Suppose a step function voltage V_A was suddenly applied across the anchors of a 2 µm thick polysilicon beam and proof mass setup as shown in Figures PS1.3-1 and PS1.3-2, which also provide lateral dimensions. For polysilicon, assume the following material properties: Young's modulus E = 150 GPa, density ρ = 2300 kg/m^3, Poisson ratio ν = 0.226, sheet resistance (resistivity•thickness^{-1}) = 10 Ω/□, specific heat = 0.77 J/(g•K), and thermal conductivity = 30 W/(m•K).

(a) With what time constant will the proof mass reach its steady-state temperature after the voltage V_A steps from 0V to 1V? Give a formula and a numerical answer with units.

(b) If the final step function value of V_A is 1V, what is the steady-state temperature of the proof mass? Give a formula and a numerical answer with units.

(c) What effect do you think the applied voltage has on the resonance frequency of the structure in the z-direction (into the page)? Give a brief qualitative explanation.

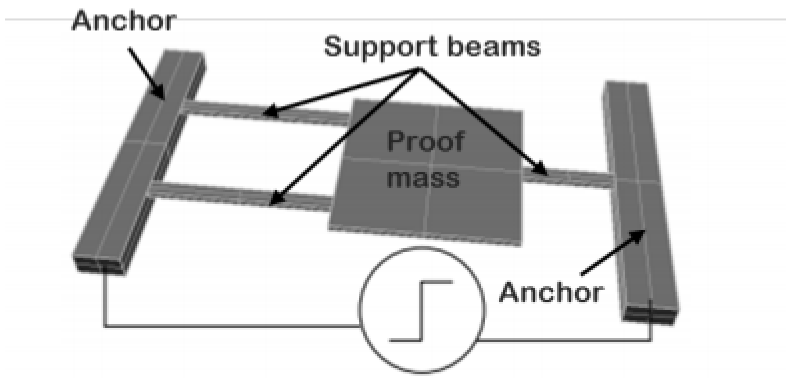

Figure PS1.3-1

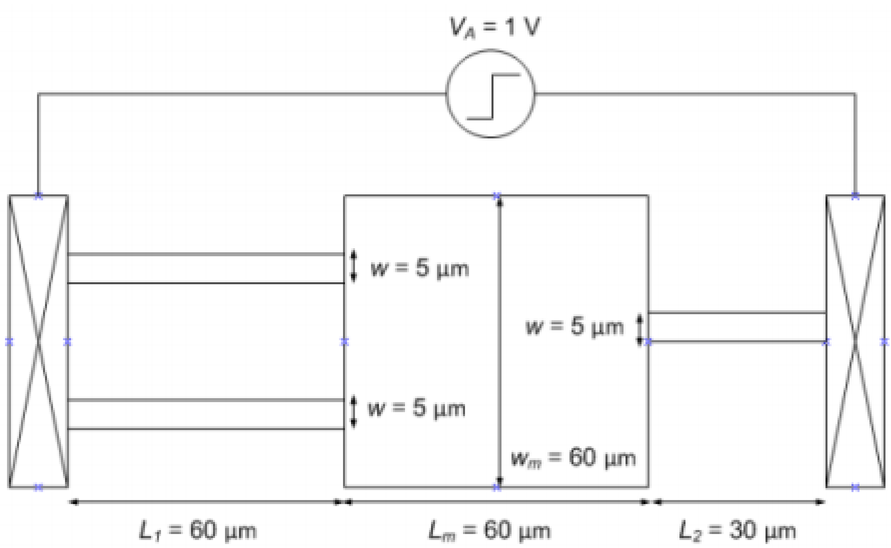

Figure PS1.3-2

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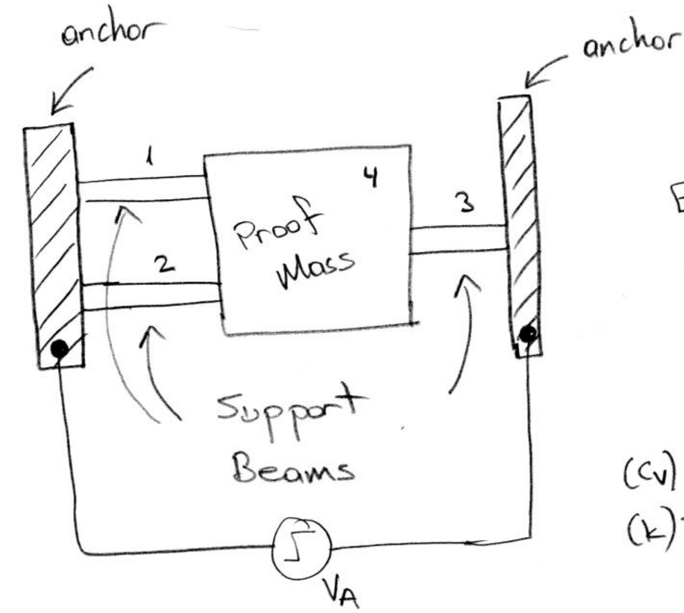


Polysilicon (2μm thick)

$E = 150\ GPa$

$\rho = 2300\ kg/m^3$

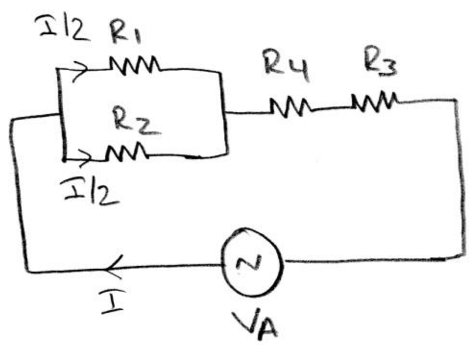
$\nu = 0.226$

Sheet Res. $= 10\Omega/\square$

(c_v) Spec. Heat $= 0.77\ J/g.K$

(k) Thermal cond $= 30\ W/m.K$

① Find the electrical circuit:



$R_1 = 12\ (10\ \Omega/\square) = 120\Omega$

$R_2 = 120\Omega$

$R_3 = 60\Omega$

$R_4 = 10\Omega$

$I = \dfrac{V_A}{R_1 \| R_2 + R_3 + R_4}$

$= \dfrac{1V}{130\Omega} = 7.7mA$

Powers:

$P_1 = \left(\frac{I}{2}\right)^2 R_1 = 1.78mW$, $P_2 = \left(\frac{I}{2}\right)^2 R_2 = 1.78\ mW$, $P_3 = I^2 R_3 = 3.56mW$, $P_4 = 0.59mW$

★ P_1-P_4 will be power inputs to the thermal circuit.

② Find the thermal circuit:

