EE C247B – ME C218
Introduction to MEMS Design
Spring 2015

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 15: Gyros, Noise, & MDS

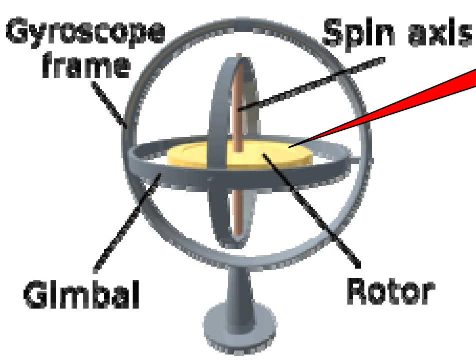
Lecture Outline

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
  - Gyro Circuit Modeling
  - Minimum Detectable Signal (MDS)
    - Noise
    - Angle Random Walk (ARW)

Gyroscopes

Classic Spinning Gyroscope

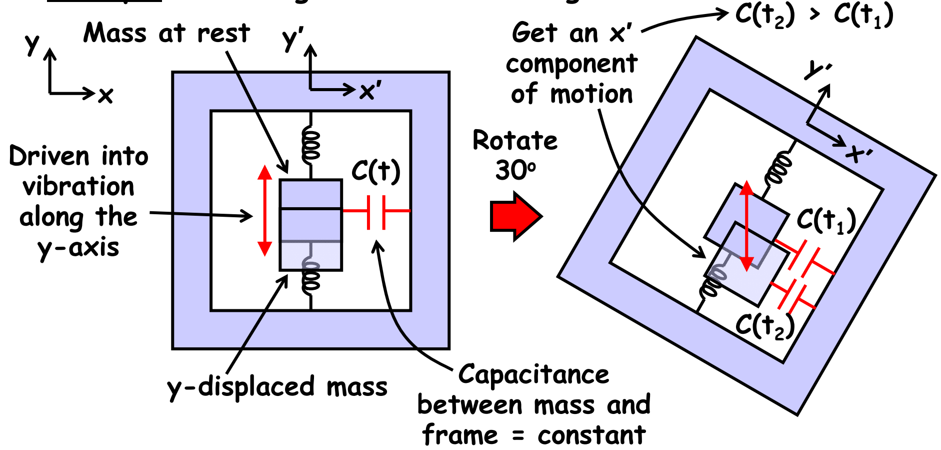
- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope



Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbled chassis

Vibratory Gyroscopes

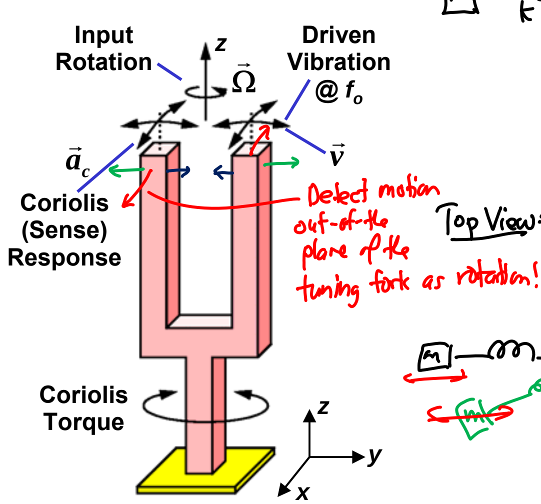
- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- Example: vibrating mass in a rotating frame

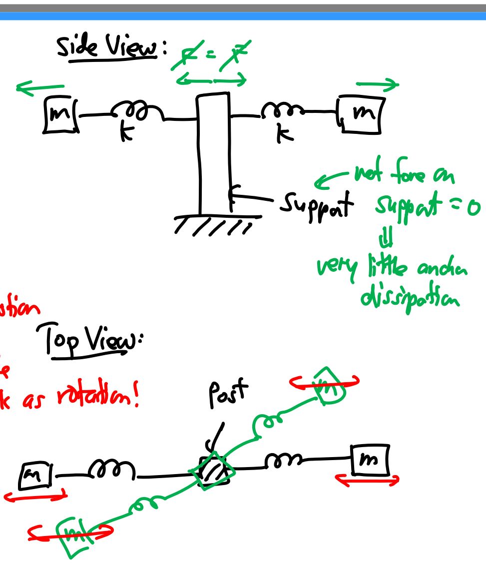


Basic Vibratory Gyroscope Operation

Principle of Operation

- Tuning Fork Gyroscope:





Basic Vibratory Gyroscope Operation

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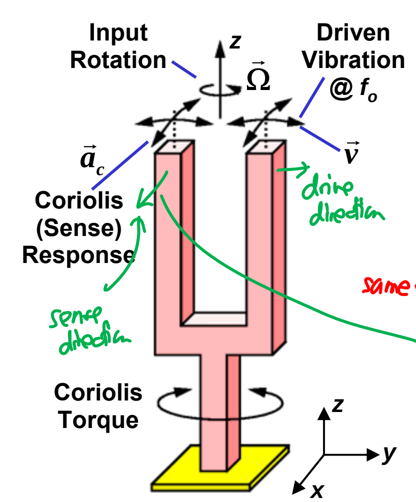
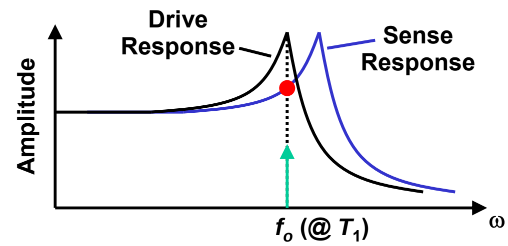


Drive/Sense Response Spectra:



Coriolis Acceleration, Driven Velocity, Rotation Rate (same frequency):

$$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

Coriolis Force, Beam Mass:

$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$$

Coriolis Displacement, Beam Stiffness (in sense direction), Sense Frequency

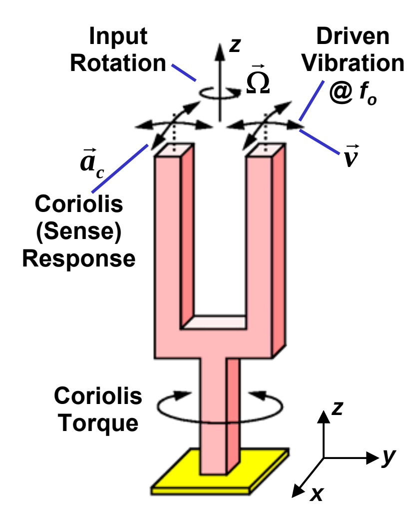
Vibratory Gyroscope Performance

Principle of Operation

- Tuning Fork Gyroscope:

