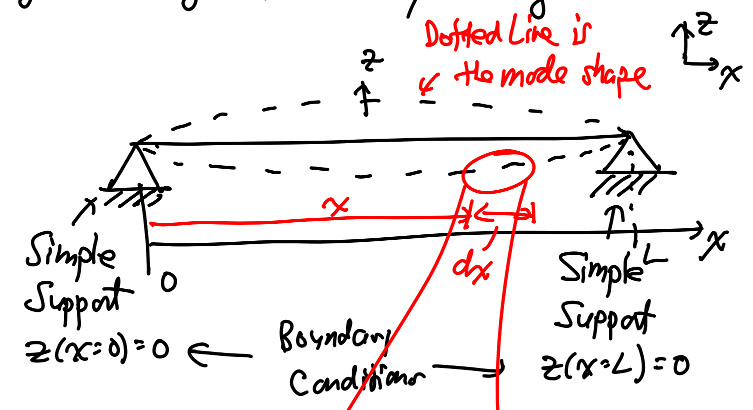
Lecture 2: Benefits of Scaling I

- **Announcements**:
- The notes from last time are online
- Modules 1 & 2 are online
- HW#1 will soon be online
- To make up for my absence in the first week, next Tuesday's lecture will go till 5:30 p.m.
  - If you can't stay, just leave
  - You can always watch the lecture on video
- ------------------------------------
- **Today**:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- ------------------------------------
- Finish Module 1
- Start going through Module 2

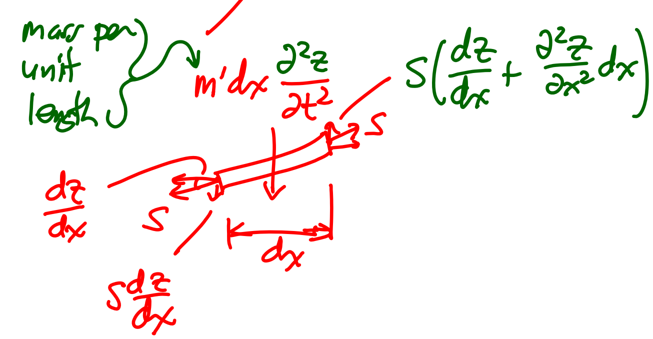
Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire



Want equation for Resonance Frequency.

Free Body Diagram



Condition: Dynamic Equilibrium

$$S\left(\frac{dz}{dx}+\frac{\partial^2 z}{\partial x^2}dx\right)-S\frac{dz}{dx}-m'dx\frac{\partial^2 z}{\partial t^2}=0$$

↓ Solve

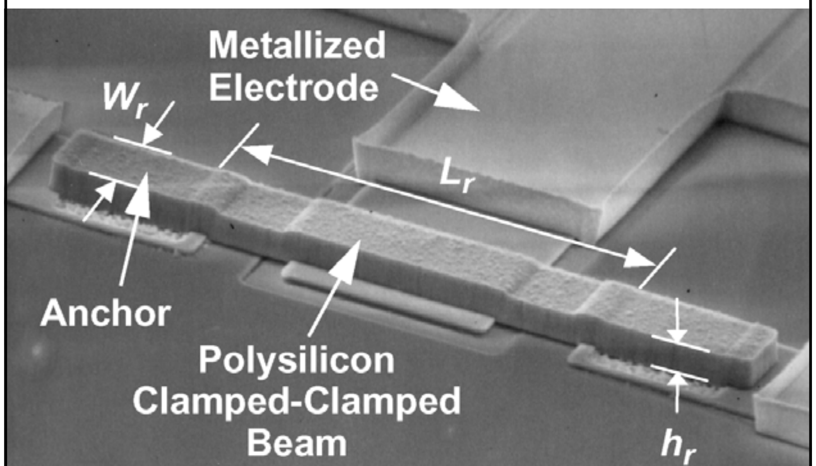
↓ Solve

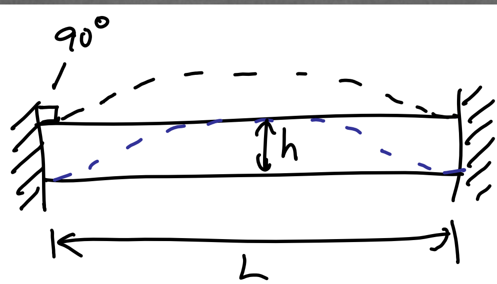
$$\boxed{f_i = \frac{\rho_i}{2L}\sqrt{\frac{S}{m'}}}$$ ← frequency

ith mode

If $L\downarrow \rightarrow f_i\uparrow$.

