UC Berkeley

EE C247B – ME C218
Introduction to MEMS Design
Spring 2014

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 7: Mechanics of Materials

UC Berkeley

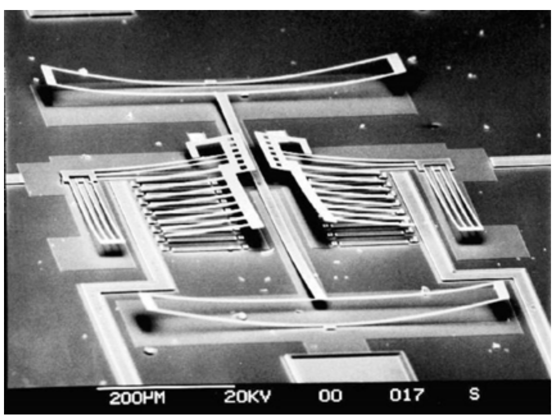
Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

UC Berkeley

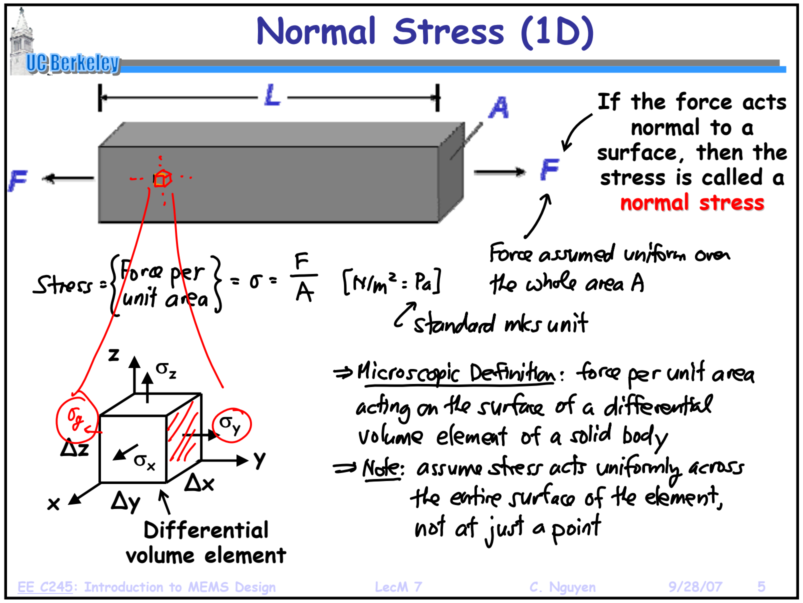
Vertical Stress Gradients

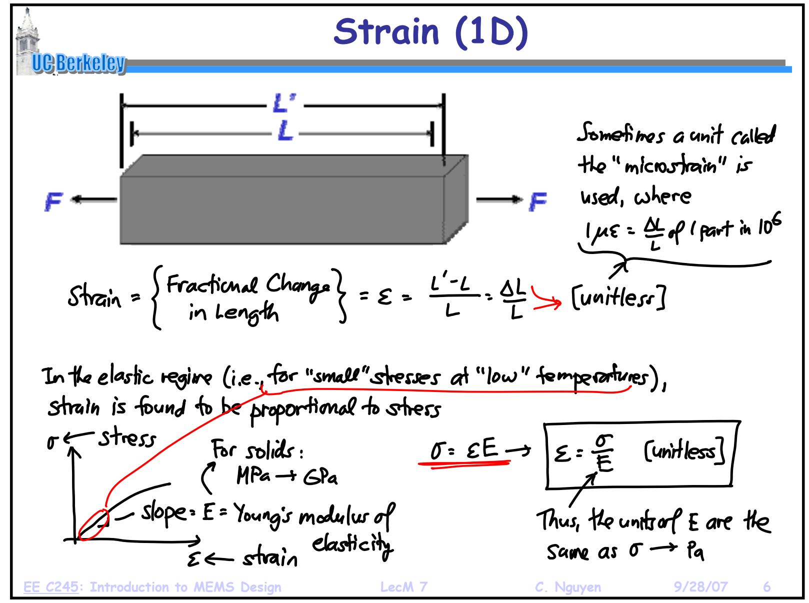
- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction