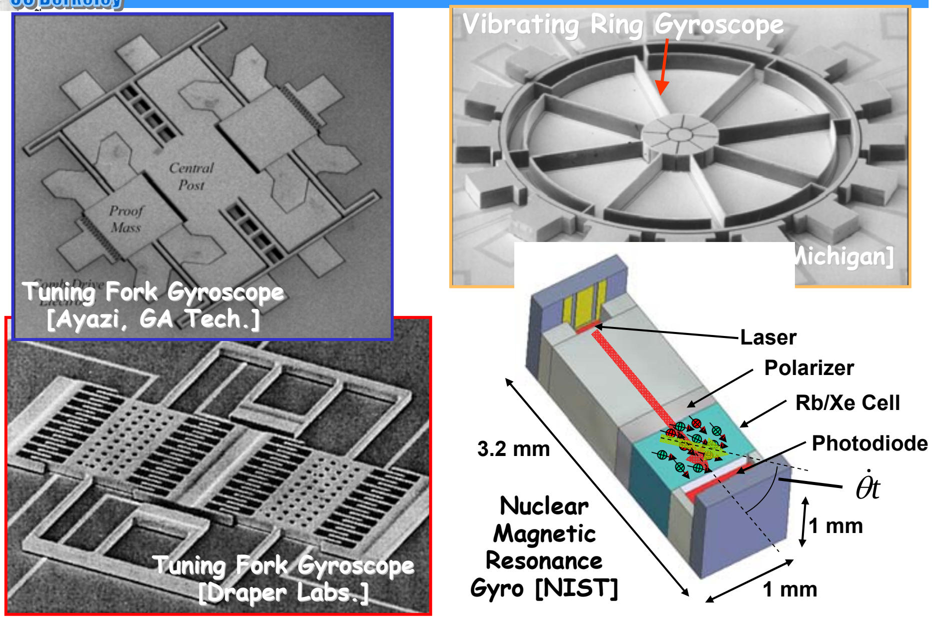
$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2} \qquad \vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

Beam Mass, Beam Stiffness, Sense Frequency, Driven Velocity



- To maximize the output signal x, need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
  - If can match drive freq. to sense freq., then can amplify output by Q times

MEMS-Based Gyroscopes



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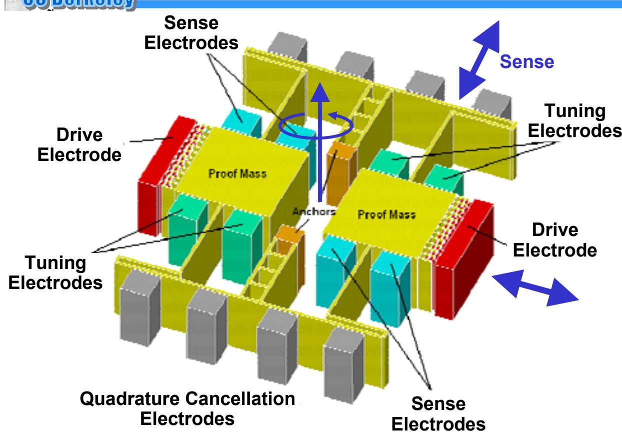
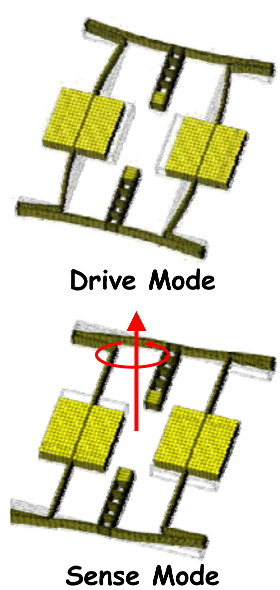
MEMS-Based Tuning Fork Gyroscope





- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]

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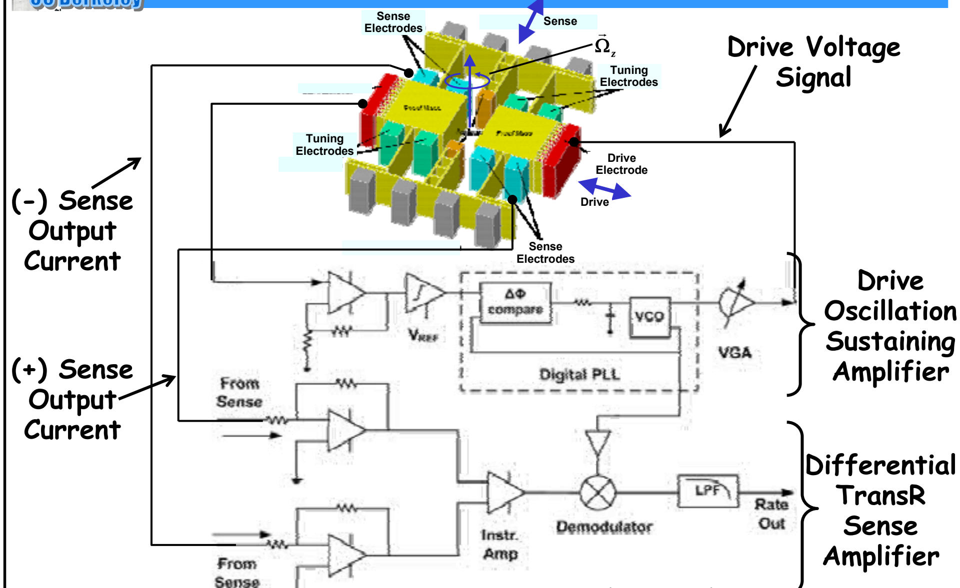
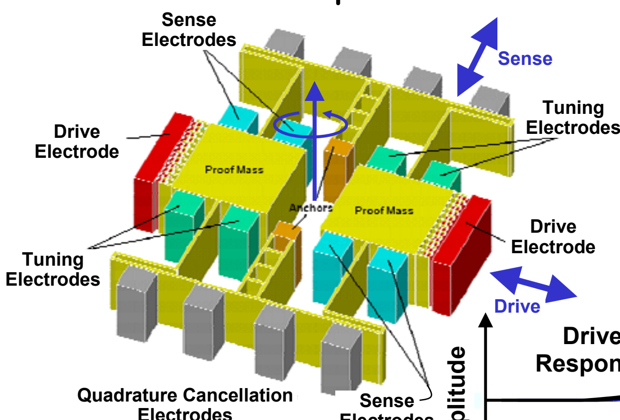
MEMS-Based Tuning Fork Gyroscope