$R_{th1} = \dfrac{l}{kA} = \dfrac{60\mu}{(30)(2\mu)(5\mu)} = 0.2\ MK/W$

$R_{th2} = R_{th1} = 0.2\ MK/W$

$R_{th3} = 0.1\ MK/W$

$R_{th4} = \dfrac{60\mu}{(30)(60\mu)(2\mu)} = 16\ kK/W$

$C_{th1} = \rho C_v V = (2300)(770)(2\mu\, 5\mu\, 60\mu) = 1.06\ nJ/K$

$C_{th2} = C_{th1} = 1.06\ nJ/K$

$C_{th3} = 0.53\ nJ/K$

$C_{th4} = 12.75\ nJ/K$

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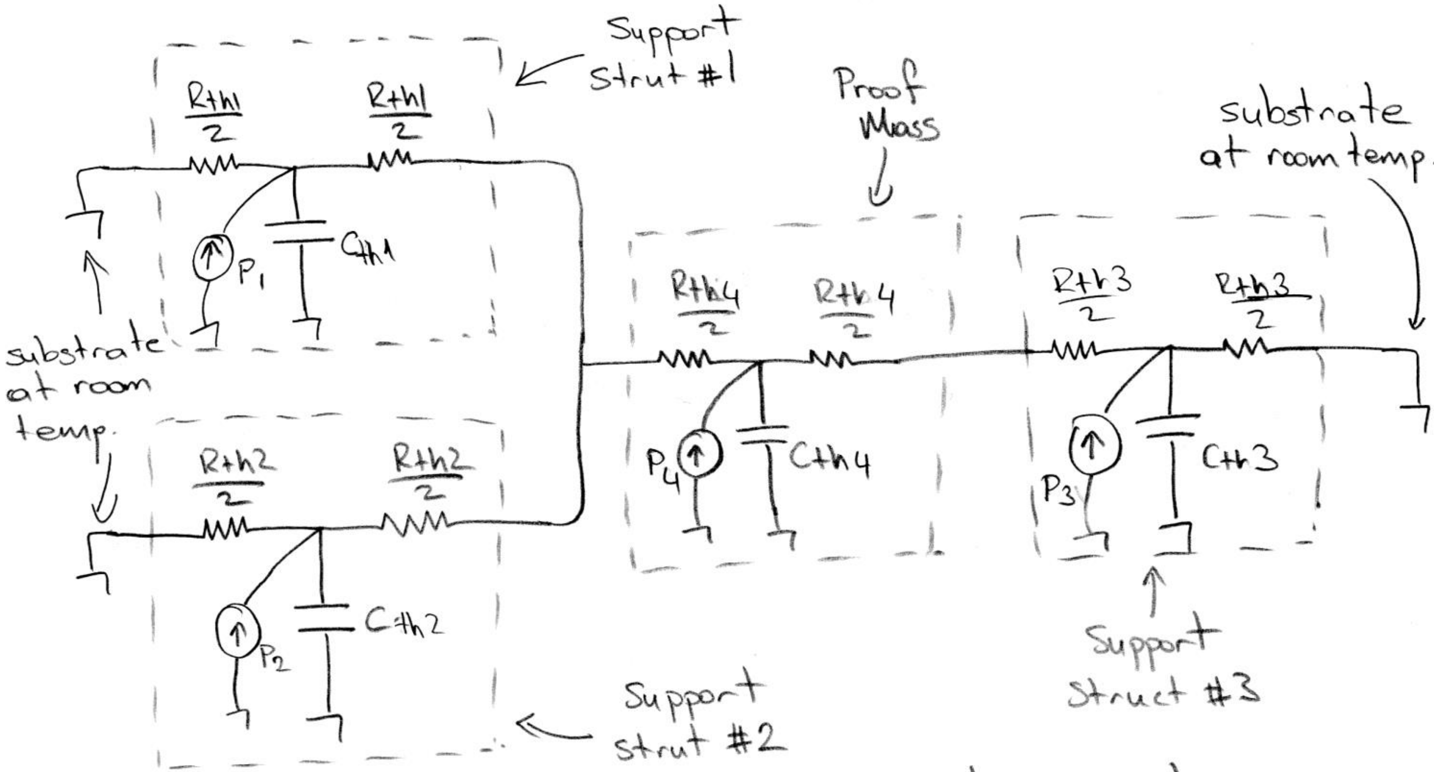


(3) Simplify the thermal circuit w/ appropriate assumptions:

$C_{th1} = 1.06\ nJ/K$

$C_{th2} = 1.06\ nJ/K$

$C_{th3} = 0.53\ nJ/K$

$C_{th4} = 12.75\ nJ/K$

$C_{th1} \ll C_{th4}$

$C_{th2} \ll C_{th4}$

$C_{th3} \ll C_{th4}$

neglect $C_{th1}, C_{th2}, C_{th3}$

$R_{th1} = 200\ kK/W$

$R_{th2} = 200\ kK/W$

$R_{th3} = 100\ kK/W$

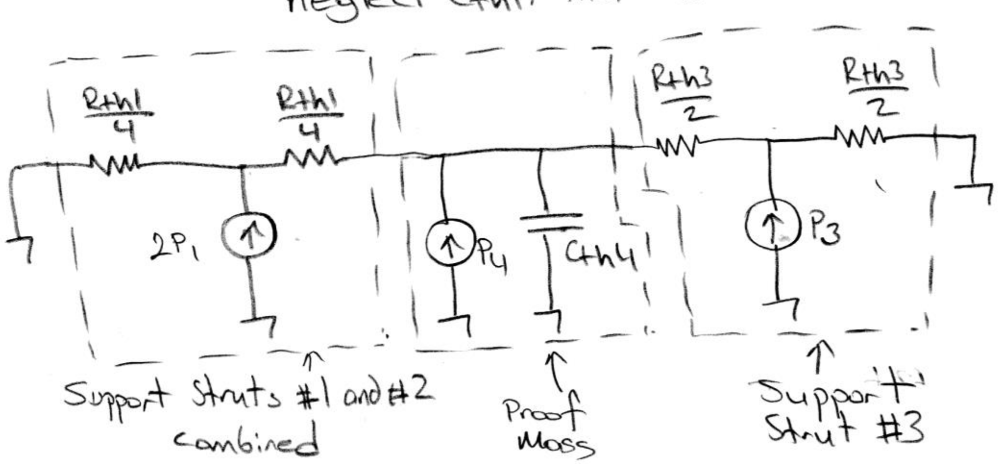
$R_{th4} = 16\ kK/W$

$R_{th4} \ll R_{th1}$

$R_{th4} \ll R_{th2}$

$R_{th4} \ll R_{th3}$

neglect R_{th4}



Simplified Steady-State Circuit

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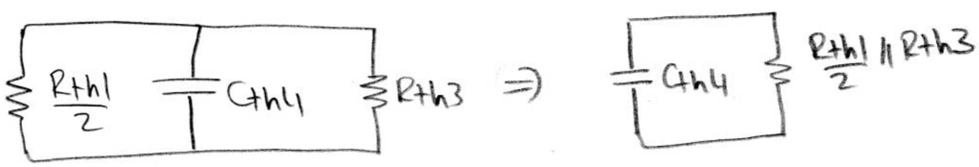
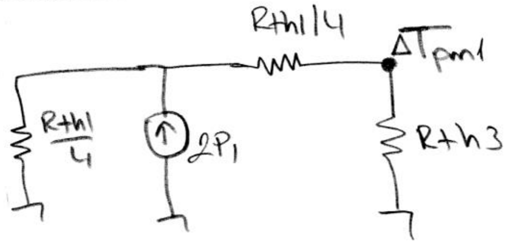
(a) Time constant circuit: kill all independent sources, i.e. short circuit voltage sources and open-circuit current sources:



$$\tau = \left(\frac{R_{th1}}{2} \parallel R_{th3}\right) C_{th4} = 0.6375\ ms$$
$$= 637.5\ \mu s$$

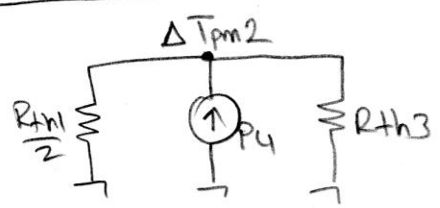
(b) Apply superposition: (@ steady-state all capacitors are fully-charged/open)

① Kill P_3, P_4:



$$\Delta T_{pm1} = 2P_1 \frac{\frac{R_{th1}}{4}}{\frac{R_{th1}}{2} + R_{th3}} R_{th3} = 89K$$

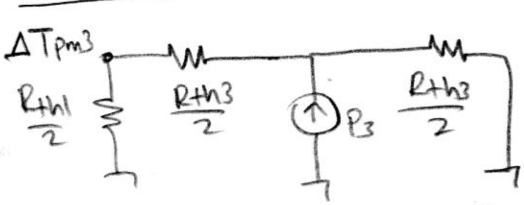
② Kill $2P_1, P_3$:



$$\Delta T_{pm2} = \left(R_{th3} \parallel \frac{R_{th1}}{2}\right) P_4 = 29.5K$$

↑ even P_4 could be neglected!

③ Kill $2P_1, P_4$:



$$\Delta T_{pm3} = P_3 \frac{\frac{R_{th3}}{2}}{R_{th3} + \frac{R_{th1}}{2}} \frac{R_{th1}}{2} = 89K$$

room temp.

$$\text{In total} \Rightarrow T_{pm} = T_0 + \sum_{i=1}^{3} \Delta T_{pmi} = 298K + 89K + 29.5K + 89K = 505.5K$$

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* What happens if we neglect P_4?

$$T_{pm}' = 298K + 89K + 89K = 476K$$

$$err = 100\left(\frac{T_{pm} - T_{pm}'}{T_{pm}}\right) = \left(1 - \frac{T_{pm}'}{T_{pm}}\right)100 = 5.84\% < 10\%$$

so you're still fine!

(c) Joule heating causes polysilicon to expand but since the anchors are fixed, compressive axial stress results in the beams. This lowers the beam stiffness and thus lowers the resonance frequency $\downarrow\omega_0 = \sqrt{\frac{\downarrow k}{m}}$