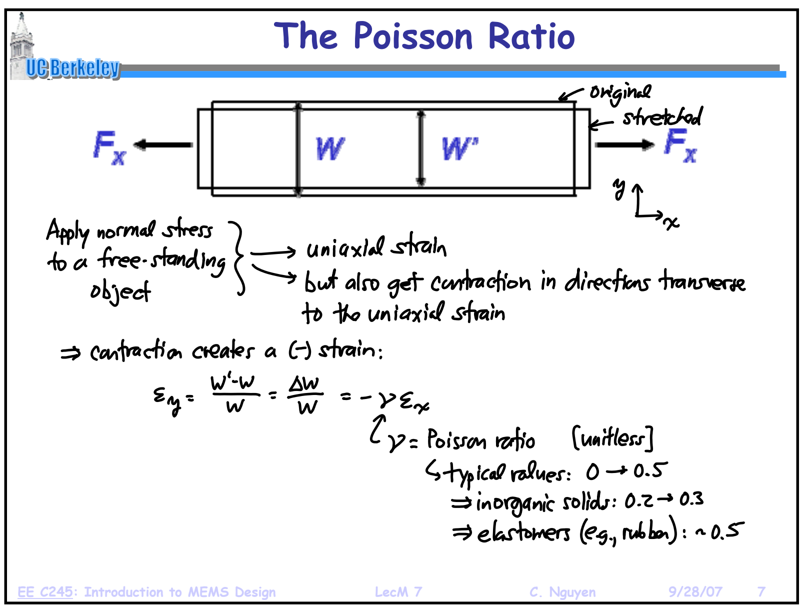


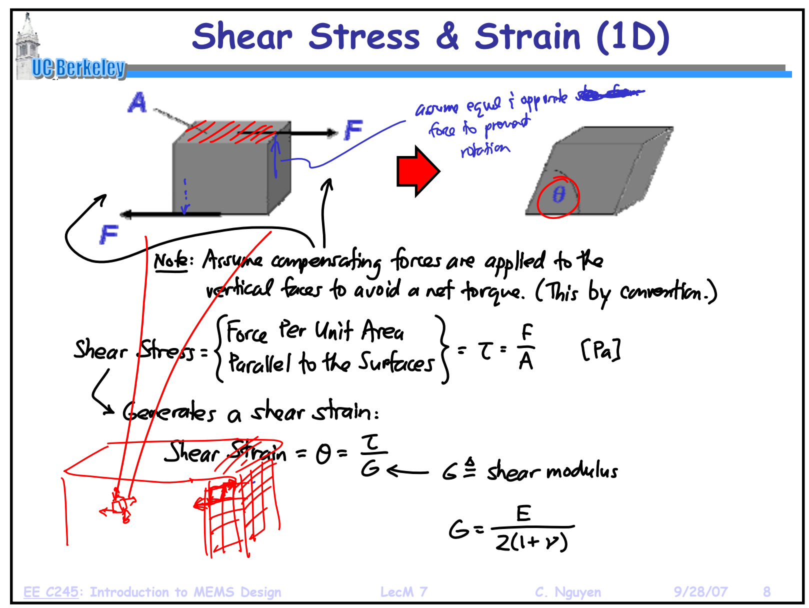
UC Berkeley

Elasticity


Normal Stress (1D)
UC Berkeley
L
A
F
F
If the force acts normal to a surface, then the stress is called a normal stress
Force assumed uniform over the whole area A
Stress = {Force per unit area} = σ = F/A [N/m² = Pa]
standard mks unit
z
σz
σx
σy
Δz
Δx
Δy
x
y
Differential volume element
⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body
⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point
EE C245: Introduction to MEMS Design
LecM 7
C. Nguyen
9/28/07
5

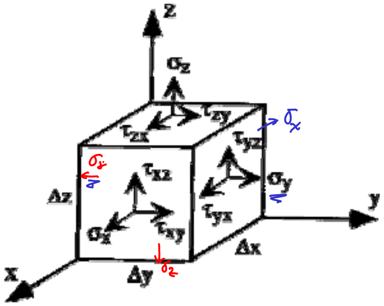

Strain (1D)
UC Berkeley
L'
L
F
F
Sometimes a unit called the "microstrain" is used, where
1με = ΔL/L of 1 part in 10⁶
Strain = {Fractional Change in Length} = ε = (L'−L)/L = ΔL/L → [unitless]
In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress
σ ← stress
For solids: MPa → GPa
slope = E = Young's modulus of elasticity
ε ← strain
σ = εE → ε = σ/E [unitless]
Thus, the units of E are the same as σ → Pa
EE C245: Introduction to MEMS Design
LecM 7
C. Nguyen
9/28/07
6


The Poisson Ratio
UC Berkeley
original
stretched
Fx
W
W'
Fx
y
x
Apply normal stress to a free-standing object → uniaxial strain
→ but also get contraction in directions transverse to the uniaxial strain
⇒ contraction creates a (−) strain:
εy = (W'−W)/W = ΔW/W = −ν εx
ν = Poisson ratio [unitless]
typical values: 0 → 0.5
⇒ inorganic solids: 0.2 → 0.3
⇒ elastomers (e.g., rubber): ~0.5
EE C245: Introduction to MEMS Design
LecM 7
C. Nguyen
9/28/07
7