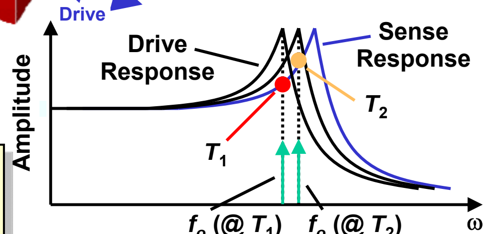
[Zaman, Ayazi, et al, MEMS'06]

MEMS-Based Tuning Fork Gyroscope

- Drive and sense axes must be stable or at least track one another to avoid output drift



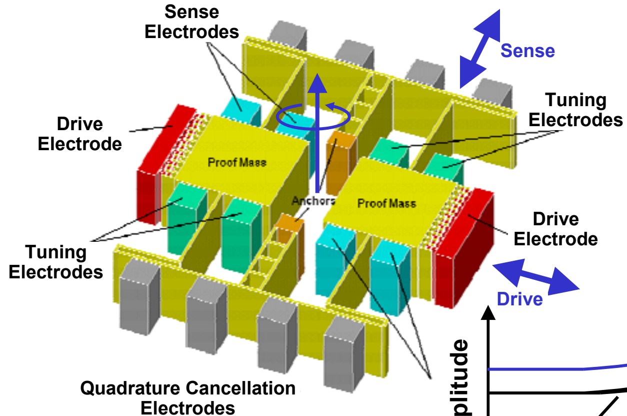
Problem: if drive frequency changes relative to sense frequency, output changes ⇨ bias drift



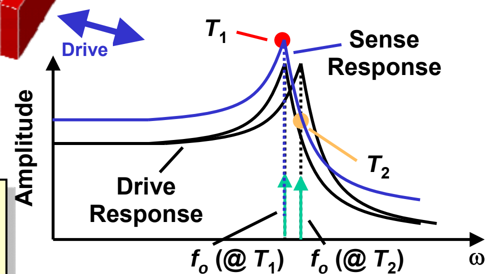
Need: small or matched drive and sense axis temperature coefficients to suppress drift

Mode Matching for Higher Resolution

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification



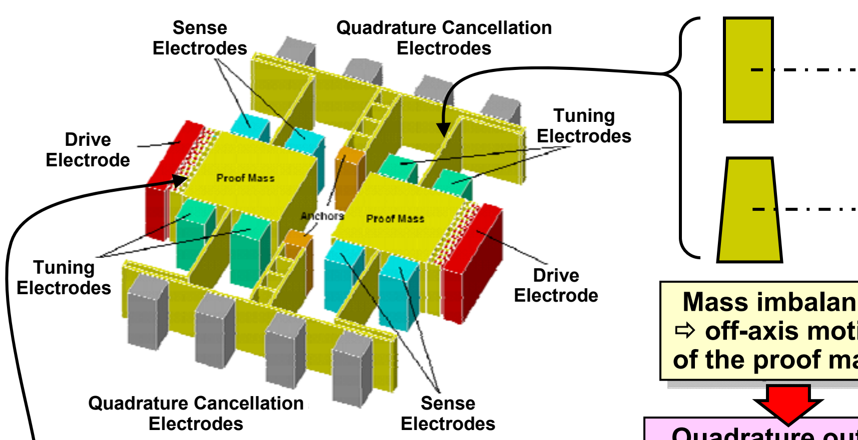
Problem: mismatch between drive and sense frequencies ⇨ even larger drift!



Need: small or matched drive and sense axis temperature coefficients to make this work

Issue: Zero Rate Bias Error

- Imbalances in the system can lead to zero rate bias error



Mass imbalance ⇨ off-axis motion of the proof mass

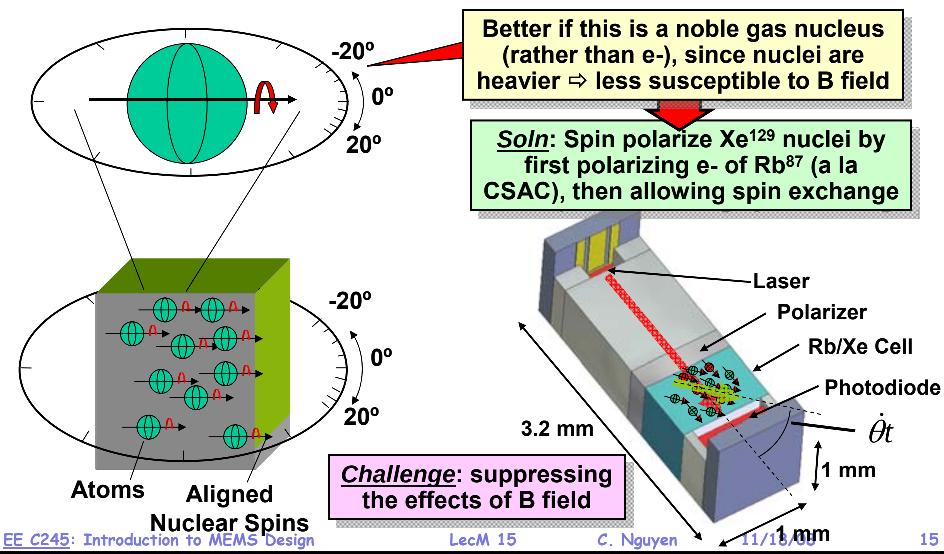
Quadrature output signal that can be confused with the Coriolis acceleration

Drive imbalance ⇨ off-axis motion of the proof mass

Output signal in phase with the Coriolis acceleration

Nuclear Magnetic Res. Gyroscope

- The ultimate in miniaturized spinning gyroscopes?
  - ↳ from CSAC, we may now have the technology to do this



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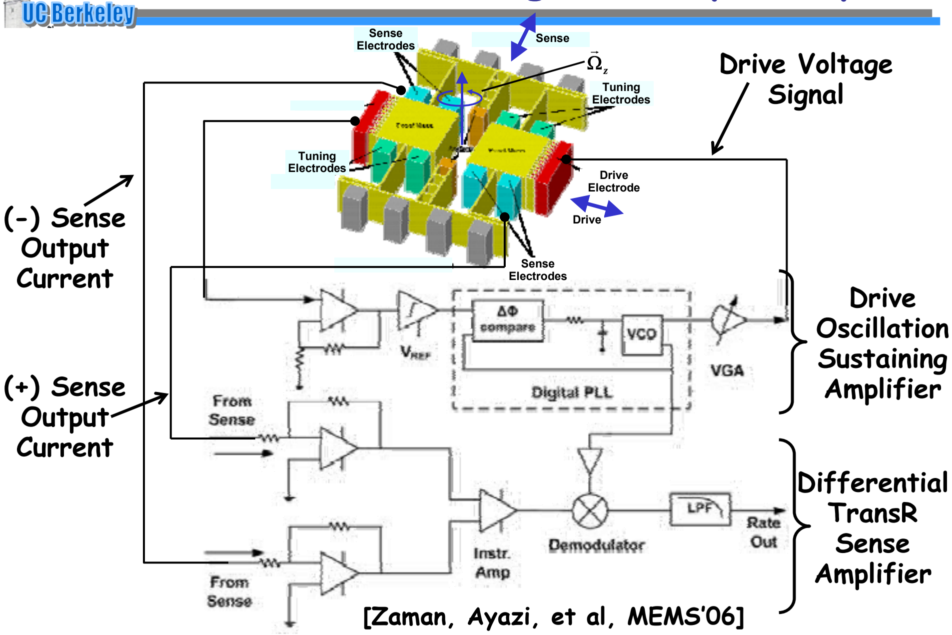
MEMS-Based Tuning Fork Gyroscope



