$\sqrt{S_o(\omega)} = \left|H(j\omega)\right|\sqrt{S_i(\omega)} \longrightarrow \text{How is it we can do this?}$$

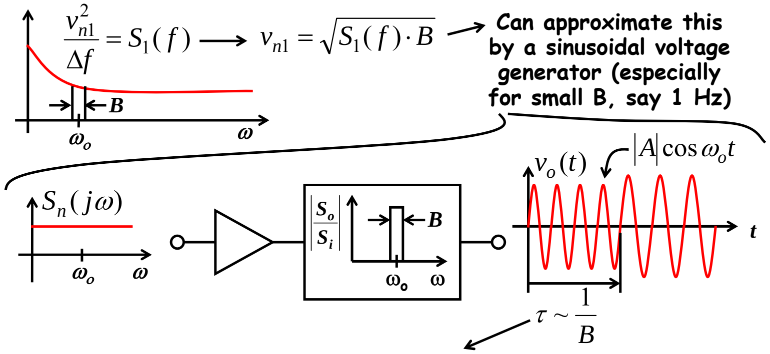
Root mean square amplitudes

Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_1(f) \longrightarrow v_{n1} = \sqrt{S_1(f)\cdot B} \rightarrow$$

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

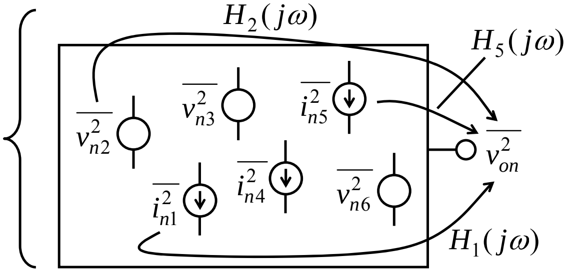


[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.

Systematic Noise Calculation Procedure

General Circuit With Several Noise Sources



- Assume noise sources are uncorrelated

1. For $\overline{i_{n1}^2}$, replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}\cdot(1\,\text{Hz})}$$

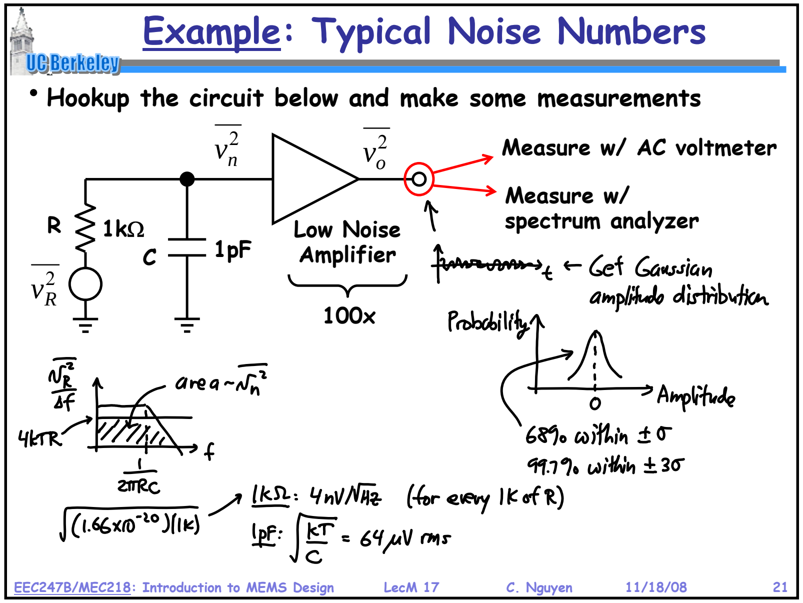
Systematic Noise Calculation Procedure

2. Calculate $v_{on1}(\omega) = i_{n1}(\omega)H_1(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2}\cdot\left|H_1(j\omega)\right|^2$
4. Repeat for each noise source: $\overline{i_{n1}^2}$, $\overline{v_{n2}^2}$, $\overline{v_{n3}^2}$
5. Add noise <u>power</u> (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