Shear Stress & Strain (1D)
UC Berkeley
A
F
F
θ
assume equal & opposite force to prevent rotation
Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)
Shear Stress = {Force Per Unit Area Parallel to the Surfaces} = τ = F/A [Pa]
Generates a shear strain:
Shear Strain = θ = τ/G ← G ≜ shear modulus
G = E/2(1+ν)
EE C245: Introduction to MEMS Design
LecM 7
C. Nguyen
9/28/07
8

2D and 3D Considerations

- **Important assumption**: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:

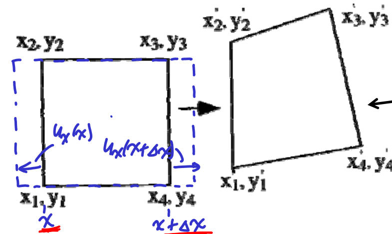


Stresses acting on a differential volume element

$\tau_{xy} = \tau_{yx}$ $\quad\quad$ $\tau_{xz} = \tau_{zx}$ $\quad\quad$ $\tau_{yz} = \tau_{zy}$

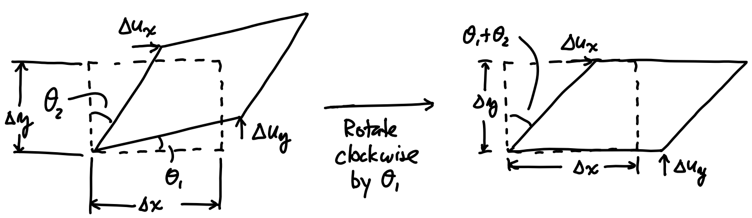
2D Strain

- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation



- Must work with displacement vectors
- Differential definition of axial strain: ⟶ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

2D Shear Strain



⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation

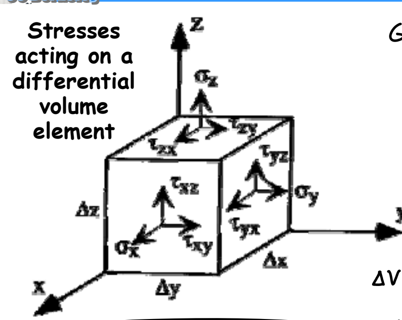
↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

↑ For small amplitude deformations.

Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element



Given an x-directed uniaxial stress, σ_x:

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume ΔV

$$\Delta V = \Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x \Delta y \Delta z$$
$$= \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains] ⇒ $\Delta V = \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$[(1+mx)^n \approx 1 + nmx]$ ⇒ $\approx \Delta x \Delta y \Delta z\left[1 + \epsilon_x - 2\nu\epsilon_x - 2\nu\epsilon_x^2 - 1\right]$ (negl.)

$$\boxed{\Delta V \approx \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

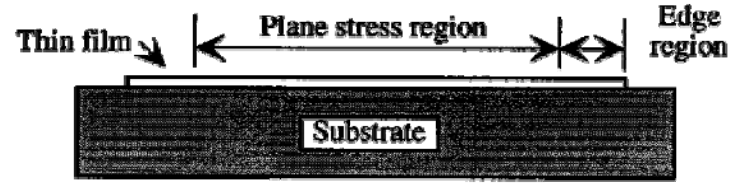
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)



- At regions more than 3 thicknesses from edges, the top surface is stress-free → $\sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

Important Case: Plane Stress (cont.)

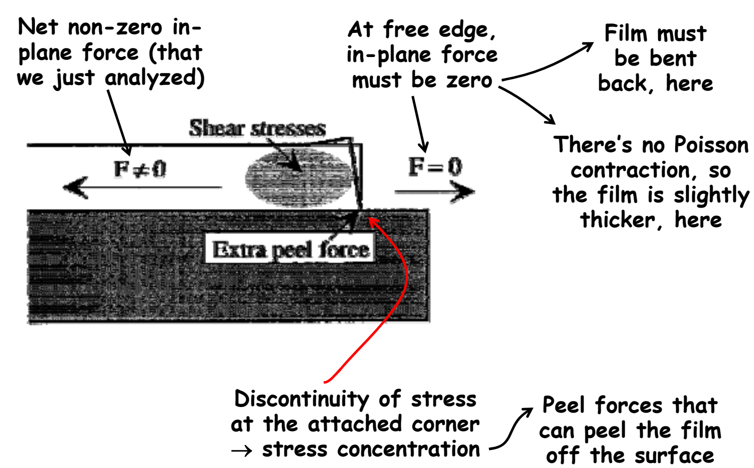
- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus} \triangleq E' = \frac{E}{1-\nu}$$