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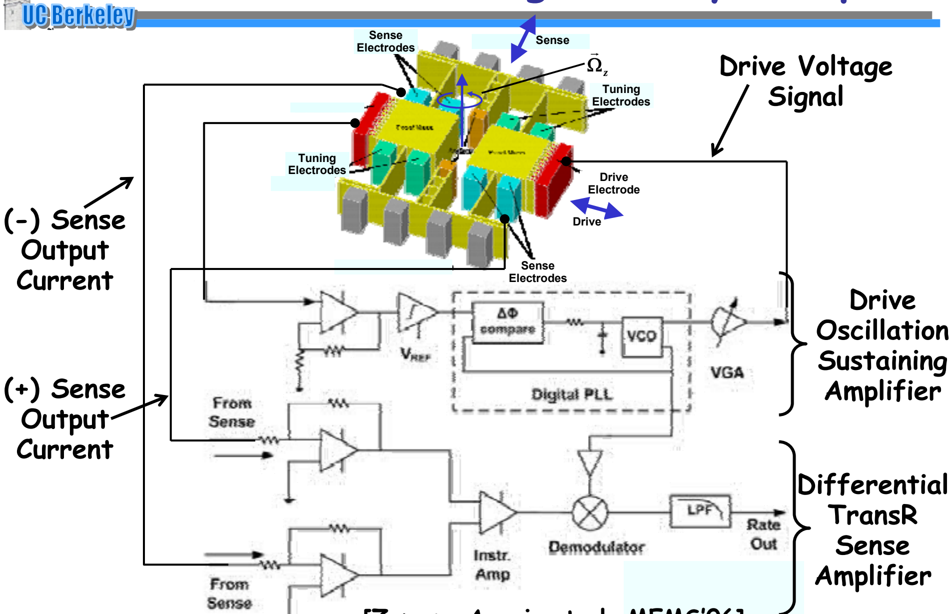
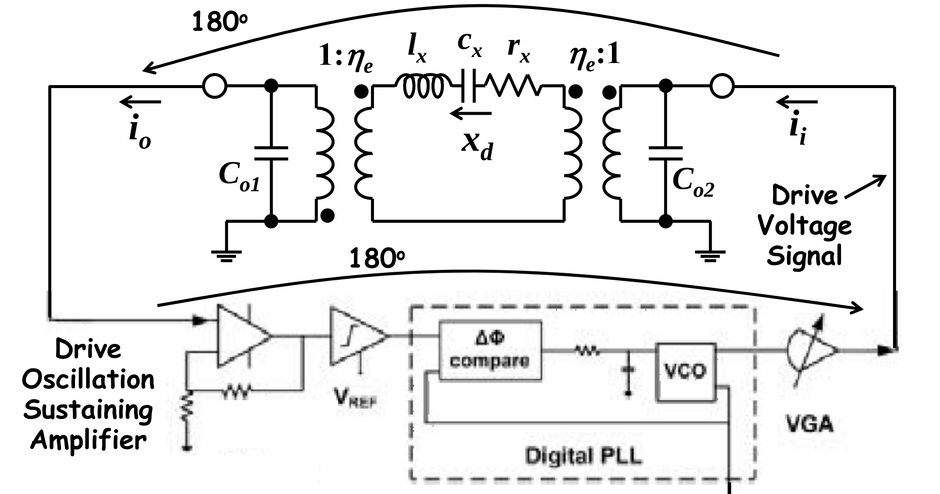
Determining Sensor Resolution

MEMS-Based Tuning Fork Gyroscope



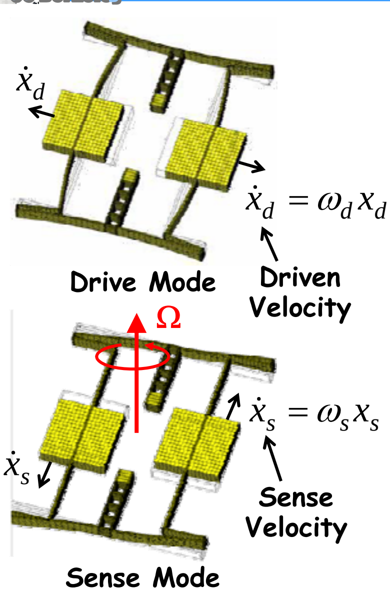
[Zaman, Ayazi, et al, MEMS'06]

Drive Axis Equivalent Circuit

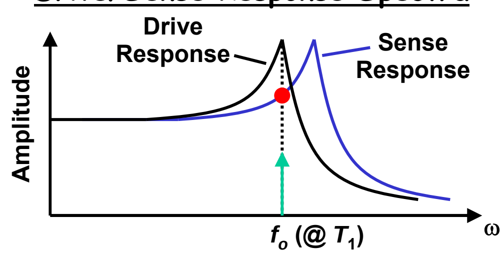


- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional

Drive-to-Sense Transfer Function



Drive/Sense Response Spectra:



Rotation-Induced Coriolis Force:

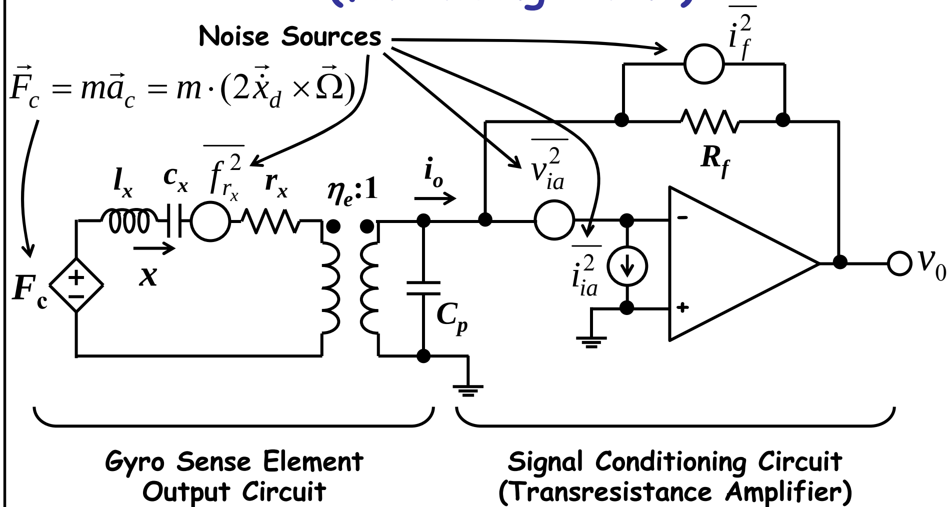
$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$

$\Rightarrow$ Acts in the sense mode direction

$a_s = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

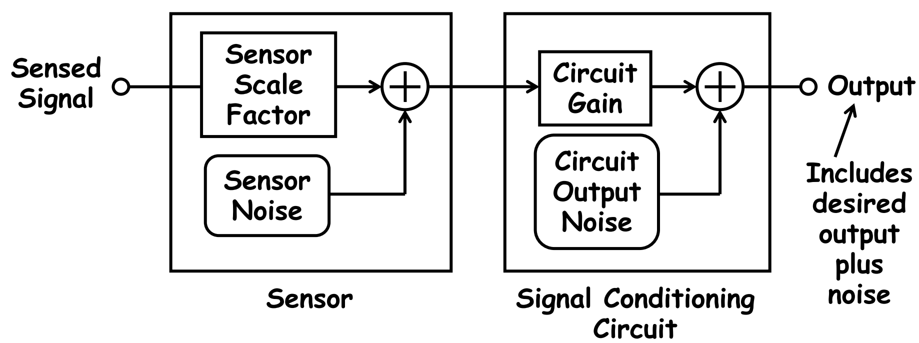
Gyro Readout Equivalent Circuit (for a single tine)



- Easiest to analyze if all noise sources are summed at a common node

Minimum Detectable Signal (MDS)

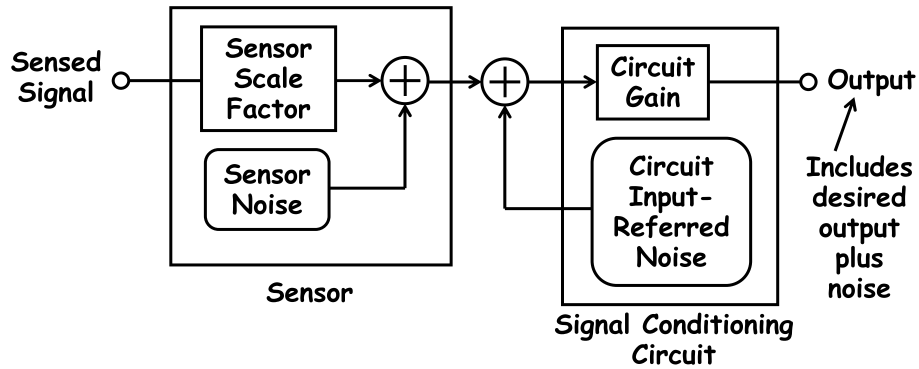
- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity



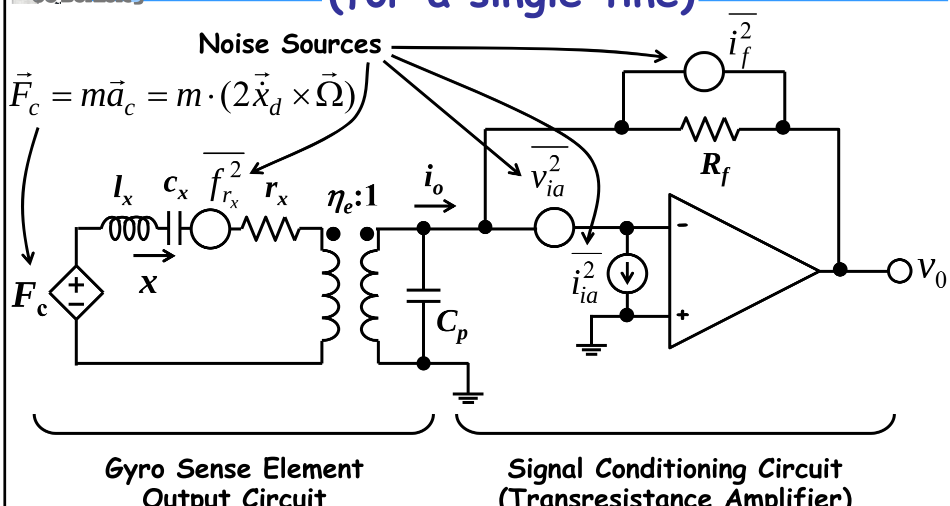
- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

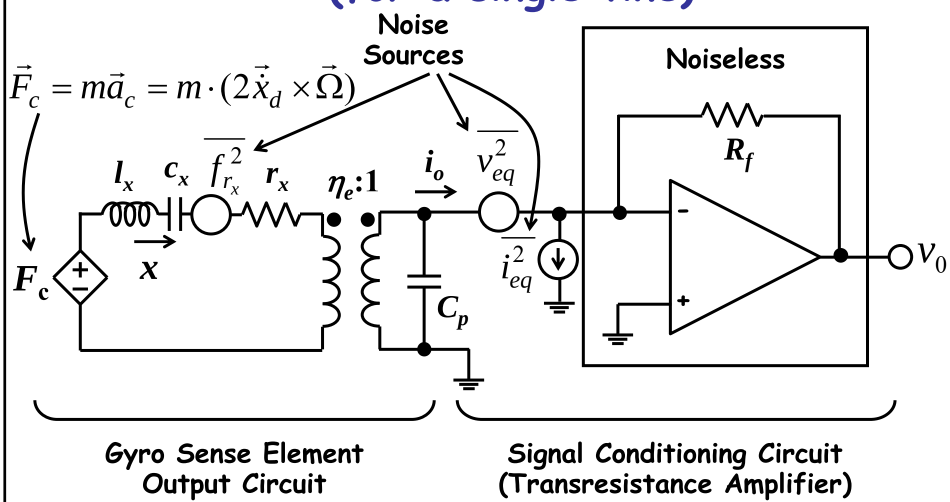


Gyro Readout Equivalent Circuit (for a single tine)