Total rms value



Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements

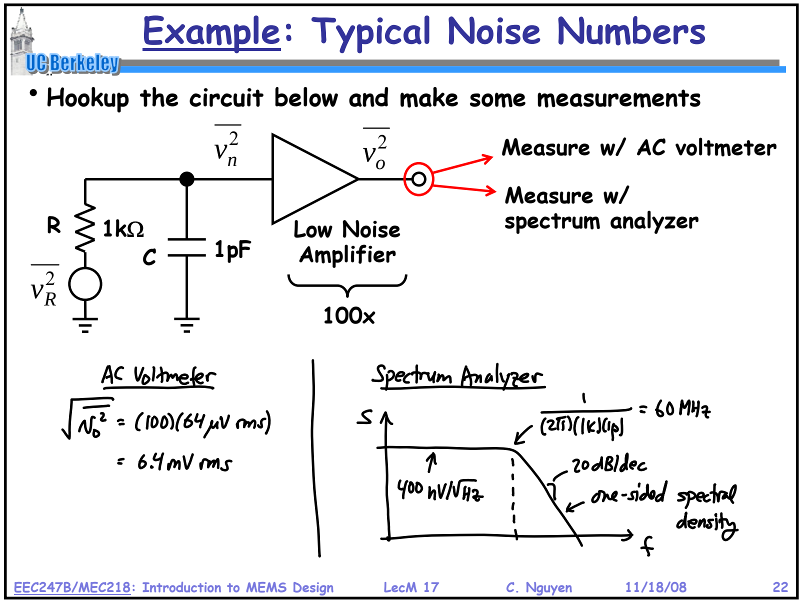
← Get Gaussian amplitude distribution

68% within ±σ

99.7% within ±3σ

$\sqrt{(1.66\times10^{-20})(1k)}$ → 1kΩ: $4nV/\sqrt{Hz}$ (for every 1K of R)

1pF: $\sqrt{\frac{kT}{C}} = 64\mu V$ rms



Example: Typical Noise Numbers

- Hookup the circuit below and make some measurements

AC Voltmeter

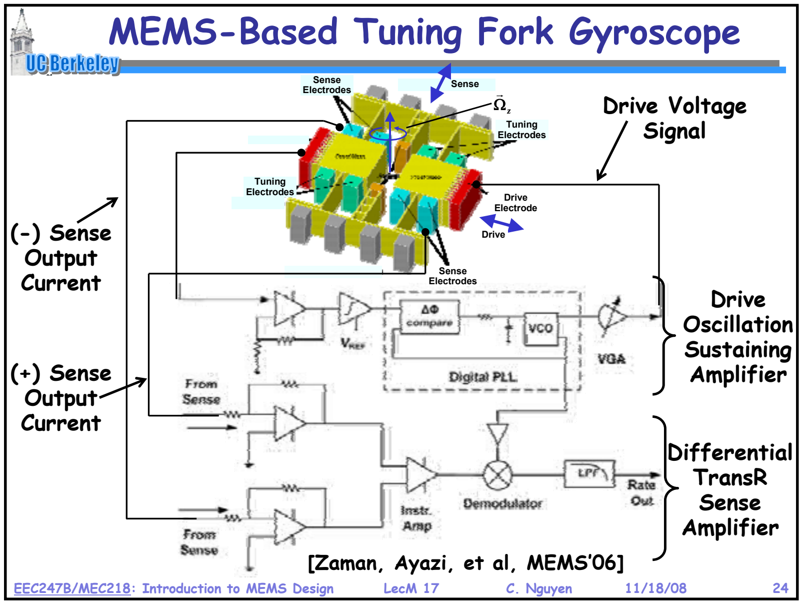
$\sqrt{\overline{v_o^2}} = (100)(64\mu V\ rms)$

