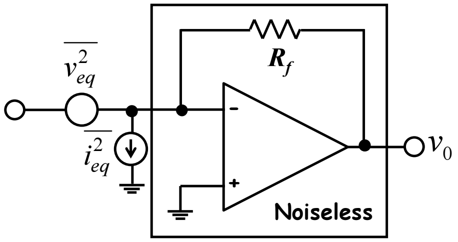
## Example: TransR Amplifier Noise (cont)

- To summarize, for a transresistance amplifier, the equivalent input-referred current and voltage noise generators are given by:



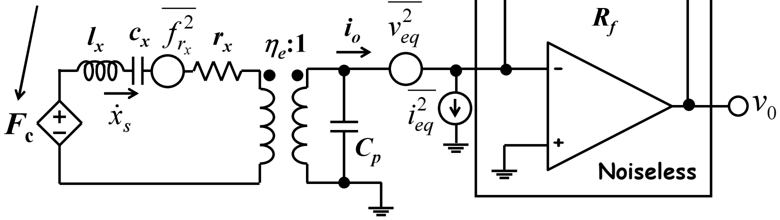
$$\boxed{\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}}$$

$$\boxed{\overline{v_{eq}^2} = \overline{v_{ia}^2}}$$

## Back to Gyro Noise & MDS

## Example: Gyro MDS Calculation

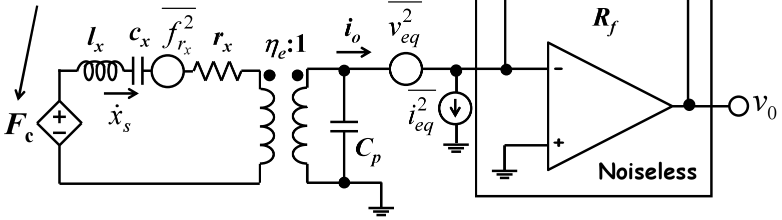
$\vec{F}_c = m\vec{a}_c = m \cdot (2\dot{\vec{x}}_d \times \vec{\Omega})$



- The gyro sense presents a large effective source impedance
  - Currents are the important variable; voltages are "opened" out
  - Must compare $i_o$ with the total current noise $i_{eqTOT}$ going into the amplifier circuit

## Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\dot{\vec{x}}_d \times \vec{\Omega})$



- First, find the rotation to $i_o$ transfer function:

$$\dot{x}_s = \frac{\omega_s Q_s}{k_s} \textcircled{H}_s(j\omega_d) F_s = \frac{\omega_s Q_s}{k_s} \cdot 2\omega_d x_d \Omega m \cdot \textcircled{H}_s(j\omega_d)$$

$$\left[F_s = F_c = 2\omega_d x_d \Omega m\right]$$

$$\frac{1}{\omega_s^2}$$

$$\dot{x}_s = 2\frac{\omega_d}{\omega_s} Q_s x_d \textcircled{H}_s(j\omega_d) \cdot \Omega$$

## Example: Gyro MDS Calculation (cont)

$$\dot{i}_o = \eta_e \dot{x}_s = \underbrace{2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d)}_{A \triangleq \text{scale factor}} \cdot \Omega \longrightarrow \boxed{\begin{array}{l} \dot{i}_o = A\Omega \\ \text{where } A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e \Theta_s(j\omega_d) \end{array}}$$

When $\Omega = \Omega_{min} \triangleq MDS$, $\dot{i}_o = \dot{i}_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/√Hz

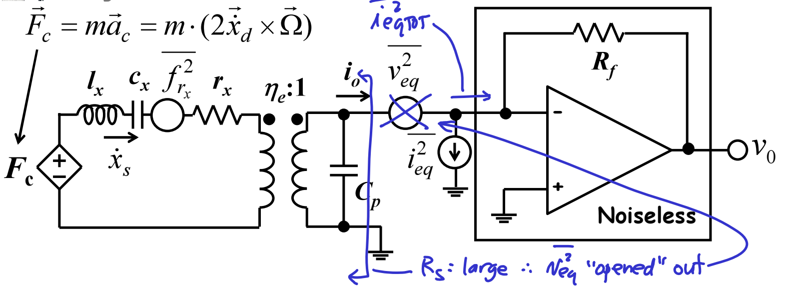
$$\therefore \dot{i}_{eqTOT} = A\Omega_{min} \rightarrow \boxed{\Omega_{min} = \frac{\dot{i}_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) \left[(^\circ/hr)/\sqrt{Hz}\right]}$$

$$\boxed{\text{Angle Random Walk} = ARW = \frac{1}{60}\Omega_{min} \quad [^\circ/\sqrt{hr}]}$$

↳ Easier to determine directional error as a function of elapsed time.

## Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\dot{\vec{x}}_d \times \vec{\Omega})$



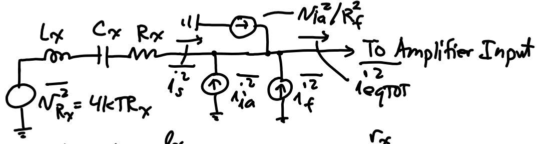
- Now, find the $i_{eqTOT}$ entering the amplifier input:

$$\dot{i}_{eqTOT} = \dot{i}_s + \dot{i}_{eq} \rightarrow \overline{\dot{i}_{eqTOT}^2} = \overline{\dot{i}_s^2} + \overline{\dot{i}_f^2} + \overline{\dot{i}_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2} \qquad \frac{\overline{f_{r_x}^2}}{\Delta f} = 4kTr_x$$

↑ Brownian motion noise of the sense element → determined entirely by the noise in $r_x$ → $\overline{f_{r_x}^2}$

↳ easiest to convert to an all electrical equiv. ckt.

## Example: Gyro MDS Calculation (cont)



where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore \dot{i}_s = v_{R_x}\left(\frac{1}{R_x}\right)\Theta_s(j\omega_d) \rightarrow \frac{\overline{\dot{i}_s^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)|\Theta_s(j\omega_d)|^2$$

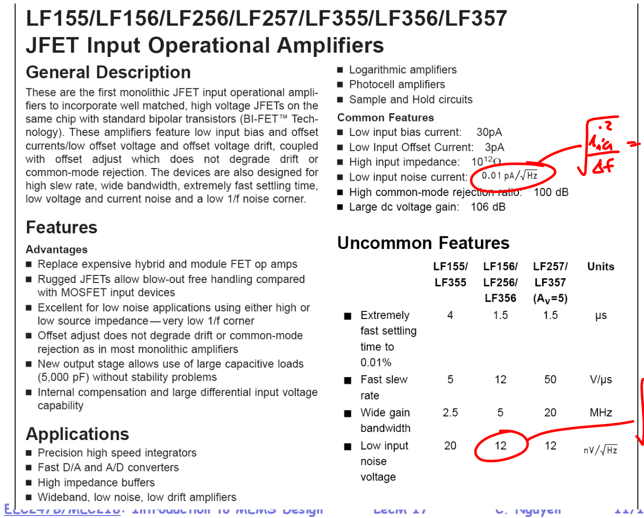
$$\Rightarrow \boxed{\frac{\overline{\dot{i}_s^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2}$$

Thus:

$$\boxed{\frac{\overline{\dot{i}_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{\dot{i}_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)}$$

Learn to get these from EE240.
↳ or just get them from a data sheet…

## LF356 Op Amp Data Sheet



### LF155/LF156/LF256/LF257/LF355/LF356/LF357
### JFET Input Operational Amplifiers

#### General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

#### Features

**Advantages**

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

#### Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers
- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits

**Common Features**

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01\ pA/\sqrt{Hz}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

#### Uncommon Features

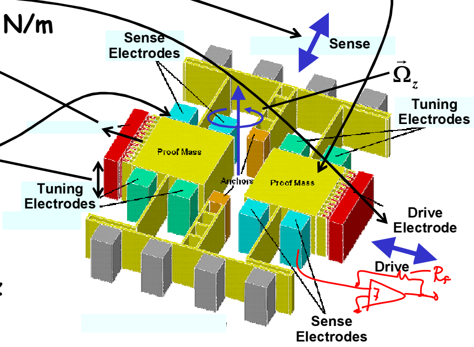
| | LF155/<br>LF355 | LF156/<br>LF256/<br>LF356 | LF257/<br>LF357<br>($A_V$=5) | Units |
|---|---|---|---|---|
| Extremely fast settling time to 0.01% | 4 | 1.5 | 1.5 | μs |
| Fast slew rate | 5 | 12 | 50 | V/μs |
| Wide gain bandwidth | 2.5 | 5 | 20 | MHz |
| Low input noise voltage | 20 | 12 | 12 | $nV/\sqrt{Hz}$ |