- Easiest to analyze if all noise sources are summed at a common node

Gyro Readout Equivalent Circuit (for a single tine)

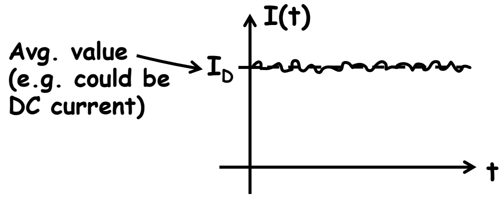


- Here, $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ are equivalent input-referred voltage and current noise sources

Noise

Noise

- Noise: Random fluctuation of a given parameter I(t)
- In addition, a noise waveform has a zero average value



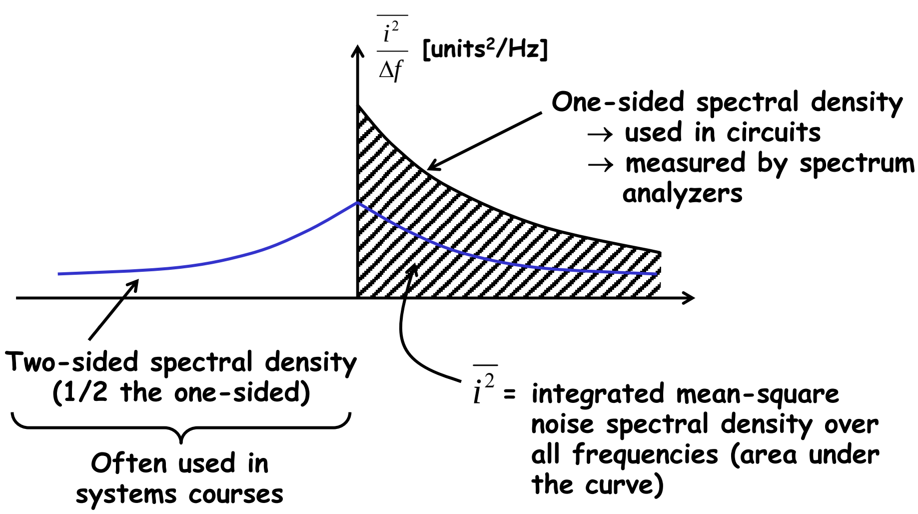
- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

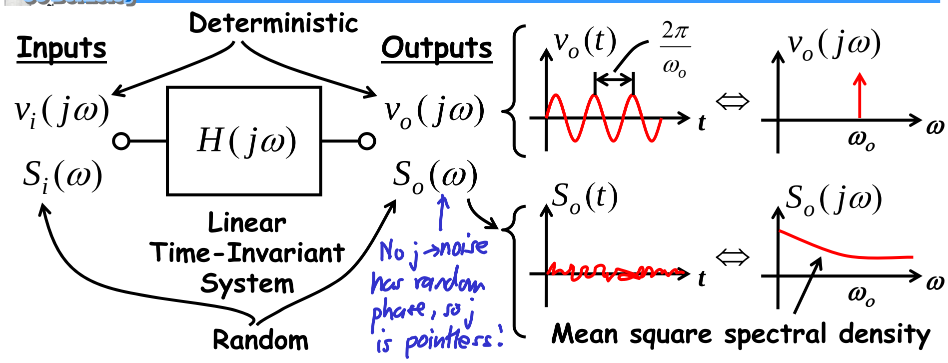
Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T |I - I_D|^2 dt$

Noise Spectral Density

- We can plot the spectral density of this mean-square value:



Circuit Noise Calculations



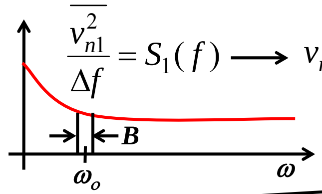
- **Deterministic:** $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- **Random:** $S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right]S_i(\omega) = \left|H(j\omega)\right|^2 S_i(\omega)$

$$\sqrt{S_o(\omega)} = \left|H(j\omega)\right|\sqrt{S_i(\omega)} \longrightarrow$$ How is it we can do this?

Root mean square amplitudes

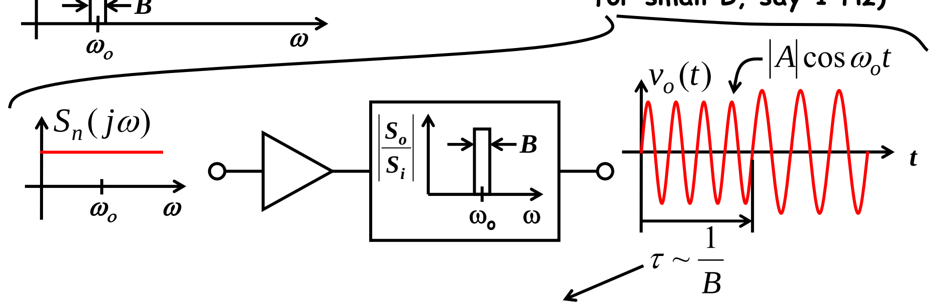
Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)



$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_1(f) \longrightarrow v_{n1} = \sqrt{S_1(f)\cdot B}$$

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

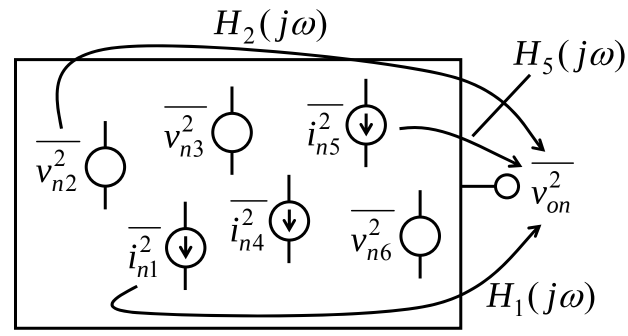


Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter]

Systematic Noise Calculation Procedure

General Circuit With Several Noise Sources



- Assume noise sources are uncorrelated

1. For $\overline{i_{n1}^2}$ replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f} \cdot (1\,\text{Hz})}$$

Systematic Noise Calculation Procedure

2. Calculate $v_{on1}(\omega) = i_{n1}(\omega) H(j\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
4. Repeat for each noise source: $\overline{i_{n1}^2}$, $\overline{v_{n2}^2}$, $\overline{v_{n3}^2}$
5. Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