$= 6.4mV$ rms

Spectrum Analyzer

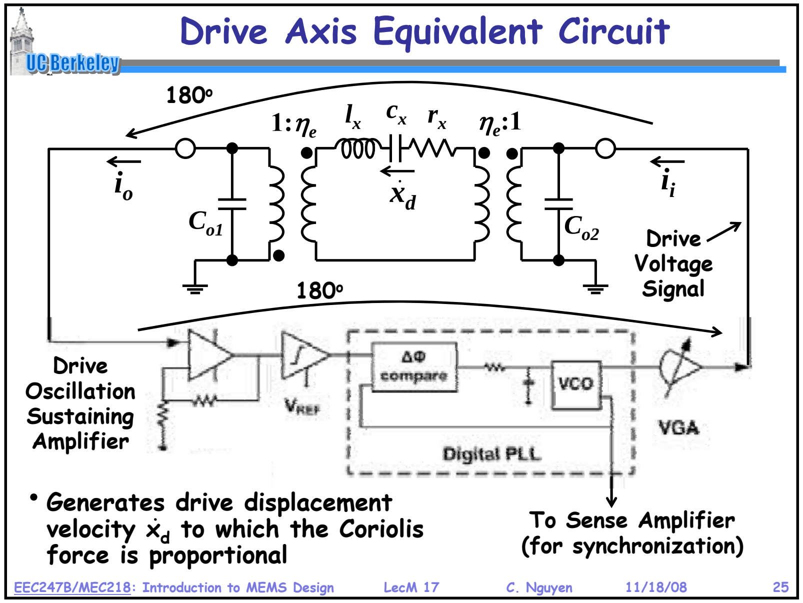
$\frac{1}{(2\pi)(1k)(1p)} = 60MHz$

Back to Determining Sensor Resolution



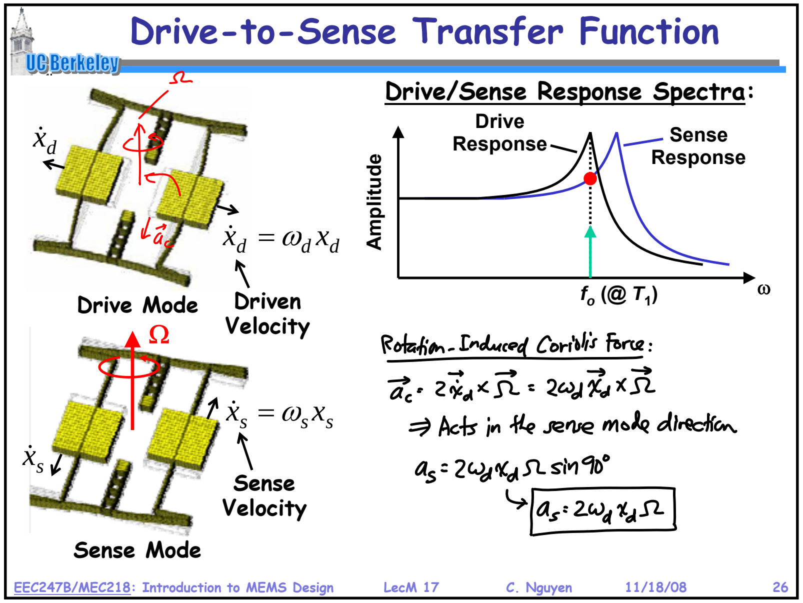
MEMS-Based Tuning Fork Gyroscope

[Zaman, Ayazi, et al, MEMS'06]