$$\sqrt{\frac{\overline{\dot{i}_{ia}^2}}{\Delta f}} = 0.01\ pA/\sqrt{Hz}$$

$$\sqrt{\frac{\overline{v_{ia}^2}}{\Delta f}} = 12\ nV/\sqrt{Hz}$$

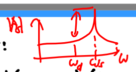
## Example ARW Calculation

- **Example Design:**
  - Sensor Element:
    - $m = (100\mu m)(100\mu m)(20\mu m)(2300kg/m^3) = 4.6\times10^{-10}kg$
    - $\omega_s = 2\pi(15kHz)$
    - $\omega_d = 2\pi(10kHz)$
    - $k_s = \omega_s^2 m = 4.09\ N/m$
    - $x_d = 20\ \mu m$
    - $Q_s = 50,000$
    - $V_P = 5V$
    - $h = 20\ \mu m$
    - $d = 1\ \mu m$
  - Sensing Circuitry:
    - $R_f = \cancel{100k\Omega}$ $1M\Omega$
    - $i_{ia} = 0.01\ pA/\sqrt{Hz}$
    - $v_{ia} = 12\ nV/\sqrt{Hz}$



## Example ARW Calculation (cont)

Get rotation rate to output current scale factor:



$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e|\Theta_s(j\omega_d)| = 2\left(\frac{10k}{15k}\right)(50K)(20\mu)(5)(2000\epsilon_o)(0.000024) = \underline{2.83\times10^{-12}C}$$

$$\left[\Theta_s(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10K)(15K)/(50K)}{(15K)^2-(10K)^2+\frac{j(10K)(15K)}{50K}} = \frac{j(3K)}{1.25\times10^8+j(3K)}\right.$$

$$\left.\rightarrow |\Theta_s(j\omega_d)| = \frac{3K}{\sqrt{(1.25\times10^8)^2+(3K)^2}} = 0.000024\right]$$

$$\left[\frac{\partial C}{\partial x} = \frac{C_o}{d} = \frac{\epsilon_o h W_p}{d} = \frac{\epsilon_o(20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_o \rightarrow \eta_e = V_P\frac{\partial C}{\partial x} = \overbrace{5(2000\underset{\nearrow\, 8.854\times10^{-12}F/m}{\epsilon_o})}^{8.854\times10^{-8}F/m}\right]$$

Assume electrode covers the whole sidewall.

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta_s(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)$$

## Example ARW Calculation (cont)

$$\left[R_x = \frac{\omega_s m}{Q_s\eta_e^2} = \frac{2\pi(15K)(4.6\times10^{-10})}{(50K)(8.854\times10^{-8})^2} = 110.6k\Omega\right]$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(0.000024)^2}_{8.64\times10^{-35}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

sensor element noise insignificant

Noise from $R_f$ dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68\times10^{-26}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{1.30\times10^{-13}}{2.83\times10^{-12}}(3600)\left(\frac{180}{\pi}\right) = \underline{9448\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(9448) = \boxed{157\ ^\circ/\sqrt{hr} = ARW}$$

⇒ Almost turned around in 1 hour!

## What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15KHz$, then $|\Theta_s(j\omega_d)| = 1$ and

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e|\Theta_s(j\omega_d)| = 2Q_s x_d \eta_e = 2(50K)(20\mu)(5)(2000\epsilon_o) = \underline{1.77\times10^{-7}C}$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(1)^2}_{1.51\times10^{-25}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Now, the sensor element dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.67\times10^{-25}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 4.08\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{4.08\times10^{-13}}{1.77\times10^{-7}}(3600)\left(\frac{180}{\pi}\right) = \underline{0.476\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(0.476) = \boxed{0.0079\ ^\circ/\sqrt{hr} = ARW}$$

⇒ Navigation grade!