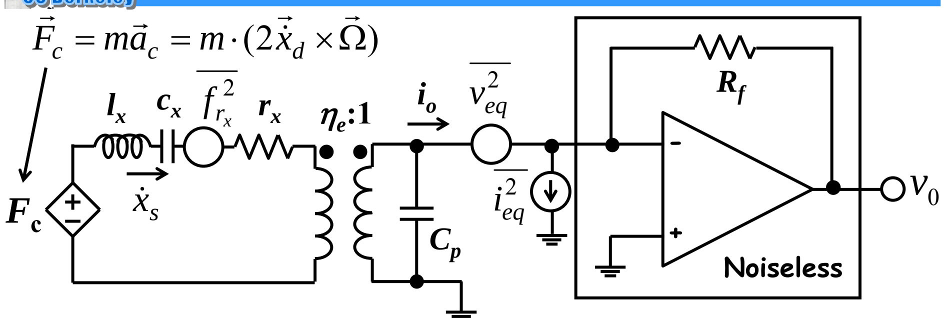
$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

Total rms value

Determining Sensor Resolution

Example: Gyro MDS Calculation

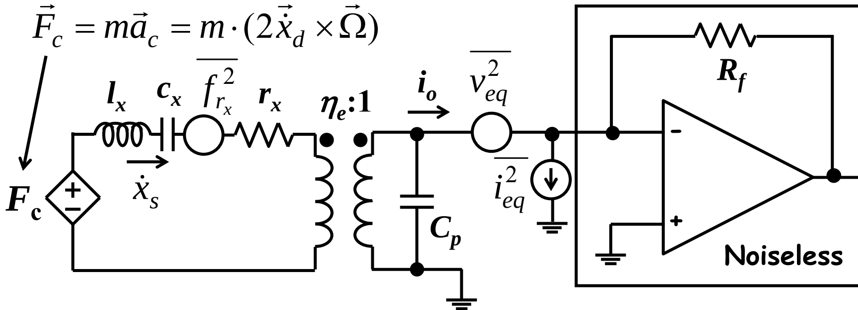
$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{\dot{x}}_d \times \vec{\Omega})$$



- The gyro sense presents a large effective source impedance
  - Currents are the important variable; voltages are "opened" out
  - Must compare i_o with the total current noise i_{eqTOT} going into the amplifier circuit

Example: Gyro MDS Calculation (cont)

$$\vec{F}_c = m\vec{a}_c = m\cdot(2\vec{\dot{x}}_d \times \vec{\Omega})$$



- First, find the rotation to i_o transfer function:

$$\dot{x}_s = \frac{\omega_s Q}{k_s}\Theta_s(j\omega_d)F_s = \frac{\omega_s Q}{k_s}\cdot 2\omega_d x_d \Omega m \cdot \Theta(j\omega_d)$$

$\frac{m}{k_s} = \frac{1}{\omega_s^2}$

$[F_s = F_c = 2\omega_d x_d \Omega m]$

$$\dot{x}_s = 2\frac{\omega_d}{\omega_s}Q x_d \Theta(j\omega_d)\cdot\Omega$$

Example: Gyro MDS Calculation (cont)

$$i_o = \eta_e \dot{x}_s = \underbrace{2\frac{\omega_d}{\omega_s}Q x_d \eta_e \Theta(j\omega_d)}_{A \triangleq \text{ scale factor}}\cdot\Omega \rightarrow \boxed{\begin{array}{l} i_o = A\Omega \\ \text{Where } A = 2\frac{\omega_d}{\omega_s}Q x_d \eta_e \Theta(j\omega_d) \end{array}}$$

When $\Omega = \Omega_{min} \triangleq$ MDS, $i_o = i_{eqTOT}$ ← input-referred noise current entering the sense amplifier → in pA/$\sqrt{Hz}$

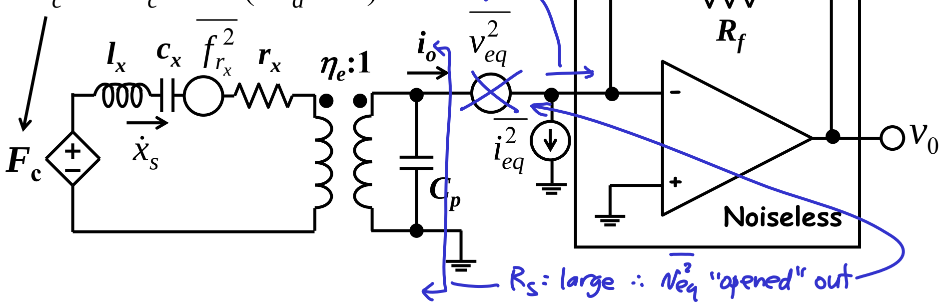
$$\therefore\ i_{eqTOT} = A\Omega_{min} \rightarrow \boxed{\Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600\text{s}}{\text{hr}}\right)\left(\frac{180^\circ}{\pi}\right)\ \left[(^\circ/\text{hr})/\sqrt{\text{Hz}}\right]}$$

$$\boxed{\text{Angle Random Walk} = \text{ARW} = \frac{1}{60}\Omega_{min}\ \left[^\circ/\sqrt{\text{hr}}\right]}$$

↳ Easier to determine directional error as a function of elapsed time.

Example: Gyro MDS Calculation (cont)

$$\vec{F}_c = m\vec{a}_c = m\cdot(2\dot{\vec{x}}_d \times \vec{\Omega})$$



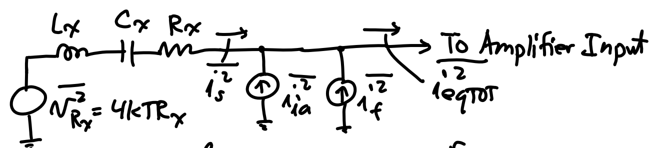
• Now, find the i_{eqTOT} entering the amplifier input:

$$i_{eqTOT} = i_s + i_{eq} \rightarrow \overline{i_{eqTOT}^2} = \overline{i_s^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{v_{ia}^2}}{R_f^2} \qquad \frac{\overline{f_{r_x}^2}}{\Delta f} = 4kTr_x$$

Brownian motion noise of the sense element → determined entirely by the noise in $r_x \rightarrow \overline{f_{r_x}^2}$

↳ easiest to convert to an all electrical equiv. ckt.

Example: Gyro MDS Calculation (cont)



where $L_x = \frac{l_x}{\eta_e^2}$, $C_x = \eta_e^2 c_x$, $R_x = \frac{r_x}{\eta_e^2}$

$$\therefore\ i_s = v_{R_x}\left(\frac{1}{R_x}\right)\Theta(j\omega_d) \rightarrow \frac{\overline{i_s^2}}{\Delta f} = 4kTR_x\left(\frac{1}{R_x^2}\right)|\Theta(j\omega_d)|^2$$

$$\Rightarrow \boxed{\frac{\overline{i_s^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2}$$

Thus:

$$\boxed{\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)}$$

Learn to get these from EE240.