Edge Region of a Tensile (σ>0) Film



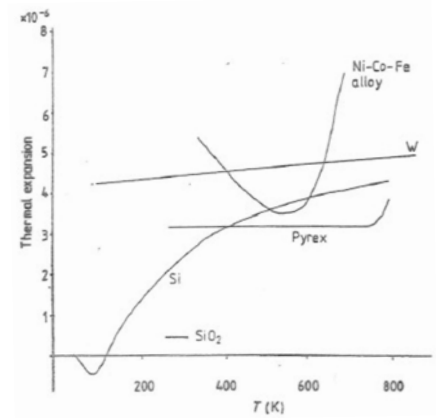
Linear Thermal Expansion

- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

Linear thermal expansion coefficient $\triangleq \alpha_T = \dfrac{d\varepsilon_x}{dT}$ [Kelvin^{-1}]

Remarks:
- α_T values tend to be in the 10^{-6} to 10^{-7} range
- Can capture the 10^{-6} by using dimensions of μstrain/K, where 10^{-6} K^{-1} = 1 μstrain/K
- In 3D, get volume thermal expansion coefficient ⟶ $\dfrac{\Delta V}{V} = 3\alpha_T \Delta T$
- For moderate temperature excursions, α_T can be treated as a constant of the material, but in actuality, it is a function of temperature

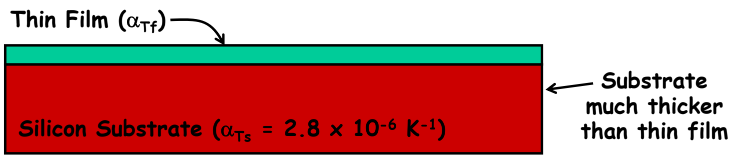
α_T As a Function of Temperature



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

Thin-Film Thermal Stress



- Assume film is deposited stress-free at a temperature T_r, then the whole thing is cooled to room temperature T_r
- Substrate much thicker than thin film → substrate dictates the amount of contraction for both it and the thin film

Thermal strain of the substrate: (in one in-plane dimension)

$$\epsilon_s = -\alpha_{Ts}\Delta T \quad , \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate: $\epsilon_{f,free} = -\alpha_{Tf}\Delta T$ ↻ over

Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate:

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

↳ Note that this is biaxial strain
↳ it can only be developed by an in-plane biaxial stress:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin-film is polyimide → $\alpha_{Tf} = 70\times10^{-6}\ K^{-1}$, $E' = 4GPa$ (e.g., SiO_2)
deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$
↳ $\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = \underline{\underline{60.5MPa}}$ ← stress is (+), ∴ tensile [(-) would be compressive]

MEMS Material Properties

Material Properties for MEMS

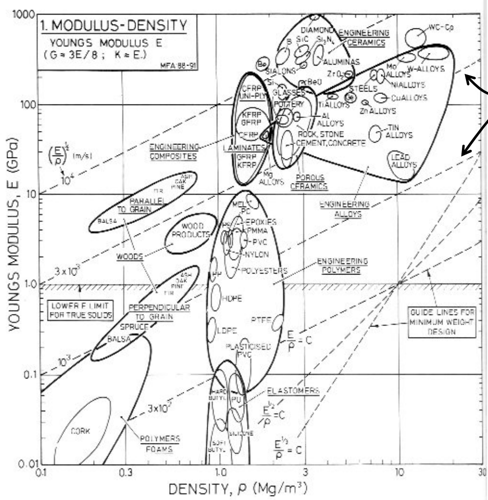
Material	Density, ρ, Kg/m³	Modulus, E, GPa	E/ ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: (m/s)^2

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

Young's Modulus Versus Density



Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

Yield Strength

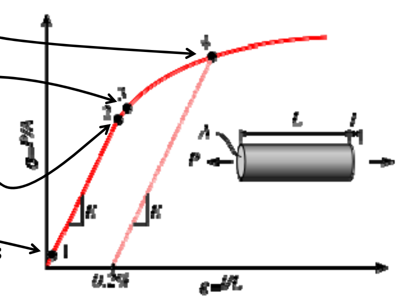
- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