Drive Axis Equivalent Circuit

- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional



Drive-to-Sense Transfer Function

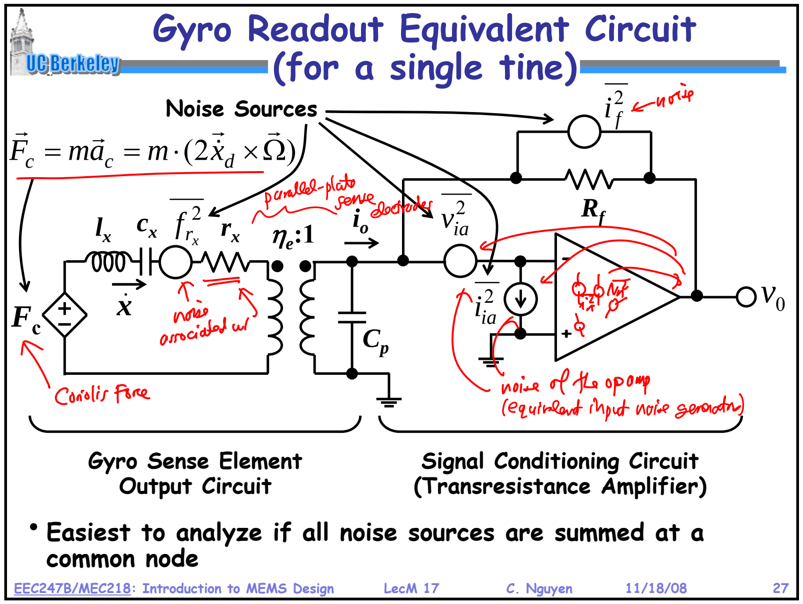
Rotation-Induced Coriolis Force:

$$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$$

⇒ Acts in the sense mode direction

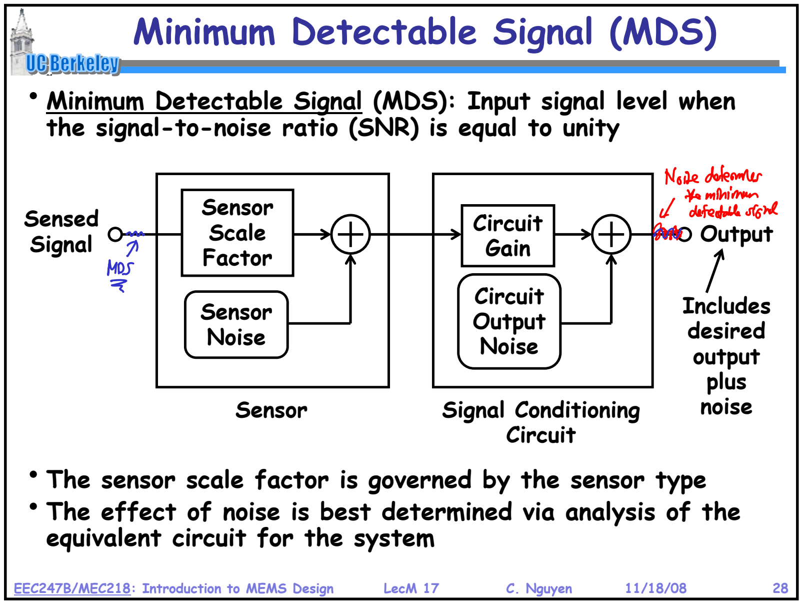
$$a_s = 2\omega_d x_d \Omega \sin 90°$$

$$\boxed{a_s = 2\omega_d x_d \Omega}$$



Gyro Readout Equivalent Circuit (for a single tine)

- Easiest to analyze if all noise sources are summed at a common node



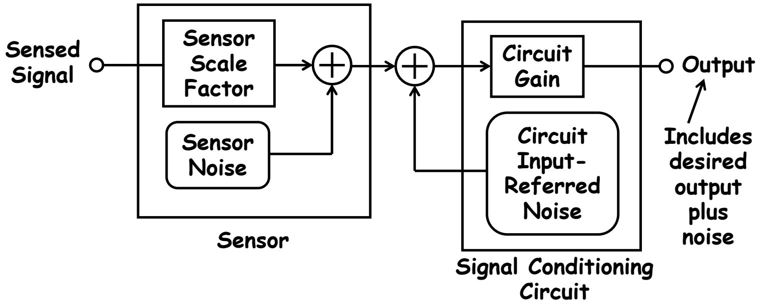
Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

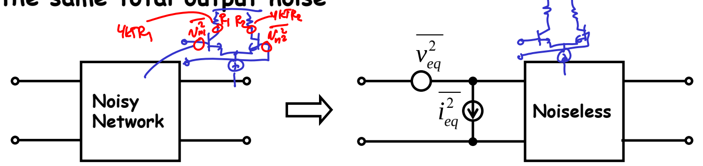
- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS



Equivalent Input-Referred Voltage and Current Noise Sources

Equivalent Input v, i Noise Generators

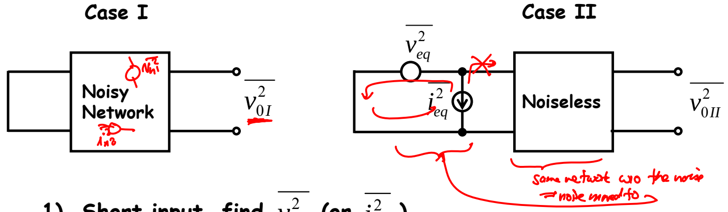
- Take a noisy 2-port network and represent it by a noiseless network with input *v* and *i* noise generators that generate the same total output noise



- **Remarks**:
  1. Works for linear time-invariant networks
  2. v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
  4. If correlation is important → easier to return to original network with internal noise sources

Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

a) To get $\overline{v_{eq}^2}$ for a two-port:



1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)

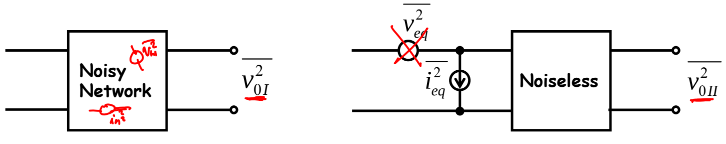
2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)

$$\overline{v_{0II}^2} = f\left(\overline{v_{eq}^2}\right) \qquad \overline{i_{0II}^2} = f\left(\overline{v_{eq}^2}\right)$$

3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}$ → solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$)

Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:



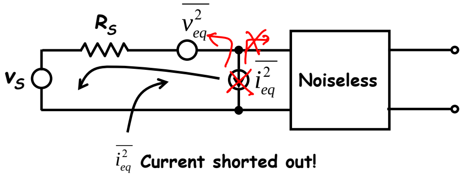
1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}\left(\overline{i_{eq}^2}\right)$ → solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}\left(\overline{i_{eq}^2}\right)$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated

Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance R_s is small compared to input resistance R_i → i.e., voltage source input
  2. Source resistance R_s is large compared to input resistance R_i → i.e., current source input

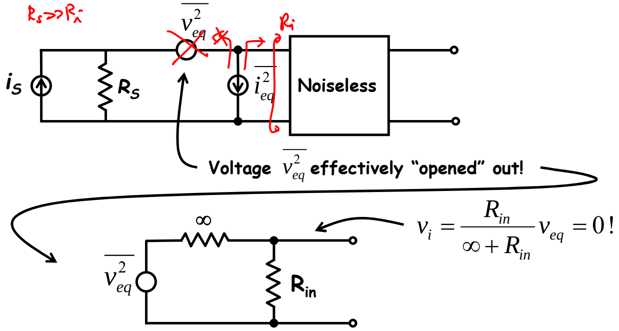
1) R_S = small (ideally = 0 for an ideal voltage source):



∴ For R_S= small, $\overline{i_{eq}^2}$ can be neglected → only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

Cases Where Correlation Is Not Important

2) R_S = large (Ideally = ∞ for an ideal current source)

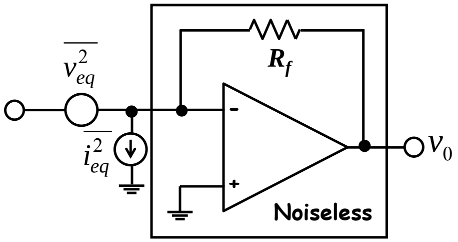


$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

∴ For R_S= large, $\overline{v_{eq}^2}$ can be neglected!
→ only $\overline{i_{eq}^2}$ is important!
(… and again, we need not deal with correlation)

Example: TransR Amplifier Noise (cont)

- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:



$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}} \qquad \boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$$