↳ or just get them from a data sheet ...

LF356 Op Amp Data Sheet

UC Berkeley

LF155/LF156/LF256/LF257/LF355/LF356/LF357
JFET Input Operational Amplifiers

General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (BI-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Features

Advantages

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

Applications

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers
- Logarithmic amplifiers
- Photocell amplifiers
- Sample and Hold circuits

Common Features

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance: $10^{12}\Omega$
- Low input noise current: $0.01\ pA/\sqrt{Hz}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

$\sqrt{\dfrac{\overline{i_{ia}^2}}{\Delta f}} = 0.01\ pA/\sqrt{Hz}$

Uncommon Features

	LF155/ LF355	LF156/ LF256/ LF356	LF257/ LF357 (A_V=5)	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	$nV/\sqrt{Hz}$

$\sqrt{\dfrac{\overline{v_{ia}^2}}{\Delta f}} = 12\ nV/\sqrt{Hz}$

Example ARW Calculation

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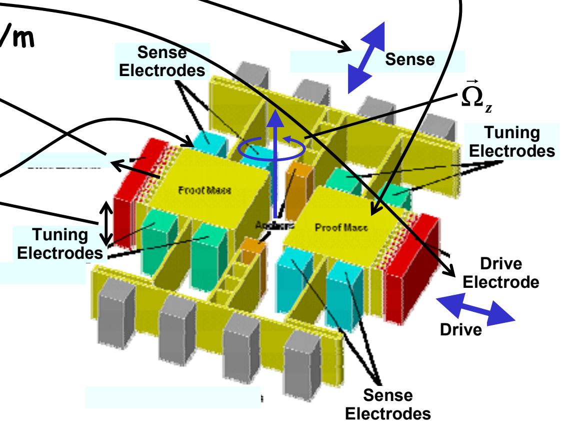
- Example Design:
  - Sensor Element:
    - $m = (100\mu m)(100\mu m)(20\mu m)(2300 kg/m^3) = 4.6\times10^{-10} kg$
    - $\omega_s = 2\pi(15kHz)$
    - $\omega_d = 2\pi(10kHz)$
    - $k_s = \omega_s^2 m = 4.09\ N/m$
    - $x_d = 20\ \mu m$
    - $Q_s = 50{,}000$
    - $V_P = 5V$
    - $h = 20\ \mu m$
    - $d = 1\ \mu m$
  - Sensing Circuitry:
    - $R_f = 100k\Omega$
    - $i_{ia} = 0.01\ pA/\sqrt{Hz}$
    - $v_{ia} = 12\ nV/\sqrt{Hz}$



Example ARW Calculation (cont)

Get rotation rate to output current scale factor:

$$A = 2\frac{\omega_d}{\omega_s}Q_s x_d \eta_e |\Theta(j\omega_d)| = 2\left(\frac{10k}{15k}\right)(50K)(20\mu)(5)(2000\epsilon_o)(0.000024) = \underline{2.83\times10^{-12}C}$$

$$\Theta(j\omega_d) = \frac{(j\omega_d)(\omega_s/Q_s)}{-\omega_d^2 + \frac{j\omega_d\omega_s}{Q_s} + \omega_s^2} = \frac{j(10k)(15k)/(50K)}{(15k)^2-(10k)^2 + \frac{j(10k)(15k)}{50K}} = \frac{j(3k)}{1.25\times10^8 + j(3k)}$$

$$\rightarrow |\Theta(j\omega_d)| = \frac{3k}{\sqrt{(1.25\times10^8)^2+(3k)^2}} = 0.000024$$

$$\frac{\partial C}{\partial x} = \frac{C_o}{d} = \frac{\epsilon_o h W_p}{d} = \frac{\epsilon_o(20\mu)(100\mu)}{(1\mu)^2} = 2000\epsilon_o \rightarrow \eta_e = V_P\frac{\partial C}{\partial x} = 5(2000\epsilon_o)$$

Assume electrode covers the whole sidewall. 8.854×10^{-12} F/m

Then, get noise:

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \frac{4kT}{R_x}|\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{\overline{i_{ia}^2}}{\Delta f} + \frac{\overline{v_{ia}^2}}{\Delta f}\left(\frac{1}{R_f^2}\right)$$

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Example ARW Calculation (cont)

$$R_x = \frac{\omega_s m}{Q_s \eta_e^2} = \frac{2\pi(15K)(4.6\times10^{-10})}{(50k)(8.854\times10^{-9})^2} = 110.6k\Omega$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(0.000024)^2}_{8.64\times10^{-35}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Sensor element noise insignificant

Noise from R_f dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.68\times10^{-26}A^2/Hz \rightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 1.30\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180°}{\pi}\right) = \frac{1.30\times10^{-13}}{2.83\times10^{-12}}(3600)\left(\frac{180}{\pi}\right) = \underline{9448\ (°/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(9448) = 157\ °/\sqrt{hr} = ARW$$

→ Almost turned around in 1 hour!

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What if $\omega_d = \omega_s$?

If $\omega_d = \omega_s = 15\text{KHz}$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e |\Theta(j\omega_d)| = 2Q_s x_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_0) = \underline{1.77\times10^{-7}C}$$

$$\frac{\overline{i_{eqTOT}^2}}{\Delta f} = \underbrace{\frac{(1.66\times10^{-29})}{(110.6K)}(1)^2}_{1.51\times10^{-25}A^2/Hz} + \underbrace{\frac{(1.66\times10^{-29})}{1M}}_{1.66\times10^{-26}A^2/Hz} + \underbrace{(0.01p)^2}_{1\times10^{-28}A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44\times10^{-28}A^2/Hz}$$

Now, the sensor element dominates!

$$\therefore \frac{\overline{i_{eqTOT}^2}}{\Delta f} = 1.67\times10^{-25}A^2/Hz \longrightarrow i_{eqTOT} = \sqrt{\frac{\overline{i_{eqTOT}^2}}{\Delta f}} = 4.08\times10^{-13}A/\sqrt{Hz}$$

$$\therefore \Omega_{min} = \frac{i_{eqTOT}}{A}\left(\frac{3600s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) = \frac{4.08\times10^{-13}}{1.77\times10^{-7}}(3600)\left(\frac{180}{\pi}\right) = \underline{0.476\ (^\circ/hr)/\sqrt{Hz}}$$

And finally:

$$ARW = \frac{1}{60}\Omega_{min} = \frac{1}{60}(0.476) = 0.0079\ ^\circ/\sqrt{hr} = ARW$$

$\Rightarrow$ Navigation grade!

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