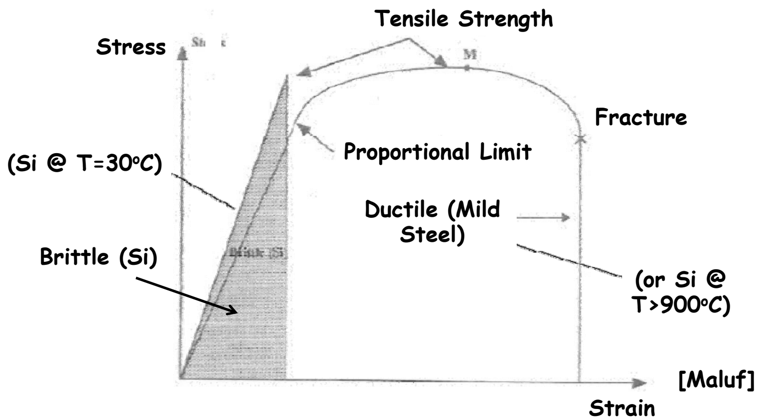
Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move



Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials



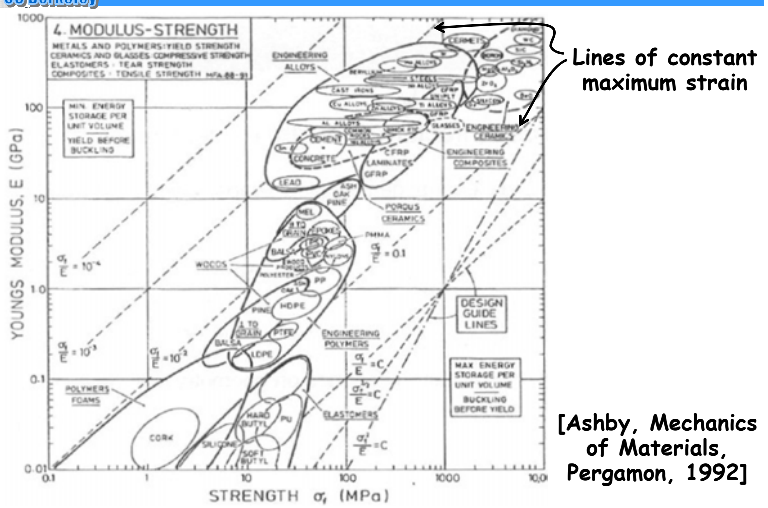
Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f, MPa	$\frac{\sigma_f}{E}$ (-) x 10^{-3}	$\frac{\sigma_f^2}{E}$ MJ/m^3
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

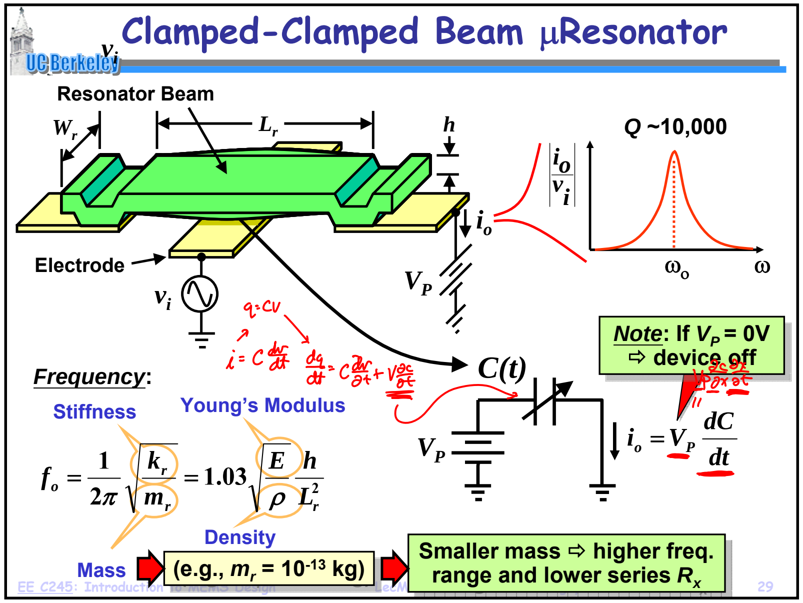
From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

Young's Modulus Versus Strength



[Ashby, Mechanics of Materials, Pergamon, 1992]

Quality Factor (or Q)


Clamped-Clamped Beam μResonator
UC Berkeley
Resonator Beam
W_r
L_r
h
Q ~10,000
$\left|\frac{i_o}{v_i}\right|$
ω_o
ω
i_o
Electrode
v_i
V_P
$C(t)$
Note: If V_P = 0V ⇨ device off
Frequency:
Stiffness
Young's Modulus
$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$
Density
Mass
(e.g., m_r = 10^{-13} kg)
Smaller mass ⇨ higher freq. range and lower series R_x
$i_o = V_P \frac{dC}{dt}$