$\boxed{\text{Clamped-Clamped Beam}}$





⇒ Eq. for Resonance Freq:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \qquad (1)$$

where $E \triangleq$ Young's modulus of elasticity [GPa]

$\rho \triangleq$ density [kg/m³]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example: $L = 40\mu m$, $h = 2\mu m$

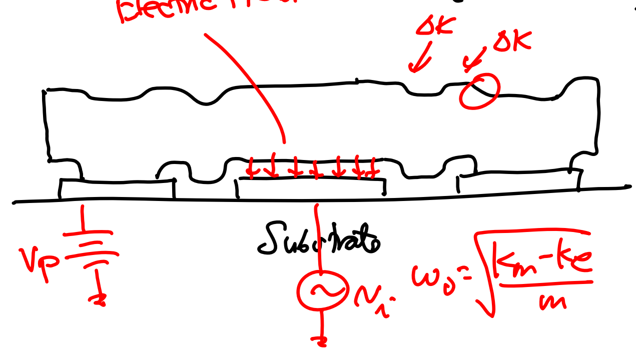
polySi → $E = 150GPa$, $\rho = 2300\ kg/m^3$

$$\therefore f_o = (1.03)\sqrt{\frac{150G}{2300}}\frac{2\mu}{(40\mu)^2} \Rightarrow f_o = 10.4MHz$$

Why isn't this 8.5MHz? (as measured)

generates electrical stiffness

← Electric Field



$$\omega_o = \sqrt{\frac{k_m - k_e}{m}}$$

Scaling: $2x, \frac{1}{2}x$

① Scale all dimensions equally by factor S:

$$f_o \sim \frac{S}{S^2} = \frac{1}{S}$$

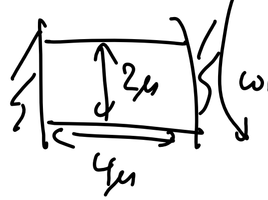
② If scale L only: $f_o \sim \frac{1}{S^2}$ → even faster rise of f_o!

But... problems!...

Example.

$$L = 4\mu m \rightarrow f_o = (1.03)\underbrace{(8076)}_{m/s}\frac{2\mu}{(4\mu)^2} = \boxed{1.04\ GHz}$$

(acoustic velocity)



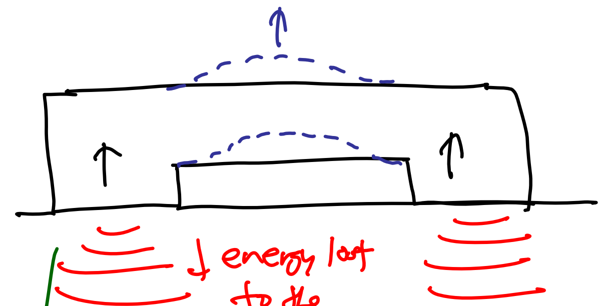
wrong equation when $L \approx h$

in actuality, $f_o \sim 800MHz$

Remarks.

① Eq. (1) not accurate when $L \approx h$.

② When $L \sim h$ (or when it isn't more than $10\times h$), get anchor loss problems that lower Q



energy lost to the substrate!

8MHz → Q~6,000 anchor loss! → $Q = \frac{\text{Energy per cycle}}{\text{Energy lost per cycle}}$

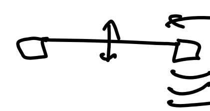
70MHz → Q~300

Q↓

Great example of pro & con of scaling!

③ Solutions: nanodimensions ✓

this means: h = 300nm, L ~ 1μm



smaller stiffness

less

Q remains high

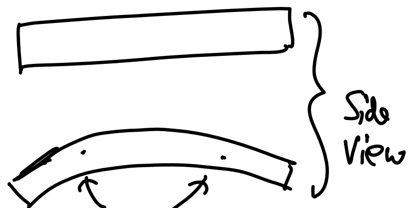
But: Problem: very small power handling

smaller → less power handling

Solution: use many of them → array

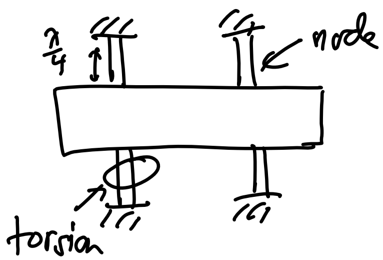
(4) Better Solution: other geometries! ✓✓

Free-Free Beam:



nodal points (where transverse motion = 0)

Top Views:



← put anchors @nodes

↓

Energy Loss ↓